HENRI THEIL'S CONTRIBUTIONS TO DEMAND ANALYSIS

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1. Introduction

Henri Theil has made numerous, influential contributions to economics and econometrics. These include the theory of economic policy, information theory, forecasting, econometric methodology and demand analysis, to name just a few areas of major contributions. There is no question that Theil's work has a style all of its own: It is characterized by insight, ingenuity, elegance and an almost unique combination of theory and measurement. Most (if not all) of his research in econometric methodology, for instance, has applications centrally in mind. A good example is the 2SLS estimator. The early work of the Cowles Commission on the estimation of simultaneous equation systems represented a major watershed in econometrics. However, the estimation methods they proposed were demanding to the point of being almost impractical given the state of computing techniques at that time. Theil solved this problem in his usual style by devising the 2SLS estimator which involves simply running two LS regressions.

As it is impossible to provide a review of all of Theil's work, we have chosen to highlight his contributions in the area of demand analysis. Even in this narrower area, Theil's work is voluminous and much of it has had a lasting impact. Accordingly, we have had to be somewhat selective in what follows. Our starting point (Section 2) is the Rotterdam model which, after almost a quarter of a century, continues to attract attention. Then follows in Section 3 an overview of Theil's work on the troublesome question of aggregating demand equations over consumers. Sections 4 and 5 deal with hypothesis testing and the estimation of demand equations. The final three sections cover preference independence, the so-called strong version of Engel's law, as well as some additional topics. The Appendix lists Theil's books and published papers dealing with demand analysis.

2. The Rotterdam Model

The Rotterdam model is a system of demand equations developed in the mid-60s by Theil and his doctoral student Barten. This model has had lasting durability and forms the basis for much subsequent work in the area by Theil and others. It is thus appropriate that we set out the development of this model in some detail.

It is no exaggeration to describe Barten's (1964) work as a major breakthrough in applied demand analysis. He showed how consumption theory could be incorporated in its full generality in demand equations that could be estimated. Until Barten's contribution, the state-of-the-art work in applied demand analysis was Stone (1954) who introduced the linear expenditure system (LES). That model is consistent with the utility-maximizing theory of the consumer in the sense that it satisfies the two major requirements of homogeneity and symmetry. That is, the demand equations in LES are homogeneous of degree zero in income and prices and the substitution effects are symmetric. However, LES is based on the special assumption of an additive utility function or preference independence. Barten's work requires no such assumption.

Barten formulated a comparative statics analysis of the consumer's utility-maximization problem in terms of the fundamental matrix equation,

$$(2.1) \quad \begin{bmatrix} U & p \\ p' & 0 \end{bmatrix} \begin{bmatrix} \partial q / \partial M & \partial q / \partial p' \\ -\partial \lambda / \partial M & -\partial \lambda / \partial p' \end{bmatrix} = \begin{bmatrix} 0 & \lambda I \\ 1 & -q' \end{bmatrix}.$$

In this equation, U is the $n \times n$ Hessian matrix of the utility function, n being the number of goods; $p = [p_i]$ and $q = [q_i]$ are n -vectors of prices and quantities; M is income, defined as $p'q$; λ is the marginal utility of income; and I is the $n \times n$ identity matrix. The term $\partial q / \partial M$ is an n -vector of

income derivatives of the demand functions, while $\partial \mathbf{q} / \partial \mathbf{p}'$ is an $n \times n$ matrix of price derivatives.

Solving (2.1) by partitioned inversion yields

$$\frac{\partial \mathbf{q}}{\partial \mathbf{M}} = \frac{\partial \lambda}{\partial \mathbf{M}} \mathbf{U}^{-1} \mathbf{p}, \quad \frac{\partial \mathbf{q}}{\partial \mathbf{p}'} = \lambda \mathbf{U}^{-1} - \frac{\lambda}{\partial \lambda / \partial \mathbf{M}} \frac{\partial \mathbf{q}}{\partial \mathbf{M}} \frac{\partial \mathbf{q}'}{\partial \mathbf{M}} - \frac{\partial \mathbf{q}}{\partial \mathbf{M}} \mathbf{q}'.$$

If we write u^{ij} for the $(i, j)^{\text{th}}$ element of $\mathbf{U}^{-1}$, the scalar versions of these equations are

$$(2.2) \quad \frac{\partial q_i}{\partial \mathbf{M}} = \frac{\partial \lambda}{\partial \mathbf{M}} \sum_{j=1}^n u^{ij} p_j, \quad i = 1, \dots, n,$$

$$(2.3) \quad \frac{\partial q_i}{\partial p_j} = \lambda u^{ij} - \frac{\lambda}{\partial \lambda / \partial \mathbf{M}} \frac{\partial q_i}{\partial \mathbf{M}} \frac{\partial q_j}{\partial \mathbf{M}} - \frac{\partial q_i}{\partial \mathbf{M}} q_j, \quad i, j = 1, \dots, n.$$

Equation (2.3) decomposes the total substitution effect of a price change into three terms, (i) the specific effect λu^{ij} ; (ii) the general effect

$$- \frac{\lambda}{\partial \lambda / \partial \mathbf{M}} \frac{\partial q_i}{\partial \mathbf{M}} \frac{\partial q_j}{\partial \mathbf{M}};$$

and (iii) the income effect $-(\partial q_i / \partial \mathbf{M}) q_j$. Equation (2.3) can be expressed in elasticity form by multiplying by p_j / q_i ,

$$(2.4) \quad \frac{\partial(\log q_i)}{\partial(\log p_j)} = \eta_{ij} - \phi \eta_i \eta_j^w - \eta_i^w \eta_j,$$

where $\eta_{ij} = \lambda u^{ij} p_j / q_i$; $\phi = [\partial(\log \lambda) / \partial(\log \mathbf{M})]^{-1}$ is the reciprocal of the income elasticity of the marginal utility of income, the income

flexibility for short; $\eta_i = \partial(\log q_i)/\partial(\log M)$ is the income elasticity of good i ; and $w_j = p_j q_j / M$ is the budget share of j .

