FORECASTING BETA: HOW WELL DOES THE
'FIVE-YEAR RULE OF THUMB' DO?

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Abstract

CAPM betas are widely used in practice. While they are estimated from historical data, they are generally applied to a future period. This is appropriate only if the betas are stable over time. However, there is widespread evidence that the CAPM betas vary considerably over time. This raises two questions: is the time-variation in the betas systematic and can systematic variation in the betas be used to generate forecasts which improve on the usual practice of using the betas estimated over the most recent five-year period (the “five-year rule of thumb”)? We address both of these questions. We estimate time-varying betas using both overlapping sub-periods (five-year rolling regressions) and non-overlapping two-year periods. We proceed to explain the time-variation in the betas using two different regression models which we subsequently use for forecasting. We find that, despite the forecasting equations having relatively high within-sample explanatory power, the forecasts generated by these equations are dominated, on average, by the five-year rule of thumb. Further experimentation shows that the five-year rule of thumb is itself dominated by two-, three-, four-, six- and seven-year rules of thumb, with the three-year rule being optimal for our sample.

1 Introduction

CAPM betas are widely used in practice. In general, they are used for some future period, e.g. for calculating required returns or expected future returns or for cost of capital computations as an input into project evaluation. However, since future betas are not known, it is common practice to use betas estimated over some recent period. Underpinning this application of the CAPM betas is the assumption that the betas estimated over a recent period are appropriate as an input into calculations that require projections into the future. While this procedure would be acceptable if the betas were known to be stable over time, there is widespread evidence to the contrary – rather than being constant, the betas are time-varying. This is reflected in the very fact that in practice the betas are generally estimated over relatively short periods, five years being common.

Using recently-estimated betas to apply to some future period presumes that the historical betas are good predictors of the future betas and, in particular, that they outperform alternative forecasting schemes. What little evidence there is on this question suggests that this is generally not so. However, beta-forecasting methods have generally been quite *ad hoc* and have not used the information contained in the time-variation in estimated betas. We set out in this paper to assess the forecasting performance of regression models estimated using time-varying betas.

We begin by estimating the time-varying betas using a rolling-regression procedure with a five-year window and use the estimated betas as dependent variables in three different forecasting models. The first and simplest model is an AR(1) model. The second model is based on the regression of the time-varying betas on a time trend, allowing for a break in level and in trend at October 1987. The regressors in this forecasting equation are deterministic which makes the model attractive for forecasting because the future values of the explanatory variables are known with certainty and so do not themselves need to be forecast before the betas can be forecast. The third forecasting model is based on the regression of the time-varying betas on a set of economy-wide factors or macroeconomic variables which may affect systematic risk as measured by the betas. The model is intuitively appealing given the growing evidence of the effects of aggregate shocks on share markets. However, it has the disadvantage from a forecasting perspective that the regressors are stochastic so that future values of the regressors themselves must be forecast before the model can be used to forecast the betas. We compare the performance of these three models

to the use of historical betas estimated over the most recent five-year period (the ‘five-year rule of thumb’).

To anticipate our results, we find that the estimated time-varying betas show considerable evidence of instability and that the fluctuations are well explained by both our regression models. On the forecasting front, we find that the five-year rule of thumb, on average, outperforms all other models. Thus the common application of the most recently estimated beta to some future time period is vindicated, at least relative to the alternative models we consider and for our data set. This result does not apply to all sectors uniformly, however; there are some sectors for which the rule of thumb can be improved upon by a variety of alternative forecasting models.

An important qualification of the results outlined above is that they are conditional on the particular method of estimating the time-varying betas. While the rolling regression procedure with a five-year window is a natural extension of the common five-year rule of thumb, it does produce estimated betas which have a high degree of autocorrelation due to the large number of common observations used to estimate successive betas. This is bound to make the betas behave very much like a random walk and it is well known that for this type of variable the current value is the best predictor of any future value and it is therefore not altogether surprising that the five-year rule of thumb outperforms the regression-based models.

To assess the influence of this overlapping-observations problem on the outcome of the forecasting comparisons, we also experimented with betas estimated over non-overlapping sub-periods. The resulting betas also show considerable variation over time which can be well explained by the regression models used previously. There is, however, no evidence of autocorrelation in the betas, confirming our conjecture that it was caused by the overlapping observations problem in the rolling-regression betas. Despite this, the forecasting performance of the five-year rule of thumb was superior to that of the other models providing some evidence of the robustness of our original conclusion. Finally, we address the issue of whether 5 is an optimal value for N in the N -year rule of thumb by experimenting with various values of N . We find that, while the five-year rule of thumb dominated the forecasting performance of the regression-based models as well as that of the AR(1) model, the five-year rule is, in turn, dominated by two-, three-, four-, six- and seven-year rules of thumb, with the three-year rule of thumb being optimal for our sample.

The practical implications of our results are simple. While we confirm that the CAPM betas are, indeed, time-varying, the nature of the time-variation is such that for most sectors the standard N-year rule of thumb works well relative to more sophisticated alternatives although for some sectors the rule of thumb can be improved upon by alternative forecasting models. The best value for N in the rule of thumb is an empirical matter and will possibly change from one sample to another.

The structure of the paper is as follows. We begin in section 2 by reviewing three strands of literature. The first deals with the evidence for instability of CAPM betas. The second strand deals with methods of estimating the market model with time-varying betas. The third deals with methods for forecasting betas. In section 3 we describe the data used in our empirical work and in section 4 we report the results of testing constancy of the betas using our data. In section 5 we present our time-varying betas estimated using both the rolling-regression procedure and the two-year non-overlapping sub-samples. We briefly report the results of estimating the regression models for the rolling-regression betas in section 6. In section 7 we use each of the models, separately and together, as well as the AR(1) model and the five-year rule of thumb to forecast betas and compare the forecasting performance of the different models. In section 8 we present the time-varying betas estimated using the two-year sub-periods and the models used for forecasting them. Forecasts of the two-year betas are presented in section 9 and we report the results of our experimentation with alternative rules of thumb in section 10. Conclusions are presented in section 11.

2. Literature Review

Betas are typically estimated as a constant parameter in the market model using OLS or similar estimation techniques. Yet, it is a stylised fact of empirical finance that betas are not stable over time, a fact reflected in the universal practice of estimating betas only over short time periods. Early evidence on beta-instability for the US dates from the early 1970s: Blume (1971, 1975), Gonedes (1973), Meyers (1973), Levy (1974) and Baesel (1974). More recent evidence using more sophisticated tests is reported in, e.g., Bos and Newbold (1984) and Gonzalez-Rivera (1997); recent Australian evidence is reported in Faff, Lee and Fry (1992), Brooks, Faff and Lee (1992, 1994) and Brooks, Faff and Josev (1997).

In addition to the instability of the betas over time, there is a general finding that the extent of the instability depends on the length of the period over which the

betas are estimated and on the length of the period to which they are applied, the “forecast period”. Although there is no unanimity on the optimal length of the estimation period, five years of monthly data appears to be the most common choice. There is also strong evidence that the betas are less stable for individual securities than they are for portfolios and that as the size of the portfolio increases, betas become more stable reflecting the effects of diversification; see, e.g., Alexander and Chervany (1980) using US data. Brooks, Faff and Lee (1994), however, find little evidence of this effect using Australian data.

Given that the betas are generally found to be unstable over time, it is desirable to accommodate this instability in the procedure used to estimate the betas. Numerous methods have been used. The most straightforward and most common is to assume that betas are constant over relatively short periods of time and to estimate them over these periods using an estimation technique (such as OLS) suitable for a constant-coefficient model. As stated above, five years of monthly data is a common sample period to use although others have been used – Chang and Weiss (1991), for example, use daily data over three and six-monthly time-spans.

