THE RELIABILITY OF ML ESTIMATORS OF
SYSTEMS OF DEMAND EQUATIONS:
EVIDENCE FROM 18 COUNTRIES

by

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Abstract

Theil (1987) shows that, in large demand systems, when the unknown error covariance matrix is approximated by its ML estimator, the coefficient estimates suffer from 'two perils', (i) the asymptotic standard errors severely understate the sampling variability of the estimates; and (ii) the efficiency of the ML coefficient estimates is greatly impaired. In this paper we evaluate the performance of an alternative estimator of the covariance matrix. Using time-series data for 18 OECD countries we find that there is a spectacular improvement.

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1. INTRODUCTION

Maximum likelihood (ML) estimators of systems of demand equations are consistent and asymptotically efficient. As these properties are asymptotic, their application in small-sample situations is, however, open to question (see Barten, 1977; Bera et al., 1981; Bewley, 1983; Bewley, 1986; Byron, 1970; Deaton, 1972; Laitinen, 1978; and Meisner, 1979). Much progress has recently been made in analysing this question; these developments have been synthesized and extended by Theil (1987). Theil shows that, in large systems, when the unknown error covariance matrix Σ is approximated by its usual ML-estimator S , the matrix of mean squares and cross products of the residuals, the estimates suffer from two problems. The first is that the asymptotic standard errors severely understate the sampling variability of the estimates. The second problem is that the ML coefficient estimates have an impaired efficiency. Theil refers to these difficulties as the 'two perils' of estimating large demand models. He shows that the cause of the two perils is not knowing the true value of Σ . (By contrast, however, E.A. Selvanathan, 1987, finds that the two perils are not so serious for smaller demand systems.)

Selvanathan (1987a) estimates Working's (1943) model under preference independence for 10 commodities using ML for 18 OECD countries. As the number of annual observations is less than 25 in most countries, the model can probably be considered to be large. In an attempt to avoid the above-mentioned two perils, rather than using S , Selvanathan (1987a) proposes an alternative specification for the covariance matrix. The main aim of this paper is to verify whether or not this alternative specification avoids the two perils. To do this we follow Theil (1987) and use Monte Carlo simulation procedures.

The structure of this paper is as follows. In Section 2 we briefly describe the OECD database. Section 3 describes the demand equations.

Section 4 presents the alternative specification for the covariance matrix and the estimation results for the 18 OECD countries. In Section 5 we estimate the demand equations under the assumption that the parameters are the same across countries. Sections 6-9 describe the simulation procedure and discuss the simulation results. Finally, in Section 10 we give our concluding comments.

2. THE DATABASE

We use time-series data from 18 OECD countries. The countries considered are the U.S. (where there are 21 annual observations after lagging), Canada (21), Sweden (17), Switzerland (21), Denmark (15), Australia (21), France (17), Germany (21), Belgium (21), Norway (17), the Netherlands (25), Iceland (13), Finland (17), Austria (17), Japan (11), the U.K. (17), Spain (13) and Italy (17).

For 15 countries, goods and services are classified into 10 commodity groups, namely food, beverages, clothing, housing, durables, medical care, transport, recreation, education and miscellaneous. Switzerland has 9 commodities as education is included in recreation. The two remaining countries, Germany and Japan, have 8 commodities, as beverages are included in food and education is included in recreation. For further details of the data, see Selvanathan (1987c).

3. THE DEMAND EQUATIONS

Let p_{it}^c be the price and q_{it}^c the per capita quantity consumed of good i during year t in country c . Let there be n^c goods in country c . Total expenditure is then $M_t^c = \sum_{i=1}^{n^c} p_{it}^c q_{it}^c$ for $c=1, \dots, 18$ countries and $t=1, \dots, T^c$, where T^c is the sample size for country c . The proportion of total expenditure devoted to commodity i , called the budget share of i , is $w_{it}^c = p_{it}^c q_{it}^c / M_t^c$.