Let $q_i = q_i(M, p_1, \dots, p_n)$ be the demand equation for good i . Writing D for the log-change operator, Barten takes a first-order Taylor expansion in logarithms of this equation,

$$\begin{aligned} Dq_i &= \frac{\partial(\log q_i)}{\partial(\log M)} DM + \sum_{j=1}^n \frac{\partial(\log q_i)}{\partial(\log p_j)} Dp_j \\ &= \eta_i DM + \sum_{j=1}^n (\eta_{ij} - \phi \eta_i \eta_j w_j - \eta_i w_j) Dp_j \\ &= \eta_i \left[DM - \sum_{k=1}^n w_k Dp_k \right] + \sum_{j=1}^n (\eta_{ij} - \phi \eta_i \eta_j w_j) Dp_j, \end{aligned}$$

where the second step is based on (2.4). The term in square brackets is the change in real income, with the income effect of the price changes, $\sum_k w_k Dp_k$, acting as the deflator.

To simplify the substitution term in the demand equation, note that (2.2) implies

$$(2.5) \quad \sum_{j=1}^n \eta_{ij} = \phi \eta_i, \quad i = 1, \dots, n.$$

We then have

$$\sum_{j=1}^n (\eta_{ij} - \phi \eta_i \eta_j w_j) Dp_j = \sum_{j=1}^n \eta_{ij} \left[Dp_j - \sum_{k=1}^n \eta_k w_k Dp_k \right].$$

Here there is a new price deflator $\sum_k \eta_k w_k Dp_k$, the general substitution effect. The weights in this index are marginal shares as $\eta_k w_k = \partial(p_k q_k)/\partial M$.

The k^{th} marginal share answers the question, how much of a one-dollar increase in income is spent on good k ? By contrast, the weights in the income deflator are budget (or average) shares.

Barten's demand equation thus takes the form

$$(2.6) \quad Dq_i = \eta_i \left[DM - \sum_{k=1}^n w_k Dp_k \right] + \sum_{j=1}^n \eta_{ij} \left[Dp_j - \sum_{k=1}^n \eta_k w_k Dp_k \right].$$

It can be seen that η_{ij} is interpreted as the elasticity of demand for good i with respect to the deflated price of good j . In the special case when the utility function is additive in the n goods, the Hessian U and its inverse are both diagonal. This then implies that the price elasticity $\eta_{ij} = 0$ for $i \neq j$, so that each demand equation contains only the own-relative price; the other prices only enter the deflator. Similar sorts of results hold for less restrictive cases such as block independence. This direct link between the structure of the utility function and the price elasticities is a major attraction of (2.6).

The elasticities in (2.6) for $i = 1, \dots, n$ are treated as constants to be estimated. It follows from the budget constraint that a budget-share-weighted average of the income elasticities is one,

$$(2.7) \quad \sum_{i=1}^n w_i \eta_i = 1.$$

The price elasticities are subject to (2.5) and

$$(2.8) \quad w_i \eta_{ij} = w_j \eta_{ji}, \quad i, j = 1, \dots, n.$$

Constraint (2.8) follows from $\eta_{ij} = \lambda u^{ij} p_j / q_i$ and $u^{ij} = u^{ji}$.

It is to be noted that the budget shares in equations (2.6)-(2.8) are all time dependent. For estimation, Barten replaces them with sample means. Theil (1965, 1967) suggests an ingenious reparametrization of the model which deals with the budget shares more satisfactorily. Consider first the income deflator $\sum_k w_k Dp_k$. Theil replaces this with the Divisia price index,

$$(2.9) \quad DP_t = \sum_{i=1}^n \bar{w}_{it} Dp_{it},$$

where $\bar{w}_{it} = \frac{1}{2}(w_{it} + w_{i,t-1})$ is the arithmetic average of the budget share in periods t and $t-1$. The use of $\bar{w}_{it}$ seems natural given that everything is formulated in terms of a comparison of prices and quantities of this period with those of the preceding one. Also, (2.9) can be shown to be a quadratic approximation to the constant-utility price index.

Next, Theil multiplies both sides of the demand equation (2.6) by $\bar{w}_{it}$, so that the left-hand variable becomes $\bar{w}_{it} Dq_{it}$. The income elasticity η_i on the right becomes $\bar{w}_{it} \eta_i$ which is the marginal share $\partial(p_i q_i) / \partial M$, to be denoted by θ_i . This θ_i becomes the weight in the price deflator in the substitution term. Finally, η_{ij} becomes $\bar{w}_{it} \eta_{ij}$, to be denoted by ν_{ij} . The revamped demand equation is thus

$$(2.10) \quad \bar{w}_{it} Dq_{it} = \theta_i (DM_t - DP_t) + \sum_{j=1}^n \nu_{ij} (Dp_{jt} - DP_t'),$$

where $DP_t' = \sum_{i=1}^n \theta_i Dp_{it}$ is the Frisch price index. The coefficients θ_i and ν_{ij} are taken to be constants.