A simple extension of five-year estimation periods is to use a rolling regression technique which shifts the five-year period forward by one month at a time to obtain a monthly time series for beta. A related technique is recursive regression which progressively increases the length of the sample by one observation at a time; see, e.g., Groenewold and Fraser (1999) for an application of these two techniques to Australian data.

A model which explicitly departs from the assumption of a constant coefficient is the random-coefficient model which has been applied to US data by Fabozzi and Francis (1978), in Bayesian form by Chen and Lee (1982) and McDonald (1983) and, to Australian data, by Brooks, Faff and Lee (1992, 1994) and Brooks, Faff and Josev (1997). This method does not, however, produce time-series estimates of the betas but is focussed more on measuring the extent of the randomness in the estimated coefficients.

A more general time-varying coefficients model is that of Cooley and Prescott (1976) which has been applied to the estimation of betas by Dotan and Ofer (1984) who assume that beta is generated by a random walk and estimate the resulting time-varying coefficients model using Bayesian methods.

Finally, an alternative and general technique for estimating time-varying coefficient models is the Kalman Filter procedure which has been applied to US data by Fisher and Kamin (1985), to UK data by Black, Fraser and Power (1992), to Swedish data by Wells (1994) and to Australian data by Brooks, Faff and McKenzie (1998) and Groenewold and Fraser (1999). A related technique which assumes the coefficients to be generated by a general linear stochastic process is applied to US data in Gonzalez-Rivera (1997). Generally these techniques have been used not so much to obtain time series for the betas as to provide further evidence of the time-varying nature of the betas.

In practical applications of the CAPM, betas are applied to future periods and, given the evidence that betas are not constant over time, it is not surprising that various authors have attempted to forecast betas in some way. The simplest forecasting procedure is to assume that betas are constant over relatively short periods so that recently-estimated betas can be used for application to the near future. A common procedure is to use a beta estimated over the most recently available five-year period. For future reference we call this the “five-year rule of thumb”.

Early empirical work found that there was considerable sampling variability in estimated betas, particularly for individual securities and attempts have been made to moderate the influence of this variability on forecast betas by using historical betas in cross-section regressions and using the generated values of the betas from these regression equations as forecasts; see Blume (1971) and Vasicek (1973) who uses a Bayesian adjustment procedure. These and similar *ad hoc* methods are compared for the US by Klemkosky and Martin (1995), for Belgium by Hawawini, Michel and Corhay (1985) and for Australia by Brooks and Faff (1997). Another strand of the literature has used accounting variables as a basis for the forecasting of betas; see Beaver, Kettler and Scholes (1970) for an early application and Young, Berry, Harvey and Page (1991) for a more recent application of this method to macroeconomic risk factors obtained from an APT model of the sort first estimated by Chen, Roll and Ross (1986). The evidence suggests that these *ad hoc* adjustment methods generally produce forecasts which are superior to the five-year rule of thumb, in many cases to a considerable extent.

All the forecasting methods discussed above are *ad hoc* and do not take advantage of the information in the betas generated by techniques of estimating time-varying betas. Thus the literature which estimates time-varying coefficient models for

the betas generally uses the results simply as a basis for tests of time-variation and not for forecasting purposes. We propose to combine the time-varying beta estimation results and forecasting. We estimate a time series for the beta for each of a set of portfolios using a time-varying estimation technique and use these estimated betas as inputs into the estimation of regression models which are then used for (out-of-sample) forecasting of the betas. This has the advantage of using as much information on the nature of time-variation as possible in the forecasting process but is, naturally, conditional on the method used to estimate the time-varying betas.

In our empirical work, the results of which are reported below, we use a rolling-regression technique with a five-year window for estimating time-varying betas. We choose this procedure because it is a natural extension of estimating betas over five-year samples. Experimentation in an earlier paper, Groenewold and Fraser (1999), with the use of recursive estimation and the Kalman Filter produced beta estimates which varied greatly at the beginning of the sample period when few observations are used and became progressively less variable as more observations were used in the estimation process so that by the end of the sample the estimated beta was effectively constant. This effect adds an artificial element to the resulting parameter estimates which is overcome by the use of the rolling-regression technique which uses a constant window size (of 60 observations in our case). A disadvantage of all three methods is the overlapping observations problem – each successive estimated beta is based on substantially the same observations as the previous betas causing autocorrelation in the resulting time-series for beta. We therefore also experiment with betas estimated over relatively short non-overlapping sub-periods.

3. The Data

We used monthly data for 18 Australian industrial sectors for the period January 1973 to June 1998 obtained from Datastream. For each sector we calculated the continuously-compounded rate of return which includes both capital gains and dividends. The sectors for which data are available for the entire sample period are listed in Table 1. Datastream has a further 10 sectors at this level of disaggregation but for none of them are data available for the whole sample period. Since we wished to estimate betas over a set of non-overlapping sub-periods as well as by rolling regression techniques, it was important to have as long a sample period as possible so that we used only the 18 sectors for which data were available from 1973. To

calculate the market return we used Datastream's Total Market index, again using an index which includes both capital gains and dividends.

Table 1 contains summary statistics for the returns for the 18 sectors. The skewness and excess kurtosis statistics are both asymptotically standard-normally distributed under the null hypothesis of normality of returns. The normality statistic is the goodness-of-fit version; it is $\chi^2_{(27)}$ distributed under the null of normality. The next three columns in the table report the previous three statistics for the sample omitting the October 1987 observation. It is often found that the events surrounding the Crash have a disproportionate effect on the properties of the data. The final two columns of the table report ARCH(6) and ADF statistics; the ARCH (6) statistic is $\chi^2_{(6)}$ distributed in the absence of heteroskedasticity.

The results in Table 1 show widespread departures from normality in the returns - there is significant skewness in 16 of the 18 sectors and excess kurtosis in all 18 sectors. The goodness-of-fit test rejects normality for 10 of the 18 sectors. There is a noticeable effect on these results of the October 1987 observation, however; the incidence of skewness falls to 6/18 while the excess kurtosis statistics fall markedly in magnitude although there is still evidence of excess kurtosis for all sectors. The normality statistic is still significant in nine of the 18 sectors. There is some evidence of heteroskedasticity, at least of the ARCH type, with six of the series exhibiting significant ARCH(6) effects. The ADF results indicate that the null hypothesis of non-stationarity in the returns can be rejected for all sectors.

In addition to share-price index and dividend-yield data, we also used macro variables in our empirical work. As explained in the introductory section, one approach to modelling the time-variation in the betas and generating forecasts is to use aggregate macroeconomic variables in our forecasting equations. Our choice of macro variables for this purpose was based on the hypothesis that at the aggregate level, risk is influenced by three classes of factors - real domestic activity, nominal domestic factors and foreign variables. Changes in any of these variables may conceivably influence agents' risk perceptions and therefore the betas.