The demand equation for good i of Working's model under preference independence takes the form (Clements, 1987)

$$(3.1) \quad \bar{w}_{it}^c (Dq_{it}^c - DQ_t^c) = \alpha_i^c + \beta_i^c DQ_t^c + \phi^c (\bar{w}_{it}^c + \beta_i^c) \left[Dp_{it}^c - \sum_{j=1}^{n^c} (\bar{w}_{jt}^c + \beta_j^c) Dp_{jt}^c \right] + \epsilon_{it}^c.$$

where $\bar{w}_{it}^c = \frac{1}{2}(w_{it}^c + w_{i,t-1}^c)$ is the arithmetic average of the budget share of commodity i in years $t-1$ and t ; $Dq_{it}^c = \log q_{it}^c - \log q_{i,t-1}^c$ and $Dp_{it}^c = \log p_{it}^c - \log p_{i,t-1}^c$ are the quantity and price log-changes of commodity i ; $DQ_t^c = \sum_{i=1}^{n^c} \bar{w}_{it}^c Dq_{it}^c$ is the Divisia volume index; ϵ_{it}^c is the disturbance term of the i^{th} equation; α_i^c is the i^{th} constant term; β_i^c is the i^{th} income coefficient; and ϕ^c is the income flexibility, the reciprocal of the income elasticity of the marginal utility of income.

4. SPECIFYING THE COVARIANCE MATRIX AND THE ESTIMATION RESULTS

In general, the contemporaneous covariance matrix Σ^c of the disturbance terms of the demand equations for country c is unknown. It is common practice to approximate Σ^c by its consistent estimator S^c , the matrix of mean squares and cross products of the ML residuals. Laitinen (1978) showed that for large demand systems without a sufficiently large sample, S^c becomes singular or ill-conditioned. Laitinen also proved that a necessary and sufficient condition for the non-singularity of S^c is that $T^c - 2n^c \geq 0$, where T^c is the sample size and n^c is the number of equations in the system. Going back to Section 2, it can be easily seen that for 11 of our 18 OECD countries $T^c - 2n^c$ is negative; that is, S^c will be singular for those countries. Also among all countries, the largest sample size is 25 (for the Netherlands), but even this cannot be considered a large sample for a model with 10 equations. Therefore, the two perils would probably cause serious problems in such large systems if we used S^c .

To avoid these problems we specify the covariance matrix for country c as

$$(4.1) \quad \Sigma^{c*} = \lambda^{c2} \Omega^c,$$

where $\Omega^c = \bar{W}^c - \bar{w}^c \bar{w}^{c'};$ $\bar{W}^c = \text{diag}[\bar{w}_1^c, \dots, \bar{w}_{n^c-1}^c];$ $\bar{w}^c = [\bar{w}_1^c \dots \bar{w}_{n^c-1}^c]';$ $\bar{w}_i^c = (1/T^c) \sum_{t=1}^{T^c} \bar{w}_{it}^c$ is the sample mean of $\bar{w}_{it}^c$; and λ^{c2} is a parameter to be estimated. It can be easily shown that specification (4.1) allows for larger error variances for goods which occupy larger shares of the budget. (See Selvanathan, 1987a, for details.) This specification has also been previously considered by Deaton (1975) and E.A. Selvanathan (1985).

Since model (3.1) for $i=1, \dots, n^c$ is nonlinear in the parameters Selvanathan (1987a) estimates it by ML for the 18 countries with Σ^{c*} . For comparison, Selvanathan also estimates (3.1) with S^c for the $18 - 11 = 7$ countries where that matrix is non-singular.

Table 4.1 presents the ML estimates of the income coefficients ($\hat{\beta}_i^c$), the income flexibility ($\hat{\phi}^c$) and the constants ($\hat{\alpha}_i^c$) for the 7 countries with non-singular S^c . Tables 4.2 and 4.3 present the estimation results for all 18 countries when $\Sigma^c = \Sigma^{c*}$. Looking at the estimates of the income coefficients in Tables 4.1 and 4.2 we see that for food and housing the values are negative for all countries for both specifications of the covariance matrix, implying that these commodities are necessities. Also we see that the income flexibilities are all negative (as expected) in both cases; and that the β_i^c 's for durables are always positive implying that they are a luxury. Also, all but one of the income coefficients for clothing are positive. An entry-by-entry comparison of the estimates for the seven countries common to Tables 4.1, 4.2 and 4.3 shows that in most cases they are not too different. Consequently, in general, the use of Σ^{c*} does not seem to have any radical, systematic implications.