The coefficients of (2.10) satisfy

$$(2.11) \quad \sum_{j=1}^n \nu_{ij} = \theta_i, \quad i = 1, \dots, n,$$

$$(2.12) \quad \sum_{i=1}^n \theta_i = 1,$$

$$(2.13) \quad \nu_{ij} = \nu_{ji}, \quad i, j = 1, \dots, n,$$

which follow from (2.5), (2.7) and (2.8). The attraction of treating the θ_i 's and ν_{ij} 's as constants in (2.10) is that the constraints (2.11)-(2.13) involve only these constants, not the (time-dependent) budget shares.

The variable on the left of (2.10), $\bar{w}_{it}Dq_{it}$, has two interpretations. First, it is the quantity component of the change in the i^{th} budget share ($\Delta w_{it} = \bar{w}_{it}Dp_{it} + \bar{w}_{it}Dq_{it} - \bar{w}_{it}DM_t + \text{higher-order terms}$). Second, $\bar{w}_{it}Dq_{it}$ is the contribution of good i to the Divisia volume index of the change in real income,

$$(2.14) \quad DQ_t = \sum_{i=1}^n \bar{w}_{it}Dq_{it}.$$

In view of (2.14), if we sum both sides of (2.10) over $i = 1, \dots, n$, we get DQ_t on the left. On the right we get $DM_t - DP_t$, which follows from (2.11) - (2.13). As DQ_t is only approximately equal to $DM_t - DP_t$, in order to make (2.10) internally consistent it is now common practice to replace $DM_t - DP_t$ with DQ_t . Consequently, (2.10) becomes

$$(2.15) \quad \bar{w}_{it}Dq_{it} = \theta_i DQ_t + \sum_{j=1}^n \nu_{ij}(Dp_{jt} - DP_t).$$

Parks (1969) christened (2.15) for $i = 1, \dots, n$ the Rotterdam model.

The Rotterdam model is elegant in its simplicity. The quantity component of the change in the budget share is expressed as a linear function of the change in real income and the n relative price changes. The model

highlights the three-way decomposition of the total substitution effect into the income effect and the specific and general effects. Finally, the cross-equation constraints on the coefficients are straightforward. For further details of the model, see Barnett (1979, 1984), Byron (1984), Mountain (1988) and Theil (1975/76).

3. Aggregation Over Consumers

The demand equations considered so far refer to a single consumer. Usually, however, data are obtainable only in some aggregate form (e.g., per capita or per household). Thus, it is natural to ask to what extent do the previous results carry over to the aggregate or market demand functions? Sonnenschein (1973) concludes that there is only little left of demand theory beyond homogeneity at the aggregate level. In a similar vein, Shafer and Sonnenschein (1982) consider conditions under which a market demand function has the same characteristics as that for an individual consumer. They conclude that the required conditions are strong. Theil (1968, 1971b, 1975/76) uses a completely different approach to attack the aggregation problem, an approach which uses statistical tools of analysis. His results are considerably more optimistic and thus more useful for applications. In this section we provide a brief outline of his procedure, known as the convergence approach to aggregation.

We write the micro demand equation for good i and the c^{th} consumer ($c=1, \dots, N$) as

$$(3.1) \quad q_{ic} = q_{ic}(M_c, p_1, \dots, p_n),$$

where it is assumed that each consumer faces the same prices $p_1, \dots, p_n$. Let $\bar{q}_i = (1/N) \sum_{c=1}^N q_{ic}$ be per capita consumption of i . To obtain the per capita version of (3.1) we sum both sides of that equation over $c=1, \dots, N$ and divide by N ,

$$\bar{q}_i = \frac{1}{N} \sum_{c=1}^N q_{ic}(M_c, p_1, \dots, p_n) = f_i(M_1, \dots, M_N, p_1, \dots, p_n)$$

for some function $f_i(\cdot)$. If exact aggregation is possible, then for some function $g_i(\cdot)$ we can write the above equation in the form

$$(3.2) \quad \bar{q}_i = g_i(\bar{M}, p_1, \dots, p_n),$$

where $\bar{M}$ is per capita income. Equation (3.2) is the macro analogue of the micro equation (3.1).

In general, the function $g_i(\cdot)$ does not exist. This is the aggregation problem. For example, let us consider (3.1) in linear form without prices

$$(3.3) \quad q_{ic} = \alpha_{ic} + \beta_{ic} M_c.$$

The per capita version of this equation is

$$(3.4) \quad \bar{q}_i = \bar{\alpha}_i + \frac{\sum_{c=1}^N \beta_{ic} M_c}{\sum_{c=1}^N M_c} \bar{M},$$

where $\bar{\alpha}_i = (1/N) \sum_{c=1}^N \alpha_{ic}$. While equation (3.4) is linear in $\bar{M}$, the coefficient of $\bar{M}$ is not constant in general.