Table 1: Summary Statistics

No.	Sector Name	Mean	Variance	Skewness	Kurtosis	Normality	Skewness (ex. Oct ' 87)	Kurtosis (ex. Oct ' 87)	Normality (ex. Oct '87)	ARCH (6)	ADF
1	Other Mining	0.0086	0.0062	-5.819	16.711	33.178	-1.004	3.327	46.993	22.134	-5.299
2	Building Materials & Merch.	0.0080	0.0046	-4.786	13.577	26.759	-0.546	2.469	26.923	5.217	-4.565
3	Chemicals	0.0132	0.0058	-4.382	15.557	38.912	0.466	2.864	51.049	24.530	-3.986
4	Diversified industries	0.0129	0.0043	-4.985	16.411	42.703	-0.227	3.858	27.062	0.607	-4.341
5	Electronic & Elect.	0.0169	0.0075	-5.597	23.634	41.764	1.122	4.477	47.856	8.748	-4.007
6	Engineering	0.0082	0.0056	-4.499	7.145	46.235	-2.680	3.463	32.010	5.340	-4.533
7	Paper & Print	0.0104	0.0046	-2.696	12.050	36.648	1.227	3.075	32.420	16.989	-4.036
8	Brewers	0.0117	0.0085	-5.543	30.303	81.023	-0.034	18.501	59.067	1.318	-3.714
9	Food Producers	0.0096	0.0044	-5.847	14.065	32.603	-2.052	4.103	35.933	9.202	-3.964
10	Health Care	0.0158	0.0062	4.115	11.649	72.946	4.313	11.941	75.213	3.165	-4.199
11	Pharmaceuticals	0.0113	0.0053	-4.070	16.524	64.583	-0.217	8.075	60.368	6.842	-4.661
12	Tobacco	0.0164	0.0067	-8.164	20.504	31.219	-2.490	2.181	27.794	2.746	-4.017
13	Media	0.0180	0.0158	-8.398	21.745	69.429	-3.951	9.192	74.542	5.684	-4.519
14	Support Services	0.0144	0.0051	-1.000	5.219	40.309	0.288	3.517	34.634	5.880	-5.328
15	Transport	0.0124	0.0044	-3.166	11.888	23.267	0.145	4.756	25.521	16.796	-3.849
16	Banks Retail	0.0121	0.0048	-1.669	11.197	29.637	-0.227	9.739	25.982	56.112	-4.864
17	Other Financial	0.0101	0.0030	-27.430	148.592	77.524	0.903	5.449	61.300	2.511	-5.380
18	Property	0.0140	0.0082	-8.401	28.985	79.000	-2.222	11.053	70.014	19.344	-4.776

Critical Values (5%): skewness, kurtosis ($N(0,1)$): 1.96; Normality (χ^2_{27}): 40.11; ARCH (χ^2_6):12.59; ADF(10%): -2.57

Data limitations restricted the choice of macro variables since several obvious choices (such as GDP, CPI and average weekly earnings) are not available at the monthly frequency of our index data. Therefore, in the first group of macroeconomic factors we experimented with an index of production, employment and the unemployment rate; for the nominal domestic influences we used an index of manufacturing prices, award wages, M3, M6 and the 90-day bank-accepted bill rate; foreign influences were captured by three alternative exchange-rate measures (in terms of the US dollar, the Japanese Yen and a trade-weighted basket of currencies) and the deficit on the current account of the balance of payments. In our estimates of beta based on non-overlapping sub-samples, we used two-year averages of macro variables and extended our set to include real GDP and the CPI inflation rate.

The variables chosen were broadly similar to those used in studies of the macro-factor arbitrage-pricing models such as Chen, Roll and Ross (1986) for the US, Clare and Thomas (1994) for the UK, Ariff and Johnson (1990) for Singapore, Martikainen (1991) for Finland and Groenewold and Fraser (1997) for Australia.

4. Time-Varying Betas: The Evidence

Before turning to the matter of the estimation and evaluation of alternative forecasting models, we briefly assess the extent of beta instability for our data set since the motivation for forecasting betas is that they are unstable over time. The simplest way to examine and account for beta-instability is to estimate betas over short sub-samples and compare the resulting estimates across time-periods. Table 2 reports betas for each sector estimated for the full sample period, 1973(1) to 1998(6), as well as for five five-year sub-periods (although the last sub-period has an extra 6 observations) and two sub-samples divided by the October 1987 observation. All betas were obtained from OLS estimation of the market model, the linear relationship between sector returns and the return to the market portfolio.

The betas reported in Table 2 show considerable variation over sub-samples. Focussing just on the five-year periods, we find that a third of the sectors have betas which change from less than one to greater than one or *vice versa* over the period. Some betas change by very large amounts; e.g. Food Producers changed from a figure in excess of 0.8 to approximately 0.2 and back to 0.8 in successive periods; the beta for the Health Care sector increased from 0.29 to 0.56 to 0.80 in successive five-year periods and the beta for the Other Financial sector increased from 0.2634 to 0.9108 in successive periods. More than half the sectors had betas which remained below one for all five five-year periods while only one sector (Mining) had

betas which were greater than one for all sub-periods. The variability appears to be less over the two longer sub-periods divided by the October 1987 observation, reflecting the common

Table 2: Sub-Sample Betas

No. Sector		Sample							
		Full	1973-77	1978-82	1983-87	1988-92	1993-98	Pre-Oct 87	Post-Oct 87
1	Other Mining	1.1188	1.0831	1.1781	1.0493	1.2848	1.1572	1.1206	1.2163
2	Building Materials & Merch.	0.9042	0.8603	0.9588	0.9170	0.8801	0.8753	0.9273	0.8901
3	Chemicals	0.7406	0.6898	0.4667	0.8623	0.8395	1.0615	0.6303	0.9173
4	Diversified industries	0.6924	0.6390	0.3855	0.7904	0.8351	1.0999	0.5769	0.9590
5	Electronic & Elect.	0.7757	0.7433	0.4993	0.9416	0.8451	0.9245	0.6387	0.8409
6	Engineering	0.7190	0.5703	0.5367	0.8742	0.6227	1.1846	0.6578	0.9154
7	Paper & Print	0.7383	0.6454	0.6471	0.7492	0.9236	0.9904	0.6667	0.9403
8	Brewers	0.8880	0.8171	0.8393	0.9859	0.9970	0.6864	0.8131	0.8292
9	Food Producers	0.6841	0.8244	0.2112	0.8111	0.8391	0.8228	0.6031	0.8495
10	Health Care	0.3316	0.3292	0.0936	0.2905	0.5624	0.8019	0.2214	0.6750
11	Pharmaceuticals	0.4482	0.5586	0.2704	0.6017	0.2399	0.1974	0.4149	0.1782
12	Tobacco	0.6672	0.7327	0.0933	0.9557	0.7053	0.7426	0.5301	0.7077
13	Media	1.2191	1.3816	0.6204	1.4831	1.3905	1.0268	1.1063	1.2255
14	Support Services	0.6108	0.6757	0.3567	0.6134	0.7859	0.9213	0.5455	0.8462
15	Transport	0.8102	0.8392	0.7397	0.7853	0.9078	0.9179	0.7984	0.9132
16	Banks Retail	0.8001	1.1801	0.4767	0.6401	0.9958	0.8891	0.8088	0.9528
17	Other Financial	0.5962	0.5315	0.2634	0.9108	0.5396	0.5643	0.4411	0.5538
18	Property	1.0658	1.4860	0.6657	1.1034	0.8990	0.8948	1.0593	0.8980

finding that there is less instability in betas estimated over longer periods. Nevertheless, nine of the 18 sectors show changes in betas of at least 0.2 from one sub-period to the other which, at an equity-premium of 5% pa (approximately the difference between the change in the aggregate accumulation index and the 90-day bill rate), makes for a change of 1 percentage point in the equilibrium return predicted by the CAPM.

The informal evidence presented in Table 2, therefore, shows that there is considerable variability of the betas over time, particularly over shorter time-period. This suggests that the market model with constant coefficients is misspecified. Thus, we now turn to more formal diagnostic tests of the market model which are reported in Table 3. Column 2 contains the estimated betas and corresponds to column 2 of Table 2.

All the betas were found to be significant and are clearly of a plausible magnitude. The market model generally explains a substantial proportion of the variation in returns over the sample period as measured by R^2 in the following column. There are some exceptions, however: the Health Care and Pharmaceuticals sectors have returns which are clearly only weakly related to the return to the market as a whole.

The Durbin-Watson statistic is often used as an informal indicator of omitted (autocorrelated) variables. For our sample there is no evidence of first-order autocorrelation in the errors, with the test statistic being in the uncertain range (at the 5% significance level) only for the Mining sector; otherwise the null hypothesis of no autocorrelation can not be rejected.