TABLE 4.1

ML ESTIMATES OF DEMAND PARAMETERS FOR 10 COMMODITIES IN 7 COUNTRIES

(Asymptotic standard errors are in parentheses)

Country	Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous	Income flexibility
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
<u>Income coefficients $\hat{\beta}_i^c$ and income flexibility $\hat{\phi}^c$</u>											
1. U.S.	-.058 (.017)	-.034 (.004)	.058 (.015)	-.111 (.011)	.072 (.010)	-.078 (.014)	.149 (.034)	.041 (.011)	-.007 (.004)	-.032 (.014)	-.689 (.053)
2. Canada	-.011 (.013)	-.027 (.006)	.007 (.014)	-.172 (.014)	.078 (.014)	.025 (.036)	.032 (.020)	.069 (.013)	-.004 (.006)	.045 (.019)	-.904 (.070)
3. Switzerland	-.020 (.019)	.045 (.015)	.058 (.015)	-.181 (.022)	.087 (.011)	-.042 (.010)	.083 (.030)	-.000 (.013)		-.030 (.015)	-.589 (.050)
4. Australia	-.139 (.019)	-.009 (.010)	.016 (.020)	-.005 (.024)	.052 (.041)	.053 (.026)	-.013 (.035)	.074 (.028)	.002 (.008)	-.031 (.015)	-.404 (.046)
5. Germany	-.101 (.025)		.056 (.014)	-.131 (.021)	.049 (.016)	-.009 (.005)	.162 (.028)	-.001 (.011)		-.024 (.008)	-.627 (.082)
6. Belgium	-.092 (.021)	-.001 (.013)	.042 (.014)	-.104 (.010)	.194 (.026)	-.146 (.010)	-.020 (.021)	.005 (.010)	-.002 (.000)	.025 (.017)	-.608 (.050)
7. Netherlands	-.138 (.031)	-.017 (.010)	.132 (.027)	-.046 (.015)	.058 (.037)	-.010 (.010)	.051 (.015)	.011 (.010)	-.016 (.007)	-.024 (.018)	-.782 (.084)
<u>Constant terms $\hat{\alpha}_i^c$ ($\times 100$)</u>											
1. U.S.	-.102 (.062)	.043 (.015)	-.264 (.055)	.383 (.034)	-.255 (.038)	.397 (.040)	-.229 (.117)	-.065 (.040)	.041 (.011)	.050 (.042)	
2. Canada	-.153 (.062)	.048 (.029)	-.098 (.071)	.552 (.062)	-.299 (.055)	-.213 (.149)	.039 (.093)	-.143 (.060)	.077 (.027)	.190 (.059)	
3. Switzerland	-.070 (.060)	-.148 (.047)	-.267 (.046)	.381 (.068)	-.328 (.031)	.168 (.022)	.030 (.080)	.077 (.036)		.159 (.033)	
4. Australia	.104 (.068)	-.099 (.033)	-.213 (.063)	.249 (.074)	-.077 (.153)	-.111 (.069)	.177 (.118)	-.057 (.080)	-.028 (.017)	.055 (.044)	
5. Germany	-.086 (.099)		-.264 (.053)	.451 (.066)	-.186 (.062)	.004 (.018)	-.140 (.110)	.060 (.043)		.162 (.028)	
6. Belgium	-.103 (.097)	-.043 (.054)	-.229 (.061)	.308 (.071)	-.562 (.126)	.300 (.051)	.227 (.081)	.027 (.036)	.004 (.001)	.071 (.117)	
7. Netherlands	.067 (.163)	.046 (.067)	-.786 (.153)	.223 (.078)	-.157 (.224)	.285 (.056)	-.045 (.096)	.005 (.044)	.118 (.029)	.245 (.083)	

TABLE 4.2

ML ESTIMATES OF THE INCOME COEFFICIENTS FOR 10 COMMODITIES

AND THE INCOME FLEXIBILITY IN 18 COUNTRIES

(Asymptotic standard errors are in parentheses)