Theil (1968, 1971b), however, shows that the coefficient of $\bar{M}$ can be approximated by a constant using the convergence approach which can be

described as follows. Assume that the N consumers are independent random elements of an infinite consumer population and that the parameters $\beta_{i1}, \dots, \beta_{iN}$ are identically independently distributed and come from a probability distribution having mean β_i and a finite variance σ^2 . We shall also assume that $M_1, \dots, M_N$ are nonstochastic. Then the coefficient of $\bar{M}$ in (3.4) is a random variable with expectation

$$E \left[\frac{\sum_{c=1}^N \beta_{ic} M_c}{\sum_{c=1}^N M_c} \right] = \beta_i$$

and variance

$$E \left[\frac{\sum_{c=1}^N \beta_{ic} M_c}{\sum_{c=1}^N M_c} - \beta_i \right]^2 = \sigma^2 \frac{\sum_{c=1}^N M_c^2}{\left[\sum_{c=1}^N M_c \right]^2}.$$

As $\bar{M} = (1/N) \sum_{c=1}^N M_c$ and $\sum_{c=1}^N (M_c - \bar{M})^2 = \sum_{c=1}^N M_c^2 - N\bar{M}^2$, the above variance can be written as

$$E \left[\frac{\sum_{c=1}^N \beta_{ic} M_c}{\sum_{c=1}^N M_c} - \beta_i \right]^2 = \frac{\sigma^2}{N} \left[1 + \frac{\frac{1}{N} \sum_{c=1}^N (M_c - \bar{M})^2}{\bar{M}^2} \right] = \frac{\sigma^2}{N} (1 + \lambda^2),$$

where λ is the coefficient of variation of income. When

$(1/N) \sum_{c=1}^N (M_c - \bar{M})^2 < \infty$, $\sigma^2(1 + \lambda^2)/N = 0$ as $N \rightarrow \infty$, so that

$$(3.5) \quad \lim_{N \rightarrow \infty} E \left[\frac{\sum_{c=1}^N \beta_{ic} M_c}{\sum_{c=1}^N M_c} - \beta_i \right]^2 = 0.$$

In view of result (3.5), Chebyshev's inequality implies that the 'coefficient' of $\bar{M}$ in equation (3.4), $\sum_{c=1}^N \beta_{ic} M_c / \sum_{c=1}^N M_c$, converges in probability to β_i . Consequently, for a large population of consumers (N), equation (3.4) becomes

$$\bar{q}_i \approx \bar{a}_i + \beta_i \bar{M}.$$

This is a linear macro analogue of the micro equation (3.3). It involves the per capita variables $\bar{q}_i$ and $\bar{M}$ and has constant coefficients. Consequently, under the convergence approach, there is no problem of aggregation bias.

Using the convergence approach, Theil (1971b, 1975/76) shows that (2.15), the micro equation of the Rotterdam model, can be aggregated into the corresponding macro form. For further results, see Barnett (1979) and E. A. Selvanathan (1988).

4. Testing Homogeneity and Symmetry

The general restrictions of consumption theory are homogeneity and symmetry. In contrast to the LES, these are testable hypotheses in the context of the Rotterdam model. This section contains a discussion of Theil's work in this important area.

Equation (2.15), the Rotterdam model, is formulated in terms of relative (or deflated) prices. For homogeneity and symmetry testing, it is

convenient to reformulate the model in absolute (or undeflated) prices by substituting $\sum_{i=1}^n \theta_i Dp_{it}$ for the Frisch price index DP'_t in (2.15). Using (2.11), this yields

$$(4.1) \quad \bar{w}_{it} Dq_{it} = \theta_i DQ_t + \sum_{j=1}^n \pi_{ij} Dp_{jt},$$

where

$$(4.2) \quad \pi_{ij} = \nu_{ij} - \phi \theta_i \theta_j$$

is the $(i,j)^{th}$ Slutsky coefficient. This coefficient deals with the total substitution effect, whereas ν_{ij} is concerned with the specific part only. Equation (4.1) is known as the absolute price version of the Rotterdam model, while (2.15) is the relative price version.

Summation of both sides of (4.2) over $j = 1, \dots, n$ gives $\sum_{j=1}^n \pi_{ij} = \sum_{j=1}^n \nu_{ij} - \phi \theta_i$, in view of (2.12). Using (2.11), we have

$$(4.3) \quad \sum_{j=1}^n \pi_{ij} = 0, \quad i = 1, \dots, n.$$

This constraint is known as demand homogeneity, which means that an equiproportional increase in all prices has no effect on consumption when real income is held constant. It is to be noted that constraint (4.3) has the attractive feature of being linear in the Slutsky coefficients.

Given that the ν_{ij} 's are symmetric in i and j [see (2.13)], equation (4.2) implies that the Slutsky coefficients share this property,

$$(4.4) \quad \pi_{ij} = \pi_{ji}, \quad i, j = 1, \dots, n.$$

This is known as Slutsky symmetry, i.e. the total substitution effect is symmetric in i and j . This constraint is also linear in the π_{ij} 's.

Adding a disturbance term ϵ_{it} to (4.1), this equation can be written for $t = 1, \dots, T$ (the sample size) as

$$(4.5) \quad y_i = X \beta_i + \epsilon_i ,$$

where $y_i = [\bar{w}_{it} Dq_{it}]$ is a T -vector; X is a $T \times (n+1)$ matrix whose t^{th} row is $(DQ_t, Dp_{1t}, \dots, Dp_{nt})$; $\beta_i = [\theta_i \ \pi_{i1} \dots \pi_{in}]'$; and $\epsilon_i = [\epsilon_{it}]$. The homogeneity constraint for good i then takes the form $v' \beta_i = 0$, where $v = [0 \ 1 \dots 1]'$. The budget constraint means that we need only consider (4.5) for $i = 1, \dots, n-1$, which, using an obvious notation, we write as

$$(4.6) \quad y = (I_{n-1} \otimes X) \beta + \epsilon .$$

Homogeneity of the entire system then takes the form

$$(4.7) \quad R\beta = 0 ,$$

where $R = I_{n-1} \otimes v'$.