The next two columns report statistics based on the recursive residuals which are commonly used to test model stability. The first of these, Harvey's t-test, is based on the CUSUM and has a t-distribution with $(N-2)$ degrees of freedom under the null hypothesis that the error process in the market model is normal. The second, Harvey's F-test, is a similar test but based on the squared recursive residuals. It is distributed with an $F_{(1, N-2)}$ distribution under a similar null hypothesis. In both cases the null hypothesis cannot be reject for any of the sectors. There is therefore no indication from these statistics that the coefficients of the market model are unstable.

The RESET test is a standard test of linearity and proceeds by regressing the dependent variable on the original regressor and the square of the predicted dependent variable from the original regression. The RESET(2) statistic is distributed $F_{(1, N-2)}$ under the null of linearity. It is rejected for 7 of the 18 sectors reported in the table, providing some evidence of mis-specification of functional form. On the whole, however, these four tests provide surprisingly little evidence of specification problems for the market model.

TABLE 3: Market Model Diagnostics

No.	Sector	Beta	R-squared	DW	Harvey's t	Harvey's F	RESET(2)	Skewness	Excess Kurtosis	Normality
1	Other Mining	1.1188	0.8696	1.6597	-1.4149	1.7985	4.1866	-3.5172	6.6867	44.9338
2	Building Materials & Merch.	0.9042	0.7595	2.0554	0.2761	0.5791	1.7569	1.6597	1.8243	27.7553
3	Chemicals	0.7406	0.4091	2.3078	-0.3119	0.7716	1.9568	0.4319	7.6809	36.6344
4	Diversified industries	0.6924	0.4792	1.9958	0.5309	0.361	6.4604	2.8195	7.7959	24.8059
5	Electronic & Elect.	0.7757	0.3464	2.4597	-1.4226	0.4419	3.8898	1.7521	4.6098	33.0366
6	Engineering	0.7190	0.3969	2.2024	0.1052	0.4203	0.2202	0.3195	3.3927	34.1533
7	Paper & Print	0.7383	0.5154	2.2354	0.9049	0.5615	0.0161	1.5946	2.2249	27.4562
8	Brewers	0.8880	0.3989	2.3172	-0.9819	0.4798	2.8763	-2.4377	19.6270	60.1704
9	Food Producers	0.6841	0.4552	1.8575	0.7574	0.6794	3.3368	0.1103	3.5853	19.0844
10	Health Care	0.3316	0.0764	2.003	-0.0395	0.4552	0.0391	4.8302	13.6058	51.8526
11	Pharmaceuticals	0.4482	0.1640	2.1789	0.815	0.7324	16.6880	0.8044	7.3874	58.3396
12	Tobacco	0.6672	0.2852	1.9598	0.6005	0.9865	6.4857	-3.1891	2.0683	38.4607
13	Media	1.2191	0.4061	1.9902	0.9493	0.7674	9.3044	-2.9140	6.2663	43.5449
14	Support Services	0.6108	0.3135	2.1497	-0.2558	0.2767	0.1345	-0.8252	3.3809	32.1620
15	Transport	0.8102	0.6432	2.0123	0.3469	0.5757	2.1606	0.1712	0.7463	24.0023
16	Banks Retail	0.8001	0.5714	1.9953	0.2702	0.3687	4.2988	1.0380	3.2016	28.5581
17	Other Financial	0.5962	0.5199	2.0693	-0.9294	0.4307	54.0870	-10.3617	40.5282	23.4112
18	Property	1.0658	0.5980	1.9522	-0.6833	0.1999	3.4599	-0.1390	5.6741	37.2606

Critical values (5%): DW; $d_L = 1.65$, $d_U = 1.69$; Harvey's t: 1.96; Harvey's F: 3.96; RESET(2): 3.96; Skewness and Excess Kurtosis: 1.96; Normality: 40.11.

The next three columns of Table 3 relate to the normality of the error process in the market model. The first tests for skewness, the second for excess kurtosis while the third is a goodness-of-fit test for normality. The skewness and kurtosis statistics are standard - normally distributed and the third is distributed $\chi^2_{(11)}$ under the null of normality. The null hypothesis is rejected for seven, 16 and 17 of the 18 sectors respectively, indicating that there is considerable departure from normality in the errors of the market model. These rejection rates are similar to those of the original returns (corrected for the influence of October 1987) and indicate that the market model “explains” little of the non-normality in the original returns

The results reported in Table 3 therefore suggest considerable departure from normality in the market model’s errors but not the widespread specification problems. This is surprisingly at variance with the strong evidence of beta instability reported in Table 2. A more direct test of parameter stability is the Chow test which, however, requires a prior specification of the break-point. We chose breakpoints at five-year intervals corresponding to the common practice of estimating betas over five-year periods as well as an alternative break-point at October 1987. The five-year breakpoints correspond to those used in Table 2. The results are reported in Table 4.

We permit only the slope coefficient in the market model (i.e. the beta) to change at the break-point. In this situation the Chow test is equivalent to a t-test on a dummy variable interacted with the return to the market portfolio. The market model was, therefore, re-estimated with 4 dummy variables, $D_1, \dots, D_4$ interacted with the return to the market portfolio. The first dummy variable is zero except for the period 1978-1982, D_2 is zero except for the period 1983-1987 and so on. A test of significance of D_1 is therefore a test of the equality of beta between the first and second five-year sub-periods, a test of significance of D_2 is a test of equality of the beta between the first and the third sub-periods and so on. Approximately a quarter of the dummy variables are individually significant. For 11 of the 18 sectors at least one of the dummy variables was significant indicating a significant break in the sector’s beta. This was confirmed by the F-test which tests for the joint significance of the four dummy variables – the prob values in parentheses under the F-statistics indicates significance at the 5% level for 11 of the sectors.

To test for a shift in beta at October 1987, the market model was subsequently re-estimated with D_{87} interacted with the return to the market portfolio. The coefficient of this variable and its t-ratio are reported in the last column of the table which show that there is a significant break at October 1987 in the betas of five of the 18 sectors, a result which is consistent with the less formal evidence in Table 2 where we found less variability over two periods separated by October 1987 than over five-year sub-periods.