Country	Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous	Income flexibility
(1)	$\hat{\beta}_1^c$	$\hat{\beta}_2^c$	$\hat{\beta}_3^c$	$\hat{\beta}_4^c$	$\hat{\beta}_5^c$	$\hat{\beta}_6^c$	$\hat{\beta}_7^c$	$\hat{\beta}_8^c$	$\hat{\beta}_9^c$	$\hat{\beta}_{10}^c$	$\hat{\phi}^c$
1. U.S.	-.055 (.019)	-.031 (.014)	.027 (.021)	-.115 (.026)	.054 (.017)	-.059 (.021)	.209 (.024)	.014 (.018)	-.003 (.009)	-.041 (.023)	-.384 (.066)
2. Canada	-.007 (.037)	-.025 (.022)	-.015 (.029)	-.183 (.041)	.050 (.034)	.055 (.020)	.084 (.036)	.051 (.027)	.001 (.014)	-.011 (.044)	-.554 (.120)
3. Sweden	-.094 (.025)	.008 (.015)	.038 (.022)	-.190 (.032)	.067 (.021)	-.022 (.005)	.138 (.026)	.054 (.021)	-.001 (.001)	.002 (.021)	-.580 (.088)
4. Switzerland	-.007 (.025)	.033 (.017)	.055 (.016)	-.153 (.020)	.086 (.019)	-.046 (.014)	.063 (.021)	.001 (.023)		-.033 (.022)	-.551 (.075)
5. Denmark	-.123 (.032)	-.028 (.018)	.044 (.016)	-.130 (.024)	.053 (.017)	-.010 (.008)	.188 (.023)	.019 (.018)	-.009 (.005)	-.005 (.018)	-.465 (.084)
6. Australia	-.144 (.037)	-.015 (.025)	.027 (.030)	-.059 (.033)	.100 (.031)	-.018 (.025)	.074 (.040)	.072 (.024)	.013 (.012)	-.050 (.027)	-.463 (.077)
7. France	-.115 (.034)	-.024 (.014)	.024 (.023)	-.081 (.024)	.059 (.022)	-.047 (.033)	.146 (.025)	.001 (.020)	-.001 (.003)	.039 (.024)	-.527 (.077)
8. Germany	-.113 (.031)		.052 (.020)	-.133 (.020)	.053 (.020)	-.005 (.010)	.171 (.021)	.000 (.017)		-.025 (.017)	-.602 (.100)
9. Belgium	-.119 (.044)	-.001 (.025)	.011 (.028)	-.066 (.033)	.070 (.039)	-.028 (.025)	-.012 (.030)	-.002 (.021)	-.002 (.005)	.150 (.041)	-.127 (.069)
10. Norway	-.180 (.029)	.010 (.015)	.021 (.021)	-.145 (.024)	.026 (.020)	-.009 (.014)	.293 (.024)	.002 (.022)	-.003 (.006)	-.016 (.022)	-.484 (.088)
11. Netherlands	-.132 (.040)	-.024 (.013)	.157 (.028)	-.046 (.019)	.051 (.029)	-.027 (.014)	.051 (.016)	-.002 (.017)	-.006 (.012)	-.023 (.025)	-.842 (.107)
12. Iceland	-.129 (.025)	-.020 (.016)	.037 (.018)	-.198 (.022)	.089 (.018)	-.015 (.017)	.181 (.020)	.023 (.017)	-.004 (.006)	.036 (.014)	-.695 (.089)
13. Finland	-.126 (.037)	.024 (.025)	.055 (.027)	-.122 (.032)	.042 (.023)	-.009 (.015)	.109 (.029)	.053 (.021)	.002 (.010)	-.028 (.027)	-.378 (.081)
14. Austria	-.175 (.041)	-.036 (.023)	.090 (.031)	-.099 (.030)	.085 (.028)	-.060 (.018)	.282 (.034)	-.011 (.021)	-.005 (.005)	-.071 (.036)	-.154 (.093)
15. Japan	-.105 (.039)		.069 (.031)	-.130 (.038)	.073 (.022)	-.020 (.023)	.031 (.026)	.013 (.026)		.070 (.035)	-.370 (.101)
16. U.K.	-.127 (.027)	.002 (.011)	.017 (.020)	-.118 (.023)	.088 (.018)	-.003 (.005)	.087 (.020)	.027 (.018)	.000 (.009)	.027 (.023)	-.396 (.071)
17. Spain	-.049 (.061)	-.003 (.024)	.031 (.036)	-.114 (.045)	.039 (.030)	-.002 (.024)	.130 (.032)	.015 (.020)	-.012 (.012)	-.034 (.028)	-.291 (.073)
18. Italy	-.044 (.029)	-.017 (.016)	.077 (.018)	-.067 (.023)	.036 (.015)	.001 (.012)	.044 (.019)	-.024 (.016)	-.002 (.004)	-.006 (.021)	-.184 (.074)

TABLE 4.3

ML ESTIMATES OF THE CONSTANT TERMS FOR 10 COMMODITIES IN 18 COUNTRIES

(Asymptotic standard errors are in parentheses)