Let b be the least-squares (LS) estimator of β in (4.6). This b consists of $n-1$ subvectors of the form $b_i = (X'X)^{-1}X'y_i$. Then $e_i = y_i - Xb_i$ is the LS residual vector of the i^{th} equation. It is assumed that ϵ is normally distributed with zero mean and covariance matrix $\Sigma \otimes I_T$, where Σ is the covariance matrix of $(\epsilon_{1t}, \dots, \epsilon_{n-1,t})$ for each $t = 1, \dots, T$. The matrix $S = (T-n-1)^{-1} [e_i' e_j]$ is an unbiased estimator of Σ .

Under (4.7) and the above assumptions,

$$(4.8) \quad \frac{b'R'\Sigma^{-1}Rb/v'(X'X)^{-1}v}{\text{tr } \Sigma^{-1}S}$$

has an F distribution. The standard procedure used to apply (4.8) is to replace the unknown Σ matrix with S. The test statistic then reduces to

$$(4.9) \quad \frac{b'R'S^{-1}Rb}{v'(X'X)^{-1}v},$$

which has an asymptotic χ^2 distribution.

The test of symmetry given homogeneity involves expressing π_{in} as $\sum_{j=1}^{n-1} \pi_{ij}$, so that (4.1) becomes

$$(4.10) \quad \bar{w}_{it}Dq_{it} = \theta_i DQ_t + \sum_{j=1}^{n-1} \pi_{ij} (Dp_{jt} - Dp_{nt}).$$

Then, with an appropriate reinterpretation of the notation, under (4.4)

$$(4.11) \quad \frac{b'R'\{R[\Sigma \otimes (X'X)^{-1}]R'\}^{-1}Rb}{[q/(n-1)]\text{tr } \Sigma^{-1}S}$$

has an F distribution where $q = \frac{1}{2} (n-1)(n-2)$. When Σ is replaced with S, as before, (4.11) becomes

$$(4.12) \quad b'R'\{R[S \otimes (X'X)^{-1}]R'\}^{-1}Rb.$$

Asymptotically this test statistic follows a χ^2 distribution.

The first application of essentially this approach was by Barten (1967) who used Dutch data with $n = 4$ commodities. He found little evidence to reject homogeneity and symmetry. Subsequent work, however, was much less encouraging with rejections becoming very common, particularly for larger systems (larger n). This evidence was obtained not only in the context of the

~~Rotterdam model, but with other models also, e.g. the translog. See Barten~~
(1977) for a survey.

These rejections represented a major puzzle as homogeneity and symmetry would seem to represent highly plausible economic restrictions. The response of Theil, and some others, was not to take these rejections too literally. He argued in Theory and Measurement of Consumer Demand that these findings reflect

"data imperfections rather than unorthodox consumer behavior. What is needed in such a situation is an explicit recognition on the part of the analyst that his data are imperfect. He should guide these data so that they yield a sensible picture. It is unavoidable that this picture of the real world will be biased toward simplicity, but it is appropriate to realize that this kind of bias is inherent in every model." (Theil, 1975/76 pp. 250-51 of Volume 1)

As a consequence, Theil felt comfortable simply imposing the constraints, especially homogeneity. [Others have responded to the rejection puzzle by emphasizing dynamics (Anderson and Blundell, 1983), simultaneity (Attfield, 1985) and the use of nonstationary variables (Bewley and Elliott, 1989).]

Theil's scepticism about the rejections was not shared by everyone. Some took the findings seriously and concluded that consumption theory should be rejected (see, e.g., Christensen et al., 1975). Apparently, this troubled Theil sufficiently to encourage two Chicago doctoral students, Laitinen and Meisner, to study the rejections by means of Monte Carlo experiments. The idea is to simulate data sets under the null, use these data to estimate the demand equations and then test homogeneity and symmetry with (4.9) and (4.12). As the hypotheses are true by construction, the rejection rates should coincide with the nominal significance levels if the tests are working satisfactorily.

Table 1 shows Laitinen's (1978) and Meisner's (1979) results. As can be seen from columns 2 and 3, the rejections of homogeneity are far too high, with the rejection rate increasing with n . The results for symmetry (columns 4 and 5) also indicate a substantial bias against the null, but the problem is now not quite so bad for $n = 5$ or 8. The source of the problems with these tests is that they have only an asymptotic justification when Σ is replaced with S . The exact tests (4.8) and (4.11), which involve Σ rather than S , perform quite satisfactorily in simulations. See Laitinen (1978), Meisner (1979) and Theil (1987b) for further details. For related contributions to this topic, see Bera et al. (1981) and Bewley (1983, 1986).