Table 4: Chow Tests

No.	Sector	α_1	β_1	D_1	D_2	D_3	D_4	F	D_{87}
1	Other Mining	-0.0032 (-1.92)	1.0826 (22.04)	0.0981 (1.39)	-0.0275 (-0.42)	0.2031 (2.19)	0.0358 (0.38)	2.2079 (0.07)	0.0821 (1.28)
2	Building Materials & Merch.	-0.0014 (-0.71)	0.8598 (14.73)	0.0925 (1.10)	0.0611 (0.79)	0.0199 (0.18)	0.0092 (0.08)	0.3830 (0.82)	-0.0445 (-0.59)
3	Chemicals	0.0049 (1.44)	0.6883 (6.88)	-0.2368 (-1.64)	0.1939 (1.46)	0.1397 (0.74)	0.3331 (1.72)	3.5039 (0.01)	0.2609 (2.00)
4	Diversified industries	0.0050 (1.85)	0.6390 (8.00)	-0.2466 (-2.14)	0.1792 (1.70)	0.1877 (1.25)	0.3739 (2.42)	6.0243 (0.00)	0.3261 (3.10)
5	Electronic & Elect.	0.0087 (2.13)	0.7406 (6.12)	-0.2334 (-1.34)	0.2205 (1.38)	0.0607 (0.27)	0.1139 (0.49)	2.0125 (0.09)	0.1324 (0.85)
6	Engineering	-0.0004 (-0.11)	0.5688 (5.72)	-0.0158 (-0.11)	0.2975 (2.26)	0.0440 (0.23)	0.5790 (3.01)	3.7844 (0.01)	0.2216 (1.69)
7	Paper & Print	0.0020 (0.74)	0.6463 (7.94)	0.0031 (0.03)	0.1125 (1.05)	0.3090 (2.02)	0.2773 (1.76)	1.7902 (0.13)	0.2651 (2.52)
8	Brewers	0.0027 (0.64)	0.8169 (6.54)	0.0418 (0.23)	0.1853 (1.12)	0.1226 (0.52)	-0.1346 (-0.56)	0.6460 (0.63)	-0.0381 (-0.24)
9	Food Producers	0.0028 (1.04)	0.8251 (10.19)	-0.5973 (-5.12)	0.0072 (0.07)	-0.0071 (-0.05)	-0.0552 (-0.35)	9.7105 (0.00)	0.1950 (1.77)
10	Health Care	0.0116 (2.62)	0.3306 (2.52)	-0.2068 (-1.09)	-0.0154 (-0.09)	0.2262 (0.91)	0.3664 (1.45)	1.6012 (0.17)	0.3927 (2.31)
11	Pharmaceuticals	0.0077 (2.00)	0.5607 (4.89)	-0.2962 (-1.79)	0.0518 (0.34)	-0.2883 (-1.33)	-0.3845 (-1.74)	2.2726 (0.06)	-0.2147 (-1.46)
12	Tobacco	0.0098 (2.52)	0.7348 (6.35)	-0.6048 (-3.63)	0.2434 (1.59)	-0.0831 (-0.38)	-0.0101 (-0.05)	7.5940 (0.00)	0.1211 (0.62)
13	Media	0.0066 (1.19)	1.3844 (8.42)	-0.7614 (-3.22)	0.1401 (0.64)	0.0034 (0.01)	-0.4149 (-1.31)	4.8116 (0.00)	0.0752 (0.35)
14	Support Services	0.0077 (2.23)	0.6749 (6.60)	-0.3124 (-2.12)	-0.0684 (-0.51)	0.1187 (0.62)	0.2204 (1.12)	2.5201 (0.04)	0.2968 (2.23)
15	Transport	0.0039 (1.69)	0.8388 (12.14)	-0.1011 (-1.02)	-0.0514 (-0.56)	0.0832 (0.64)	0.0433 (0.32)	0.6978 (0.59)	0.1090 (1.22)
16	Banks Retail	0.0047 (1.91)	1.1816 (16.12)	-0.7012 (-6.64)	-0.5365 (-5.53)	-0.2027 (-1.47)	-0.2521 (-1.78)	13.5891 (0.00)	0.1622 (1.59)
17	Other Financial	0.0041 (2.00)	0.5305 (8.81)	-0.2444 (-2.82)	0.3615 (4.54)	0.0007 (0.01)	0.0223 (0.19)	14.8153 (0.00)	0.0829 (1.12)
18	Property	0.0048 (1.52)	1.4871 (15.72)	-0.8155 (-5.99)	-0.3740 (-2.99)	-0.6067 (-3.41)	-0.5779 (-3.16)	9.6764 (0.00)	-0.1638 (-1.27)

On the whole, therefore, the Chow test results indicate widespread shifts in the slope coefficient of the market model over our sample and this confirms the evidence derived from an inspection of the magnitudes of the sub-period betas in Table 2. The results described above are not dissimilar to those obtained by Faff, Lee and Fry (1992), Brooks, Faff and Lee (1992), Faff and Brooks (1998) and Brooks, Faff and Lee (1994). All use Australian data, some on individual shares and some portfolio data similar to those used in the present study. All test for beta stability although only Faff and Brooks (1998) use dummy variables for sub-periods. The findings are comparable to those reported above - at the 5% significance level, β stability is rejected for approximately 15-35% of assets (or portfolios).

We may summarise the evidence presented in this section by saying that there is evidence of widespread but not universal instability in betas estimated using Australian share-price data whether for individual assets or for portfolios. We proceed, therefore, to estimate an explicit time-varying parameter version of the market model, investigate the time-series properties of the resulting parameter estimates directly before going on to exploit these properties for the purposes of forecasting.

5. Estimated Time-Varying Betas

For each of the 18 sectors for which data were available for the entire sample period we estimated a time series for beta using the rolling regression technique using a window size of 60 months. They are pictured in Figure 1, together with betas estimated over non-overlapping two-year sub-periods. The rolling-regression betas for the first 60 months are set equal to their value for the month 60, the first period for which the betas were estimated. The two-year sub-period betas show wide fluctuations over time and illustrate graphically the instability of the betas. There are many cases of very large changes in the estimated betas from one two-year period to the next; e.g. the beta for the Building materials sector falls from approximately 1.35 to about 0.8 at the time of the crash of October 1987 and the beta for the Property sector falls from a figure in excess of 1.5 to one a little over 0.5 from 1975-76 to 1977-78. It should be kept in mind that part of this will be sampling variability due to the short periods over which the betas are estimated. The rolling betas are

estimated over five-year samples and should be less affected by sampling variability but still exhibit substantial changes over both short and longer periods.

Overall, the results pictured in Figure 1 confirm the intertemporal instability in the betas suggested by the statistical evidence presented in the previous section.

6. Modelling the (Rolling-Regression) Time-Varying Betas

We now proceed to address the question of whether the variation which we have observed in the estimated betas can be explained in terms of regression models before using the estimated models for forecasting purposes. We start with a model in which the fluctuations in the betas are explained only in terms of a trend and a break at October 1987. We then go on to consider the influence of economy-wide variables. It is important to distinguish between these models (although we eventually experiment with a combination of the two) for forecasting purposes since the future values of the deterministic functions of time are known with certainty which simplifies forecasting considerably compared to the use of models with macro variables, the values of which must themselves be forecast before they can be used as inputs into the process of forecasting the betas.

6.1 The betas as functions of time

In both models we proceed by using the betas estimated by the rolling-regression technique as dependent variables in regression equations. We begin by regressing the beta for each sector on a time trend (T), a dummy variable for October 1987 (D_{87}) and the interaction of these two variables ($T \cdot D_{87}$) so that we allow for a break both in level and in trend in the equation explaining each beta. The results are reported in Table 5.

The equations reported in Table 5 generally have good explanatory power. R^2 exceeds 50% for all but 4 of the sectors and is greater than 70% for almost half of the cases reported. The regressors are generally significant: the intercept is significant and positive in all cases, the trend term is significant in the equations for two-thirds of the sectors and in these it is generally positive. There is clear evidence of a break at October 1987 in both the trend and the intercept. The intercept dummy variable is significant in all but one of the equations and the interaction with the trend term is significant for 15 of the sectors. The overwhelming significance of the intercept shift