Country	Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous
(1)	$\hat{\alpha}_1^c$	$\hat{\alpha}_2^c$	$\hat{\alpha}_3^c$	$\hat{\alpha}_4^c$	$\hat{\alpha}_5^c$	$\hat{\alpha}_6^c$	$\hat{\alpha}_7^c$	$\hat{\alpha}_8^c$	$\hat{\alpha}_9^c$	$\hat{\alpha}_{10}^c$
1. U.S.	-.121 (.060)	.033 (.039)	-.131 (.069)	.384 (.078)	-.203 (.052)	.351 (.053)	-.418 (.074)	.040 (.058)	.028 (.023)	.037 (.063)
2. Canada	-.218 (.124)	.046 (.085)	.014 (.120)	.581 (.144)	-.177 (.125)	-.300 (.073)	-.103 (.133)	-.026 (.109)	.047 (.038)	.136 (.119)
3. Sweden	-.099 (.071)	-.073 (.046)	-.156 (.081)	.507 (.067)	-.142 (.051)	.067 (.024)	-.108 (.065)	.110 (.058)	.004 (.006)	-.110 (.041)
4. Switzerland	-.100 (.071)	-.122 (.050)	-.258 (.050)	.341 (.053)	-.327 (.053)	.169 (.031)	.063 (.054)	.074 (.060)		.159 (.045)
5. Denmark	-.065 (.080)	-.010 (.064)	-.228 (.059)	.649 (.072)	-.286 (.055)	.017 (.025)	-.209 (.071)	.084 (.057)	.070 (.016)	-.022 (.054)
6. Australia	.122 (.112)	-.074 (.074)	-.227 (.087)	.377 (.087)	-.234 (.111)	.017 (.053)	-.015 (.111)	-.031 (.066)	-.041 (.022)	.106 (.071)
7. France	-.011 (.133)	-.027 (.062)	-.280 (.094)	.510 (.080)	-.299 (.093)	.475 (.131)	-.311 (.097)	.060 (.092)	.001 (.011)	-.118 (.087)
8. Germany	-.043 (.120)		-.251 (.074)	.455 (.060)	-.200 (.081)	-.007 (.028)	-.172 (.081)	.055 (.063)		.161 (.055)
9. Belgium	-.008 (.161)	-.036 (.092)	-.073 (.105)	.164 (.121)	-.087 (.146)	.238 (.088)	.179 (.110)	.045 (.074)	.004 (.017)	-.426 (.155)
10. Norway	.160 (.103)	-.073 (.059)	-.204 (.075)	.502 (.083)	-.026 (.070)	.018 (.044)	-.550 (.085)	.148 (.087)	.000 (.018)	.024 (.068)
11. Netherlands	.049 (.197)	.085 (.082)	-.896 (.148)	.228 (.091)	-.129 (.174)	.316 (.057)	-.045 (.089)	.046 (.062)	.095 (.048)	.250 (.106)
12. Iceland	.033 (.187)	.106 (.133)	-.262 (.159)	.536 (.185)	-.272 (.152)	.289 (.108)	-.472 (.160)	.015 (.112)	.020 (.034)	.006 (.110)
13. Finland	-.065 (.183)	-.008 (.125)	-.441 (.135)	.419 (.169)	-.077 (.119)	.095 (.077)	-.075 (.148)	-.090 (.109)	-.058 (.045)	.300 (.124)
14. Austria	.247 (.156)	.042 (.095)	-.222 (.125)	.458 (.105)	-.270 (.115)	.158 (.059)	-.569 (.147)	.092 (.083)	.010 (.019)	.053 (.127)
15. Japan	-.140 (.173)		-.335 (.120)	.711 (.160)	-.403 (.099)	.309 (.109)	.015 (.108)	-.052 (.117)		-.106 (.136)
16. U.K.	-.090 (.073)	-.046 (.039)	-.066 (.069)	.280 (.058)	-.216 (.053)	-.002 (.015)	.055 (.059)	.080 (.050)	.015 (.022)	-.010 (.063)
17. Spain	-.349 (.287)	-.016 (.125)	-.235 (.159)	.406 (.229)	-.216 (.137)	.220 (.105)	-.155 (.156)	.009 (.085)	.054 (.050)	.282 (.128)
18. Italy	-.225 (.121)	.023 (.072)	-.301 (.073)	.161 (.089)	-.083 (.062)	.114 (.050)	.109 (.076)	.114 (.066)	-.002 (.017)	.090 (.081)

All entries are to be divided by 100.