Table 1
PERCENTAGE REJECTIONS OF HOMOGENEITY AND SYMMETRY

Number of commodities, n	Rejections of homogeneity		Rejections of symmetry	
	5 percent	1 percent	5 percent	1 percent
(1)	(2)	(3)	(4)	(5)
5	14	6	9	3
8	30	16	26	8
11	53	35	50	37
14	87	81	96	91

If the standard tests are defective, how does one proceed? For homogeneity testing, there is now no need to use asymptotics at all as Laitinen (1978) also derives an exact finite-sample test. For symmetry, Theil and Rosalsky (1984b) propose the use of nonstochastic matrices rather than S in the test statistic (4.12). These are an augmented identity matrix $\Sigma^* = [\delta_{ij} \quad 1/n]$, where δ_{ij} is the Kronecker delta; and $\Sigma^{**} = [\bar{w}_i(\delta_{ij} - \bar{w}_j)]$, where $\bar{w}_i$ is the sample mean of the budget share of good i . Simulations indicate that both these matrices perform well for symmetry testing and have at least tolerable power.

As another way of avoiding the problems of asymptotic tests, Theil (1987b) introduces a Monte Carlo technique which is distribution-free. This involves simulating a large number of values of the test statistic under the null hypothesis to construct its empirical distribution. The observed value of the test statistic is then compared to this distribution, rather than its asymptotic counterpart. This approach has great flexibility with respect to the choice of the test statistic and should attract attention in the future.

5. Estimation

System (4.6) is linear in the parameter vector β . This β is subject to the Slutsky symmetry constraint (4.4). (Note that in view of the adding-up property of the model, symmetry implies homogeneity.) The symmetry-constrained estimation of (4.6), and other demand systems linear in parameters, would seem to be a straightforward task involving seemingly unrelated regression techniques. However, influential work by Theil and his collaborators has established that this is not the case (Fiebig and Theil, 1983; Theil, 1987b; and Theil and Rosalsky, 1984a).

Recall from the previous section that when the true covariance matrix Σ is replaced with its LS estimator S , the test statistics for homogeneity and symmetry are defective as they are seriously biased against the null. Theil's group has shown that there are analogous difficulties with symmetry-constrained estimation when S is used in place of Σ . To illustrate these difficulties, we return to equation (4.10) where homogeneity is imposed. Adding an error term and then reinterpreting the notation, we write this equation for $t = 1, \dots, T$ and $i = 1, \dots, n-1$ as (4.6). The symmetry constraint (4.4) then can be expressed in the form (4.7). Let b be the unrestricted LS estimator of β and $C = \Sigma \otimes (X'X)^{-1}$. Then, the

symmetry-constrained GLS estimator is

$$(5.1) \quad \hat{\beta} = b - CR'(R'CR)^{-1}Rb,$$

with covariance matrix

$$(5.2) \quad \text{var } \hat{\beta} = C - CR'(RCR')^{-1}RC.$$

If Σ is known, as is the case in a Monte Carlo experiment, then (5.1) and (5.2) can be applied directly. Columns 4-6 of Table 2, from Fiebig and Theil (1983), show the results of such an experiment with $n = 8$ commodities. (A cross-country variant of the absolute price version of the Rotterdam model is used in this experiment.) As before, artificial data sets are generated by the true model and these are used to reestimate that model a number of times. As is clear, for each parameter bias is small and the root-mean-squared error (RMSE) is close to the root-mean-squared standard error (RMSSE). So far, so good.

When S is used in place of Σ in (5.1) and (5.2), so that C becomes $S \otimes (X'X)^{-1}$, the standard errors become only approximate. Columns 7-9 of Table 2 indicate that using S has dramatic effects. Now the RMSs of the approximate standard errors (the RMSASEs) are all substantially below the corresponding RMSEs. The effects of estimating the covariance matrix can be seen by comparing column 5 with 8 and 6 with 9. This comparison reveals that the RMSEs increase substantially when S is used, indicating an efficiency problem. In addition, the RMSs of the approximate standard errors are all below the corresponding RMSSEs of column 6. Consequently, the approximate standard errors understate the true variability of the estimates. Fiebig and Theil (1983) refer to these problems as "the two perils of symmetry-constrained estimation".

On the basis of simulation results, Theil and Rosalsky (1984a) recommend replacing S with Σ^{**} defined in the previous section. This avoids the two perils almost entirely.

Table 2
SYMMETRY-CONSTRAINED ESTIMATION, 500 TRIALS

Index		True value	Using Σ			Using S		
i	j		Bias	RMSE	RMSSE	Bias	RMSE	RMSASE
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Diagonal Slutsky coefficients π_{ii}								
1	n.a.	-1061	-26	801	823	-75	1109	636
2	n.a.	-502	-3	175	174	2	223	134
3	n.a.	-785	-11	303	304	-8	331	277
4	n.a.	-546	-19	495	480	-56	818	309
5	n.a.	-492	-3	74	77	-3	94	63
6	n.a.	-580	9	447	459	4	510	404
7	n.a.	-713	5	220	211	-15	339	148
Off-diagonal Slutsky coefficients π_{ij}								
1	2	121	9	271	270	3	385	199
1	3	203	9	314	324	16	379	272
1	4	133	22	499	495	66	805	338
1	5	119	9	183	182	1	238	140
1	6	142	-10	470	450	-28	546	362
1	7	181	-20	306	309	-4	486	209
2	3	82	8	142	145	7	176	125
2	4	54	-13	198	196	-20	313	133
2	5	48	8	83	83	16	115	63
2	6	58	4	196	201	20	243	163
2	7	73	-7	141	138	-13	212	96
3	4	90	-5	200	195	-5	256	169
3	5	80	-2	97	98	-5	120	85
3	6	96	3	257	260	0	287	225
3	7	123	-5	158	157	-5	201	138
4	5	53	0	124	127	4	184	91
4	6	63	9	283	293	24	411	222
4	7	80	12	244	244	-11	408	148
5	6	56	-15	134	141	-17	172	113
5	7	72	3	92	93	11	128	71
6	7	86	8	206	209	28	280	162

All entries are to be divided by 10^4 .