Table 5: Modelling Betas as Functions of Time

Sector	Const	T	D₈₇	T*D₈₇	R²	DWS
Other Mining	1.1791 (-80.05)	-0.0229 (-1.94)	-0.2480 (-6.90)	0.1028 (5.59)	0.2568	0.1571
Building Materials & Merch.	0.8445 (66.56)	0.1015 (9.98)	0.0741 (2.40)	-0.1186 (-7.48)	0.6498	0.1652
Chemicals	0.4443 (12.93)	0.1135 (4.12)	0.4781 (5.70)	-0.1314 (-3.06)	0.7007	0.0583
Diversified Industries	0.3692 (9.69)	0.1425 (4.66)	-0.1010 (-1.09)	0.1151 (2.42)	0.7091	0.0406
Electronic & Elect.	0.2501 (5.60)	0.2385 (6.65)	1.1292 (10.36)	-0.4203 (-7.53)	0.7468	0.0713
Engineering	0.2102 (5.36)	0.3829 (12.17)	-0.2744 (-2.87)	0.0139 (0.28)	0.6459	0.0660
Paper & Print	0.5999 (34.98)	0.0587 (4.26)	-0.4728 (-11.30)	0.2604 (12.16)	0.8471	0.1737
Brewers	1.0582 (39.04)	-0.2024 (-9.31)	0.2105 (3.18)	0.0828 (2.44)	0.6384	0.2190
Food Producers	0.5131 (8.50)	-0.0400 (-0.83)	0.8655 (5.88)	-0.2175 (-2.88)	0.4802	0.0454
Health Care	0.2512 (6.22)	-0.0591 (-1.82)	-1.6078 (-16.31)	0.8702 (17.24)	0.8018	0.0469
Pharmaceuticals	0.2610 (7.22)	0.0246 (0.85)	1.4289 (16.20)	-0.5548 (-12.29)	0.6423	0.1593
Tobacco	0.3492 (7.09)	0.0253 (0.64)	0.6458 (5.37)	-0.0620 (-1.01)	0.7745	0.0688
Media	0.6380 (6.89)	0.2386 (3.21)	0.9101 (4.03)	-0.2718 (-2.35)	0.5193	0.0308
Support Services	0.4936 (12.97)	0.0054 (0.18)	-0.6254 (-6.74)	0.3597 (7.57)	0.5764	0.0656
Transport	0.5266 (18.28)	0.1816 (7.86)	-0.3211 (-4.57)	0.0996 (2.77)	0.5428	0.0618
Banks Retail	1.2213 (20.83)	-0.4436 (-9.43)	-1.6461 (-11.51)	0.9845 (13.45)	0.4975	0.0243
Other Financial	0.2924 (7.42)	0.0770 (2.44)	1.7419 (18.12)	-0.6309 (-12.82)	0.7857	0.0898
Property	1.4361 (22.64)	-0.4696 (-9.23)	0.3923 (2.54)	0.1342 (1.69)	0.4377	0.0484

variable indicates a marked change in risk for most sectors at the time of the Crash – about half the sectors experienced an increase in their risk parameter. The significance of the interaction term indicates that there was also a widespread shift in the rate at which the betas change at October 1987. Finally, the low reported values for the Durbin-Watson (DW) statistic provide strong evidence of autocorrelation in the residuals of the equations. This makes the t-tests invalid; however, re-estimation

of the equations with robust standard errors following White (1980) change the standard errors only slightly and do not change the conclusions drawn from the results reported in Table 5.¹

We now turn from an analysis of the time-variation in the sectoral betas in terms of functions of time to the question of whether the variation in the betas is systematically related to the state of the economy by estimating models which explain the intertemporal variation in the betas to macroeconomic variables.

6.2 The betas as functions of the state of the economy

As explained in section 4, our choice of macro variables was based on the hypothesis that at the level of the economy as a whole, risk is influenced by three classes of factors - real domestic activity, nominal domestic factors and foreign variables. In the first group we experimented with an index of production, employment and the unemployment rate; for the nominal domestic influences we used an index of manufacturing prices, award wages, M3, M6 and the 90-day bank-accepted bill rate; foreign influences were captured by three alternative exchange-rate measures (in terms of the US dollar, the Japanese Yen and a trade-weighted basket of currencies) and the deficit on the current account of the balance of payments. We carried out a preliminary analysis by regressing time-varying β s on each of these variables individually and found that only the unemployment rate, the bill rate, the Yen exchange rate and the percentage change in M6 had any significant explanatory power and undertook further work only with these variables.

Table 6 reports the results of regressing each sector's beta on these four macroeconomic variables. The results show that much of the time-variation in the betas is explained by changes in aggregate economic conditions – the R^2 values exceed 50% for the majority of the sectors and approximately half have a value over 70%. Two of the variables are significant for all sectors, viz. the Yen exchange rate and the unemployment rate. For both of these variables the coefficient is negative for almost all sectors, indicating that the sector risk, as measured by its beta, falls with an appreciation of the \$A and with a rise in the unemployment rate.

¹ In fact, the generally high R^2 s and very low DW statistics provide informal evidence that the betas are non-stationary. This is not surprising given the overlapping-observations problem. We calculated ADF statistics and found that all the betas are better described by a difference-stationary process than by a trend-stationary one. Since the present paper focusses on forecasting of betas we take the pragmatic

Table 6: Modelling Betas as Functions of Macroeconomic Variables

Sector	Const.	B-Rate	Yen	%ΔM6	Unemp.	R²	DW
Other Mining	0.9633 (32.51)	-0.0123 (-0.14)	0.0457 (9.66)	0.6621 (1.06)	1.2130 (5.04)	0.3254	0.1946
Building Mat. & Merch.	0.6453 (17.97)	0.8150 (7.79)	0.0312 (5.45)	2.0222 (2.68)	1.4881 (5.10)	0.3708	0.1853
Chemicals	1.5681 (27.36)	-1.1178 (-6.69)	-0.2440 (-26.69)	-1.9232 (-1.60)	-4.0527 (-8.71)	0.8142	0.1510
Diversified Industries	1.6929 (34.37)	-1.8163 (-12.65)	-0.2774 (-35.30)	-0.0205 (-0.02)	-4.5194 (-11.30)	0.8914	0.1561
Electronic & Elect.	1.6442 (18.98)	-0.0825 (-0.33)	-0.3509 (-25.39)	-4.3724 (-2.41)	-4.0836 (-5.81)	0.7875	0.1890
Engineering	1.2752 (15.43)	-0.4277 (-1.78)	-0.2440 (-18.50)	6.7938 (3.92)	-1.9052 (-2.84)	0.6477	0.1264
Paper & Print	1.3117 (28.69)	-1.5926 (-11.95)	-0.1326 (-18.18)	0.3094 (0.32)	-2.0254 (-5.45)	0.7566	0.1495
Brewers	1.4375 (17.98)	-1.4725 (-6.32)	-0.0431 (-3.38)	-7.0359 (-4.19)	-2.9889 (-4.60)	0.2965	0.2091
Food Producers	1.8451 (15.94)	-1.8483 (-5.48)	-0.2580 (-13.97)	-6.9268 (-2.85)	-6.9740 (-7.42)	0.5720	0.0805
Health Care	1.6455 (16.77)	-4.4705 (-15.63)	-0.2329 (-14.88)	2.3462 (1.14)	-5.5227 (-6.93)	0.7391	0.1759
Pharmaceuticals	0.7627 (7.33)	1.1103 (3.66)	-0.1443 (-8.69)	-6.4555 (-2.96)	-3.0757 (-3.64)	0.3348	0.1630
Tobacco	2.3861 (28.14)	-2.7969 (-11.32)	-0.3987 (-29.47)	-6.1372 (-3.45)	-9.5402 (-13.85)	0.8508	0.1805
Media	2.5964 (17.13)	-2.3580 (-5.34)	-0.4431 (-18.32)	-6.5339 (-2.05)	-5.0815 (-4.13)	0.7119	0.0594
Support Services	1.5363 (27.17)	-2.4984 (-15.16)	-0.1902 (-21.09)	1.8220 (1.54)	-4.5660 (-9.94)	0.7907	0.2612
Transport	0.9432 (16.88)	-0.5758 (-3.54)	-0.1179 (-13.22)	3.1542 (2.69)	0.9115 (2.01)	0.6160	0.1219
Banks Retail	2.2703 (21.66)	-5.5534 (-18.17)	-0.0882 (-5.27)	0.2498 (0.11)	-9.0712 (-10.65)	0.6359	0.1471
Other Financial	1.3724 (10.85)	0.2151 (0.58)	-0.2621 (-12.98)	-9.3164 (-3.51)	-4.4930 (-4.37)	0.5064	0.1445
Property	2.5009 (17.81)	-3.0938 (-7.56)	-0.1305 (-5.82)	-8.1396 (-2.76)	-11.3143 (-9.92)	0.3717	0.0832

The bill rate is significant for all but four of the sectors and, again, the sign is generally negative indicating an fall in beta when interest rates rise. Finally, the coefficient of the rate of monetary growth is significant and negative for about half of the sectors, significant and positive for four sectors and insignificant for six sectors. The reaction of the betas is, therefore, more varied in response to a change in monetary growth than is the case for the other factors. The final statistic reported for

view that the question of stationarity is not crucial. We will revisit the stationarity question when we discuss the forecasting performance of our models.

each of the sectors is the DW statistic which indicates widespread evidence of autocorrelated residuals, as was the case in the estimates reported in Table 5. As in the previous case, this indicates strong autocorrelation in the betas which is not explained by the regressors. The presence of autocorrelation in the errors makes the standard t-tests invalid. Re-estimation using White's (1980) robust standard errors, changes the t-ratios only slightly and does not affect any of the inferences previously drawn. Alternative responses to the presence of autocorrelation is to include a lagged dependent variable as a regressor or to use an estimation method such as the Cochrane-Orcutt estimator which explicitly accounts for the autocorrelation in the error process. Both of these were experimented with; results will be discussed later when we present the forecasts.