5. ESTIMATION RESULTS FOR THE POOLED MODEL

The parameters of model (3.1) are α_i^c , β_i^c ($i=1, \dots, n^c$) and ϕ^c . It is to be noted that these differ across countries, as indicated by the country superscript. For the 15 countries with 10 commodity groups we can consider the hypothesis that the income and price responses are identical across countries by specifying that $\beta_i^c = \beta_i$, $\phi^c = \phi$, $i=1, \dots, 10$; $c=1, \dots, 15$. Then model (3.1) for these countries becomes

$$(5.1) \quad \bar{w}_{it}^c (Dq_{it}^c - DQ_t^c) = \alpha_i^c + \beta_i DQ_t^c + \phi (\bar{w}_{it}^c + \beta_i) \left[Dp_{it}^c - \sum_{j=1}^{10} (\bar{w}_{jt}^c + \beta_j) Dp_{jt}^c \right] + \epsilon_{it}^c,$$

where the constant terms (α_i^c) are allowed to vary over countries. We shall refer to (5.1) as the pooled model.

Selvanathan (1987b) estimates this model by ML with specification (4.1) of the covariance matrix. See Selvanathan (1987b) for details of the estimation procedure. Table 5.1 summarizes the estimation results. Row 1 presents the estimates of the income coefficients $\hat{\beta}_i$, $i=1, \dots, 10$; and the income flexibility $\hat{\phi}$. Rows 2-16 present the estimates of the constant terms ($\hat{\alpha}_i^c$) for the 15 countries.

6. THE SIMULATION PROCEDURE

In this section we describe the Monte Carlo simulation procedure to be used to test the reliability of the ML estimates. This procedure is from Theil (1987). We start with the ML estimates ($\hat{\alpha}_i^c$, $\hat{\beta}_i^c$, $\hat{\phi}^c$) of model (3.1) for $i=1, \dots, n^c$. We shall refer to these estimates as the true parameter values. Let $\hat{\Sigma}^c$ be a data-based consistent estimator of Σ^c . (Subsequently we shall specify $\hat{\Sigma}^c$ to be S^c and Σ^{*c} .) To obtain data sets for the

TABLE 5.1

ML ESTIMATES OF THE POOLED DEMAND MODEL FOR 15 COUNTRIES

(Asymptotic standard errors are in parentheses)

Country	Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous	Income flexibility
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Income coefficients $\hat{\beta}_i$ and income flexibility $\hat{\phi}$											
1. All countries	-.097 (.008)	-.011 (.005)	.040 (.006)	-.114 (.008)	.054 (.006)	-.012 (.003)	.122 (.007)	.017 (.005)	-.002 (.001)	.004 (.006)	-.449 (.022)
Constant terms $\hat{\alpha}_i^c$ (x100)											
2. U.S.	-.025 (.019)	-.013 (.011)	-.181 (.019)	.381 (.018)	-.208 (.014)	.264 (.006)	-.213 (.015)	.022 (.015)	.026 (.002)	-.053 (.019)	
3. Canada	-.013 (.019)	.006 (.013)	-.161 (.020)	.388 (.020)	-.173 (.017)	-.100 (.008)	-.194 (.017)	.111 (.017)	.043 (.002)	.091 (.024)	
4. Sweden	-.098 (.013)	-.045 (.007)	-.131 (.017)	.411 (.008)	-.129 (.009)	.046 (.006)	-.114 (.009)	.188 (.010)	.005 (.001)	-.133 (.022)	
5. Denmark	-.100 (.011)	-.047 (.010)	-.217 (.015)	.644 (.006)	-.288 (.008)	.019 (.004)	-.128 (.009)	.098 (.010)	.063 (.001)	-.033 (.024)	
6. Australia	.007 (.020)	-.085 (.010)	-.260 (.015)	.477 (.014)	-.079 (.018)	.006 (.005)	-.131 (.014)	.080 (.011)	-.020 (.002)	.004 (.021)	
7. France	-.066 (.030)	-.077 (.018)	-.328 (.024)	.604 (.022)	-.260 (.021)	.353 (.011)	-.229 (.022)	.004 (.022)	.004 (.003)	-.006 (.026)	
8. Belgium	-.085 (.026)	-.011 (.015)	-.207 (.022)	.324 (.020)	-.072 (.018)	.204 (.008)	-.210 (.018)	-.016 (.016)	.003 (.004)	.071 (.025)	
9. Norway	-.061 (.022)	-.012 (.012)	-.253 (.017)	.422 (.019)	-.098 (.015)	.025 (.006)	-.106 (.016)	.104 (.018)	-.001 (.002)	-.020 (.020)	
10. Netherlands	-.110 (.036)	.030 (.021)	-.335 (.027)	.451 (.028)	-.042 (.028)	.218 (.009)	-.345 (.029)	-.044 (.019)	.058 (.004)	.119 (.031)	
11. Iceland	-.166 (.035)	.057 (.023)	-.235 (.037)	.148 (.037)	-.038 (.032)	.246 (.012)	-.194 (.034)	.011 (.024)	.009 (.005)	.162 (.035)	
12. Finland	-.162 (.028)	.124 (.017)	-.387 (.026)	.383 (.033)	-.137 (.023)	.106 (.012)	-.128 (.024)	.049 (.022)	-.044 (.003)	.197 (.029)	
13. Austria	-.022 (.028)	-.088 (.018)	-.110 (.024)	.509 (.016)	-.225 (.021)	.054 (.005)	.010 (.020)	-.023 (.019)	.000 (.004)	-.104 (.032)	
14. U.K.	-.147 (.016)	-.021 (.010)	-.144 (.019)	.283 (.007)	-.150 (.011)	.013 (.005)	.019 (.011)	.096 (.010)	.019 (.001)	.032 (.022)	
15. Spain	-.152 (.037)	.014 (.025)	-.253 (.025)	.402 (.038)	-.261 (.023)	.264 (.011)	-.196 (.033)	.022 (.018)	.025 (.004)	.134 (.037)	
16. Italy	-.069 (.028)	-.036 (.021)	-.175 (.020)	.316 (.023)	-.136 (.018)	.150 (.012)	-.104 (.020)	-.052 (.019)	-.001 (.004)	.108 (.028)	