~~The material of this and the previous section shows how simulation~~
experiments can be used to analyse the performance of econometric procedures under ideal circumstances where everything is known. Theil (1987b) views simulation, particularly when applied to systems of equations, as an important new extension of econometrics. Originally, this field consisted of two components: econometric methodology and data analysis. Theil argues that simulation is now a third component of econometrics.

6. Preference Independence

When the consumer's tastes can be described by means of a utility function which can be written as the sum of n sub-utility functions, each involving one good only, then tastes are said to exhibit preference independence. Formally, the utility function is of the preference independent form if

$$(6.1) \quad u(q_1, \dots, q_n) = \sum_{i=1}^n u_i(q_i),$$

so that the marginal utility of good i is independent of the consumption of j , $i \neq j$. Consequently, the Hessian matrix of utility function (6.1) and its inverse are both diagonal.

Under preference independence, the coefficients of the other relative prices in demand equation (2.15), the ν_{ij} for $i \neq j$, are all zero. Constraint (2.11) becomes $\nu_{ii} = \phi\theta_i$, so that (2.15) takes the form

$$(6.2) \quad \bar{w}_{it} Dq_{it} = \theta_i DQ_t + \phi\theta_i (Dp_{it} - DP'_t).$$

The advantage of the preference independence assumption is that it is the simplest structure of preferences. The number of unknown coefficients to be estimated in a n -commodity demand system is something of the order n^2 (each of the n demand equations has n prices, so that there is a $n \times n$ substitution matrix). Under preference independence, the number of coefficients falls to n .

The assumption of preference is strong in that it severely restricts the substitution possibilities. Although it has been argued that the assumption will have more chance of success when the commodities are broad aggregates, in his earlier work with such aggregates Theil seemed to be skeptical of the validity of preference independence (see, e.g., Theil, 1975/76). Moreover, when the hypothesis was tested it tended to be rejected.

Perhaps because of dissatisfaction with the asymptotic basis for the tests of preference independence and a general feeling that most data are not capable of revealing too much about subtle substitution possibilities, Theil seems to have subsequently changed his position on the status of preference independence. In more recent work Theil is now willing to simply impose preference independence on a fairly routine basis (see, e.g., Theil, 1987a; Theil et al., 1989; and Theil and Suhm, 1981). It is of interest to note that very recent tests of preference independence, which do not involve asymptotics, tend to show that the hypothesis is at least not violently incompatible with the data in 18 OECD countries (S. Selvanathan, 1987).

7. The Strong Version of Engel's Law

One of the major empirical regularities in consumption economics is Engel's law, i.e., the budget share of food falls with increasing income. Theil et al. (1989) show that it is now possible to make a more precise

~~statement regarding the magnitude of the fall in this share.~~

Working (1943) observed that the budget share for food seems to be a linear function of the logarithm of income. This observation was generalized by Leser (1963) to a complete set of n commodities to yield

$$(7.1) \quad w_i = \alpha_i + \beta_i \log M, \quad i=1, \dots, n.$$

Choosing the income unit such that $M = 1$ for some household, α_i is then interpreted as the budget share of i for that household. The coefficient β_i gives 100 times the change in the budget share of i resulting from a 1 percent increase in income. We shall refer to (7.1) as Working's model.

Theil (1987a), using data from Kravis et al. (1982) for 34 countries, plots the budget share of food against the log of real per capita total consumption. Figure 1 gives such a plot (with consumption scaled such that $M = 1$ for the poorest country). The solid line is the LS regression line. As can be seen, the points are all scattered around a downward sloping line which gives strong visual support for Working's model for food. The slope of the LS line is $-.15$ (standard error $.01$) which is an estimate of β_i for food. This estimate does not take account of cross-country differences in the relative price of food; however, approximately the same value of β_i emerges when these differences are allowed for (Theil, 1987a). The estimate of β_i of $-.15$ is in remarkable agreement with other estimates of the food β_i ; see Table 3. Thus $\beta_i = -.15$ seems to be more or less a natural constant.

Figure 1

Budget Share of Food Against Scaled Total Consumption Expenditure
Per Capita in 1975 for 34 Countries

Table 3

PREVIOUS ESTIMATES OF WORKING'S INCOME COEFFICIENT FOR FOOD

Author(s)	Country	Estimate
Aasness and Rodseth (1983)	Norway	-.17
Blanciforti and Green (1983)	U.S.	-.13
Chung and Lopez (1988)	Spain	-.18 & -.16
Deaton and Muellbauer (1980)	Britain	-.16
Finke et al. (1984)	Japan	-.15
Musgrove (1985)	Dominican Republic	-.14
Theil (1987a)	Cross country	-.14 to -.16
Theil and Finke (1984)	Netherlands	-.13
Theil et al. (1987)	China	-.13

Source: Chung and Lopez (1988, Table 1).