7. Forecasting the Rolling-Regression Betas

We now use the models described above to generate forecasts of the betas for a two-year period at the end of our sample. Models reported in sub-sections 6.1 and 6.2 were estimated over the period 1978(2)-1993(6), with the data for the last five years reserved for the purpose of forecast evaluation. We used nine alternative forecasting models based on the results reported above. The models are the following:

1. The model discussed in sub-section 6.1 which uses a time trend and allows for a break in level and in trend at October 1987. (T,D87)
2. The four-variable macro model reported in sub-section 6.2. (R,Y,M,U)
3. A two-variable macro model using the Yen exchange rate and the bill rate. (R,Y)
4. A model which combines two macro variables and the time trend (with break). (R,Y,T,D87)
5. A model with two macro variables and a lagged dependent variable. (R,Y, $\beta(-1)$)
6. A model with time-trend and breaks and a lagged dependent variable. (T,D87, $\beta(-1)$)
7. A model with the time trend and break variables estimated with an AR(1) correction. (T,D87,AR(1))
8. An AR(1) model. ($\beta(-1)$)
9. The five-year rule of thumb. (β_5)

The first two models are simply the models estimated in sections 6.1 and 6.2. The third model eliminates the money growth rate and the unemployment rate from the macro-variable model since these two variables were not always significant in the equations reported in Table 6. The fourth model combines the macro and time trend variables, using just two macro variables since it turns out that the two-variable macro model outperforms the four-variable model. The fifth, sixth and seventh models take the autocorrelation in the betas into account – models 5 and 6 by adding a lagged dependent variable to the regressors and model 7 by estimating the forecasting equation with an AR(1) correction. The eighth model can be seen as a simple alternative time-series model and the final model is simply the five-year rule of thumb using the most recently estimated β .

In each of these nine cases we generate a forecast for 1998(6). We imagine a decision-maker standing at the end of the estimation period, 1993(6), and requiring a beta to apply to the following 5-year period as an input into some decision. Since the betas we have estimated are five-year rolling betas, the required beta is simply the beta for 1998(6) which has been estimated using data for the period 1993(7)-1998(6). We therefore compare the forecast beta to the actual beta for 1998(6) for each sector and calculate absolute deviations and squared deviations. Finally we report the mean across sectors of the absolute deviations (MAD) and the square root of the mean squared errors (RMSE) for each of our nine forecasting models in Table 7.

Table 7: Forecasts of the Rolling Regression Betas

Model	MAD	RMSE
1. (T, D87)	0.1608 (3)	0.2349 (6)
2. (R,Y,M,U)	0.2779 (8)	0.3359 (8)
3. (R,Y)	0.2222 (7)	0.2706 (7)
4. (R,Y,T,D87)	0.1557 (2)	0.2204 (5)
5. (R,Y, $\beta(-1)$)	0.1726 (5)	0.2167 (4)
6. (T,D87, $\beta(-1)$)	0.5108 (9)	0.6841 (9)
7. (T,D87,AR(1))	0.1656 (4)	0.2154 (3)
8. ($\beta(-1)$)	0.1804 (6)	0.2143 (2)
9. (β_5)	0.1301 (1)	0.1625 (1)

Numbers in parentheses are rankings.

The two measures of forecast performance do not produce identical rankings for the nine models. Thus model (β_5) ranks 1, model (R,Y,M,U) ranks 8 and model (T,D87, $\beta(-1)$) ranks 9 on both criteria while model (R,Y,T,D87) ranks 2 on the MAD criterion and 5 on the RMSE, model ($\beta(-1)$) ranks 6 and 2 and model (T,D87) ranks 3 and 6 on the two criteria. Presumably these different rankings are related to the property of RMSE that it penalises large errors relative to the MAD. If we examine the individual sector results, we find that, indeed, the (T,D87) model has large errors for the Engineering and Media sectors while the ($\beta(-1)$) model performs reasonably well across the board with relatively uniform and modest errors.

Consider what the results tell us about the relative performance of the forecasting models. First, the four-variable macro model (R,Y,M,U) performs very poorly although the truncated version of the model (R,Y) has somewhat lower forecasting errors on both criteria. The time-trend model (T,D87) performs very well on the basis of the MAD criterion but less well judged by its RMSE. On the basis of both criteria both the (R,Y) and (T,D87) models are improved by combining them, achieving a rank of 2 as measured by the MAD. An alternative but less successful adjustment to the (R,Y) model is to add the lagged dependent variable, $\beta(-1)$, to account for the strong autocorrelation in the betas. When the lagged dependent variable is added to the (T,D87) model the forecasting performance deteriorates to a surprising extent – the resulting model has the highest forecasting error of any of the models by a large margin. An examination of the disaggregated errors for this model reveal consistent errors across all sectors as suggested by the similar results for the two error criteria. The forecast errors for 11 of the 18 sectors exceeded 0.3 in absolute value, of which seven were in excess of 0.5 and two were greater than 1. The re-estimation of the (T,D87) model using an estimator corrected for AR(1) errors has mixed results – the forecasts improve on average on the basis of the RMSE criterion but deteriorate according to the MAD measure. The same is true of the simple AR(1) model – it achieves a rank of 2 on the RMSE measure but only 6 based on its MAD. Finally and surprisingly, the five-year rule of thumb (β_5) has the smallest forecast errors on both criteria. On further consideration, this is not so surprising since all the estimated beta series were earlier found to be first-order integrated processes (“random walks” with some autocorrelation in the innovations)

which are well forecast by the most recent actual value. This is exactly what the five-year rule of thumb does.

The superiority of the rule of thumb is true on average but not for all sectors, however. In fact, for one sector, Building Materials, seven of the eight models outperform the rule of thumb while for a further three sectors the (β_5) model has larger forecast errors than four of the eight alternative models (Other Mining, Chemicals, Diversified Industries). In these cases it is generally the models including macro variables which outperform the (β_5) model. Hence, for some sectors, at least, there is potential for improving rule-of-thumb betas forecasts. At the other extreme, there are three sectors for which the rule of thumb is superior to all alternative models (Electronic and Electrical, Tobacco, Transport).

One incongruity in the macro models used above is that the beta for a particular period is estimated from returns over the previous 60 months whereas the macro variables used as regressors in the macro equations relate to the current period. A case could be made for using a 60-month moving for the macro variables to match the time-period over which the betas are estimated. Data limitations meant that we could experiment with a moving-average form only for the Yen exchange rate and the bill rate but this was not a serious constraint since these two variables were more successful in forecasting equations than the other two.

The result was that the R^2 s of the forecasting equations fell for all but four of the 18 sectors. Forecasting performance improved for models using only macro variables but deteriorated for the models which combined the macro variables with the time-trend or with the lagged dependent variable. The overall conclusion regarding the superior forecasting performance of the models taking the autocorrelation into account and particularly of the five-year rule of thumb are not affected by the use of the macro variables in moving-average form.