simulation experiment, we generate pseudo-normal random error vectors with mean zero and covariance matrix $\hat{\Sigma}^c$. Then we use these values together with the true values of the parameters and the observed values of the independent variables to obtain new values of the dependent variables from equation (3.1) for $i=1, \dots, n^c$. These values and the observed values of the independent variables are then used to re-estimate model (3.1) by ML. This procedure is repeated 100 times.

In the above simulation experiment we assume that Σ^c is unknown for each trial. That is, we re-estimate Σ^c as well as the demand parameters in each trial. Recognizing the fact that in a simulation experiment Σ^c is known and equal to its data-based estimate $\hat{\Sigma}^c$, we repeat the simulation experiment for each country with known Σ^c .

To assess the reliability of the pooled estimates, we use the same simulation procedure with the pooled model (5.1) and the ML estimates $(\hat{\alpha}_i^c, \hat{\beta}_i, \hat{\phi})$.

7. THE SIMULATION RESULTS FOR SEVEN COUNTRIES WITH THE USUAL COVARIANCE MATRIX

In this section we apply the simulation procedure described in Section 6 using $\hat{\Sigma}^c = S^c$ for the 7 countries where S^c is non-singular. Table 7.1 presents the simulation results for the U.S. Column 2 of the table presents the data-based ML estimates from Table 4.1. Columns 3-6 give the results when S^c is treated as unknown at each iteration in the ML procedure. In column 3 we present the means over the 100 trials of the ML estimates obtained using the simulated data. As can be seen, these estimates are quite close to the corresponding true values in column 2. To

TABLE 7.1

ML ESTIMATION WITH S^C : SUMMARY RESULTS FOR 100 TRIALS, U.S.

Commodity	True value	S^C unknown				S^C known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_1^C</u>									
1. Food	-.058	-.057	.021	.012	.566	-.059	.016	.017	1.036
2. Beverages	-.034	-.035	.006	.003	.444	-.034	.004	.004	.882
3. Clothing	.058	.057	.022	.011	.498	.058	.014	.015	1.019
4. Housing	-.111	-.111	.016	.009	.544	-.110	.011	.011	.981
5. Durables	.072	.070	.013	.008	.579	.070	.011	.010	.909
6. Medical care	-.078	-.080	.021	.010	.486	-.078	.013	.014	1.068
7. Transport	.149	.152	.047	.025	.544	.152	.036	.034	.927
8. Recreation	.041	.041	.013	.008	.588	.040	.010	.011	1.021
9. Education	-.007	-.007	.005	.003	.560	-.007	.003	.004	1.098
10. Miscellaneous	-.032	-.030	.019	.011	.562	-.032	.013	.014	1.075
<u>Income flexibility ϕ^C</u>									
11.	-.689	-.701	.088	.038	.428	-.695	.061	.053	.864
<u>Constant terms α_1^C ($\times 100$)</u>									
12. Food	-.102	-.105	.065	.056	.861	-.100	.055	.062	1.133
13. Beverages	.043	.045	.018	.013	.732	.043	.016	.015	.902
14. Clothing	-.264	-.257	.068	.045	.662	-.260	.050	.055	1.091
15. Housing	.383	.382	.042	.030	.716	.380	.035	.034	.978
16. Durables	-.255	-.249	.044	.033	.764	-.249	.040	.038	.936
17. Medical care	.397	.401	.044	.036	.820	.398	.037	.039	1.054
18. Transport	-.229	-.244	.133	.103	.776	-.245	.118	.117	.989
19. Recreation	-.065	-.062	.042	.033	.790	-.060	.038	.040	1.047
20. Education	.041	.041	.013	.010	.793	.041	.012	.011	.906
21. Miscellaneous	.050	.048	.047	.036	.767	.051	.038	.042	1.091
22. Mean					.642				1.000