To analyse the implications of this β_i estimate, consider moving from one country to another. Assume that the first country's per capita income is M , while that of the second is $2M$. Therefore,

$$\Delta(\log M) = \log 2M - \log M = \log 2 = .69.$$

From (7.1) we have

$$\Delta w_i = \beta_i \Delta \log M.$$

Consequently, when $\beta_i = -.15$ the effect of doubling income on the food budget share is

$$\Delta w_i = -.15 \times .69 = -.10.$$

That is, when income doubles, the budget share declines by 10 percentage points. Theil et al. (1989) refer to this as the strong version of Engel's law.

One use of this law is to make real income comparisons across consumers or countries on the basis of the food budget share. As the food share is obviously a pure number, it is directly comparable over time, countries etc. independent of inflation, exchange rates and so on. Consequently, such indirect income comparisons could be an attractive short-cut.

8. Other Topics

In this section we briefly discuss some of Theil's other contributions to demand analysis.

Conditional Demand Analysis

The assumption of preference independence (or some other form of separability) is one way of reducing the number of unknown coefficients in the demand equations. Another is to focus on a subgroup of goods, rather than all n goods. An example is the meats subgroup comprising beef, pork, chicken and lamb. When the subgroup is separable in the utility function, the demand equations for members of the group have the same form as (2.15) or (4.1); all that is required is a reinterpretation of the notation ($\bar{w}_{it}$ becomes the budget share of i within the subgroup, DQ_t the Divisia volume index of the subgroup and so on). These are known as conditional demand equations as they refer to consumption within the subgroup.

Theil and his associates have contributed to conditional demand theory (see, e.g., Barten and Turnovsky, 1966; and Theil, 1975/76). As an illustrative application, Table 4 presents estimates of conditional demand equations for meats, from Theil (1975/76). The estimates of the conditional marginal shares imply that when the meat expenditure increase by \$1, consumers spend 75 cents of that increase on beef, 15 cents on pork, 5 cents on chicken and the remaining 5 cents on lamb. All the diagonal elements of the Slutsky matrix are negative, as they should be, and the off-diagonal elements are positive, indicating that the four meats are all pairwise substitutes.

Table 4

CONDITIONAL DEMAND EQUATIONS FOR MEATS:
HOMOGENEITY AND SYMMETRY CONSTRAINED ESTIMATES
UNITED STATES, 1950-1971

Meat	Conditional marginal share	Conditional Slutsky coefficients $\times 10$			
		Beef	Pork	Chicken	Lamb
Beef	.746	-.137	.100	.021	.016
Pork	.146		-.127	.020	.007
Chicken	.051			-.048	.007
Lamb	.057				-.030

Modeling the Disturbances of Behavioural Equations

Suppose that prices and the consumer's real income are held constant and that this consumer is subject to some form of random shock. If this shock causes consumption of good i to increase and that of j to fall, what can be said about the substitutability/complementarity relationship between i and j ? As more of good i has been compensated by less of j , this would seem to indicate that both goods satisfy the same type of want of the consumer. In other words, the goods are competitive or substitutes. This line of argument indicates that goods whose random components are negatively correlated are substitutes; complements have positive correlations.

This idea has a long history in consumption economics, going back to Allen and Bowley (1935). Theil and his associates have elaborated the concept to provide an elegant general theory for any decision maker under uncertainty, known as rational random behaviour (Barbosa, 1974; Theil, 1967, 1971a, 1974a, 1974b; and Theil and Neudecker, 1957). When applied to the

The Rotterdam model, the result which emerges is that the covariance matrix of the disturbances is proportional to the Slutsky matrix $[\pi_{ij}]$.

Rational random behaviour implies that if we plot the variances of the error terms ($\text{var } \epsilon_{it}$) against a negative multiple of the own-Slutsky coefficients ($-k\pi_{ii}$, $k>0$), the $i=1, \dots, n$ points should be scattered around a straight line coming out of the origin. If we use a double-log scale, the relationship becomes linear with a unit slope. Figure 2 (from Theil, 1975/76) presents such a scatter for Dutch data with $n = 14$. As can be seen, this evidence is quite encouraging.

Figure 2
Evidence on Rational Random Behaviour

The Preference Independence Transformation

The concept of preference independence is attractive in its simplicity as each demand equation contains only the own-relative price; see equation (6.2). However, it may not be realistic with respect to narrowly defined goods. In these cases, Brooks (1970) and Theil (1967, 1975/76) propose a transformation whereby observed goods are changed into transformed goods in such a way that the latter are preference independent. This involves taking an estimated demand system and then diagonalizing the Hessian matrix of the utility function subject to the restriction that the transformed goods satisfy the budget constraint.

The transformed goods are linear combinations of the observed goods. Thus, bread and cheese, for example, become breeese and chead (Theil, 1967). As the utility function is additive in the transformed goods, they are identified as the "basic wants" of the consumer. For further details, see Theil (1975/76, 1977) and Theil and Laitinen (1981).

APPENDIX

Theil's Publications in Demand Analysis*Books and monographs*

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System-Wide Explorations in International Economics, Input-Output Analysis, and Marketing Research. New York: Elsevier/North-Holland, Inc., and Amsterdam: North-Holland Publishing Company, 1980.

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Applied Demand Analysis: Results from System-Wide Approaches (with K.W. Clements). Cambridge, Massachusetts: Ballinger Publishing Company, 1987.

International Evidence on Consumption Patterns (with C-F. Chung and J.L. Seale, Jr.). Greenwich, Connecticut: JAI Press, Inc., 1989.

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1957

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1960

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1962

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1964

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1965

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1980

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