8. Explaining the Sub-Period Betas

The betas used in the empirical work reported in the previous two sections were estimated using a rolling-regression technique with a five-year window. This seems a natural approach given the common use a five-year sample periods to estimate β s in practice. It is not without its costs, however. One of these costs which had a marked impact on the work reported so far is the strong autocorrelation caused by the

extensive overlap between sample periods used to estimate successive betas. In this section we experiment with an alternative method of obtaining a time series for each sector's beta, viz. estimating them over non-overlapping samples. The costs of this approach is that relatively few "observations" are generated and we attempt to reduce this cost by estimating successive betas over two-year sub-samples, although we recognise that this is likely to increase the sampling variability in the estimated betas.

8.1 The estimated sub-sample betas

The estimated sub-sample betas are pictured in Figure 1. They have been described in section 5 where it was remarked that they show wide fluctuations with some period-to-period changes of close to 1. It should be kept in mind, however, that this is no doubt partly the result of sampling variability due to the relatively short 24-observation sample period over which they are estimated.

8.2 Explaining the sub-sample betas

We proceed, as we did with the rolling-regression betas, to explain the variation of the β s over time by two alternative models, the first using just a time trend and allowing for a break in trend and in level at October 1987 and the second using macroeconomic variables. In the model with a trend, break in level and break in trend the last variable was significant for only one sector so that this variable was omitted and we estimated a model with just a trend and an intercept dummy variable. Even though the regressors were often insignificant, the value of R^2 was such that for most sectors a substantial proportion of the sample variation in the estimated beta was explained by the model although less so than in for the models explaining the rolling-regression betas. Autocorrelation was found not to be a problem with the sub-sample betas suggesting that the autocorrelation found in the rolling-regression betas was largely the result of the large overlap in the observations used to estimate the successive betas.

When using macro variables to explain the variation in the betas, we deleted M6 growth and the unemployment rate from the set of possible regressors since they were not available early enough in our sample period. This is likely to have little adverse effect on the results since in the previous section forecasts actually improved

when these variables were dropped from the forecasting equation. On the other hand, we added CPI inflation and real GDP growth. These variables could not be used for the regressions involving the rolling-regression betas because they are available only at a quarterly frequency. They could, however, be used in the regressions for the two-year sub-period betas because we use the regressors in the form of two-year averages. As in the case of the models involving a time trend and an October 1987 dummy variable, the regressors were generally insignificant when they were all used together although the R^2 's indicated a high degree of variation explained – higher than in the time-trend model.

9. Forecasting the Sub-Sample Betas

We then proceeded to forecast the sub-sample betas using several variations of the models described in the previous section. In particular, we used the following five alternative forecasting schemes:

1. A model with a time trend and an intercept dummy variable for October 1987. (T,D87)
2. A model with four macroeconomic variables: the bill rate, the Yen exchange rate, the rate of real GDP growth and the rate of CPI inflation. (R,Y,G,P)
3. A model with the two “best” macro variables: real GDP growth and the rate of CPI inflation. (G,P)
4. A model with the four macro variables, a time trend and an intercept dummy for October 1987. (R,Y,G,P,T,D87)
5. A model with two macro variables, a time trend and an intercept dummy for October 1987. (G,P,T,D87)
6. The most recent “actual” beta (the “two-year rule of thumb”). (β_2)

Given the absence of evidence of autocorrelation in the estimated betas, we did not experiment with either AR(1)-corrected estimators or AR(1) forecasting models or the use of the lagged dependent variable as a regressor. Forecasts from the six models are reported in Table 8. In this case there is no conflict between the rankings based on the two alternative forecast-error measures. Given the very different method of beta estimation in this case, it is surprising that the results are broadly consistent with those obtained for the rolling-regression betas.

Table 8: Forecasts of Two-Year Sub-Sample Betas

Model	MAD	RMSE
1. T, D87	0.2637 (2)	0.3339 (2)
2. R, Y, G, P	0.3534 (5)	0.4441 (5)
3. G, P	0.3453 (4)	0.4275 (4)
4. R, Y, G, P, T, D87	0.3718 (6)	0.4621 (6)
5. G, P, T, D87	0.3403 (3)	0.4235 (3)
6. β_2	0.1958 (1)	0.2412 (1)

Numbers in parentheses are rankings

The macro models perform poorly compared to the model with just a time trend and an intercept dummy variable. Adding T and D_{87} to the macro model results in a deterioration of forecasting performance when there are four macro variables and a slight improvement when there are two. Finally, the two-year rule of thumb outperforms all others as the rule of thumb forecasts did in the previous case, notwithstanding the absence of autocorrelation in the betas.

10. Alternative Rules of Thumb

We have seen that the common approach to “beta-forecasting”, where the most recently available estimated beta is assumed (often implicitly) to apply to the future, performs well compared to alternative, more sophisticated methods. This raises the question of what the optimal time-period is, from a forecasting perspective, for the estimation of the beta used as a forecast, i.e., does a two-year rule of thumb outperform a three-year rule of thumb, and so on? This question was addressed within the context of the two-year non-overlapping betas discussed in the previous section by comparing the MAD and RMSE for one-, two-, ..., 12-year rules-of-thumb. The results are reported in Table 9.

We see that, again, the rankings produced by the two measures of forecast error are identical. It is interesting that forecast performance does not change monotonically with the length of the estimation period. There is a global optimum at 3 years and local optima at seven and 11 years. There is no support for a five-year rule as commonly used. In our sample the five-year rule is outperformed by two-,

three-, four-, six- and seven-year rules. This is only limited evidence, however. In particular, the results may be sensitive to the period for which the betas are being forecast – here we have used the period 1995(1)-1998(6) – and to the way in which the betas were estimated.

Table 9: Alternative Rules of Thumb

Estimation Period (Years)	MAD	RMSE
1	0.2406	0.3005
2	0.1858	0.2315
3	0.1606	0.1962
4	0.1892	0.2349
5	0.2209	0.2731
6	0.2010	0.2506
7	0.1752	0.2169
8	0.2475	0.2850
9	0.2471	0.2874
10	0.2397	0.2791
11	0.2239	0.2631
12	0.2259	0.2640

10. Conclusions

We can draw the following conclusions from the empirical work reported in this paper.

1. We have confirmed that betas are very unstable over time. This is clear both from formal tests and observation of graphs of betas over our sample period. The instability is obvious both for betas estimated using a rolling-regression technique and for those estimated over relatively short sub-samples.
2. Time-varying betas can be successfully explained by either a time trend and shift variables or macro variables or a combination of the two. In many cases the combination model can explain over 70% of the variation in the betas over the sample. The most successful macro variables were found to be the 90-day bill

rate and the Yen exchange rate although other variables (the unemployment rate and the rate of monetary growth) were also significant in many cases.

3. The rolling-regression betas exhibit very strong autocorrelation caused by the large number of overlapping observations used in the estimation of successive betas. This presents a problem for inference although re-estimation with robust standard errors indicated that our conclusions are not sensitive to this problem.
4. Thus use of non-overlapping sub-samples to estimate betas overcomes the autocorrelation problem. For these betas, too, we successfully explained the time-variation using either a trend or macro variables.
5. We used the estimated equations for the betas for out-of-sample forecasting and found that the common rule of thumb which uses the most recently estimated beta to apply to some future period out-performed all other models, whether they included just a time trend or just macro variables or a combination of both or a simple AR(1) model.
6. The forecasting superiority of the rule of thumb may simply reflect the high autocorrelation in the rolling-regression betas. However, the two-year non-overlapping betas were not found to exhibit any autocorrelation but they, too, were best forecast by the most recently-estimated beta.
7. We experimented with alternative estimation periods as a basis for the rule of thumb forecasts and found that, contrary to common practice, a three-year rule is optimal. Indeed the common five-year rule is dominated by at least five others.

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Figure 1: Time-Varying Betas

Figure 1 Continued: Time-Varying Betas

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