test for any bias, we use the test statistic

$$(7.1) \quad |\text{bias}| \times \frac{\sqrt{\text{Number of trials}}}{\text{RMSE}},$$

where RMSE is the root-mean-squared error computed around the true value; the RMSEs are given in column 4. The test statistic (7.1) has a t-distribution with (number of trials - 1) degrees of freedom. Using this test we conclude that the ML estimates are unbiased.

Column 5 of the table presents the root-mean-squared asymptotic standard errors (RMSASEs). The RMSASEs are obtained by summing each diagonal element of the asymptotic covariance matrix over the 100 trials, dividing by 100 and then taking the square root. Comparing column 4 with column 5 we see that the RMSASEs are always less than the RMSEs. To facilitate this comparison, column 6 gives the ratio RMSASE/RMSE. Row 22 presents the mean of these ratios. As can be seen, all entries in column 6 are far below 1. Consequently, when S^C is used the asymptotic standard errors provide an overly optimistic picture of the sampling variability of the point estimates. This confirms that these estimates do indeed suffer from Theil's (1987) first peril.

Columns 7-10 of Table 7.1 present the results when S^C is treated as known for each iteration. As before, bias is not a problem. As can be seen from column 10, the RMSASE/RMSE ratios now fluctuate around unity. This shows that the source of the first peril is in estimating the error covariance matrix. Note also that the RMSEs in column 4 are substantially higher than those in column 8. This indicates that there is a substantial loss of efficiency when the covariance matrix is estimated. This is Theil's (1987) second peril. See Appendix A1 for the results of the other 6 countries where S^C is non-singular.

Table 7.2 presents the RMSASE/RMSE ratios for the income coefficients for the 10 commodities and income flexibility in the 7 countries. Column 13 gives the means of the corresponding rows. The upper half of the table presents the results when S^C is re-estimated in each trial; the lower half presents the results when this matrix is treated as known. Rows 8 and 16 present the ratios averaged over countries for the two cases. As can be seen, the ratios are well below unity when S^C is unknown, while they are very close to one when S^C is known. This clearly shows the existence of the first peril in all 7 countries when the covariance matrix is estimated in the usual way. It should also be noted that the first peril tends to be a bit more of a problem for the ϕ -estimates than for the β_i -estimates. On average, the asymptotic standard errors of the ϕ 's are understated by about 50 percent when S^C is treated as unknown; for the β_i 's the understatement is about 40 percent.

Table 7.3 presents same ratios for the constant terms. While these results have the same general pattern as Table 7.2, the upper-half ratios (S^C unknown) are now closer to unity.

8. THE SIMULATION RESULTS FOR EIGHTEEN COUNTRIES WITH THE ALTERNATIVE COVARIANCE MATRIX

In this section we assess the reliability of the ML estimates of the individual country models obtained with the covariance matrix specification (4.1).

Table 8.1 presents the simulation results for the U.S. when Σ^C is specified as Σ^{*C} . Column 2 presents the data-based ML estimates from Tables 4.2 and 4.3. Columns 3-6 give the results when Σ^{*C} is treated as unknown. In column 3 we present the means over 100 trials of the ML estimates obtained using the simulated data. As can be seen, these

TABLE 7.2

RMSASE/RMSE RATIOS FOR INCOME COEFFICIENTS FOR 10 COMMODITIES AND
INCOME FLEXIBILITY IN 7 COUNTRIES: 100 TRIALS WITH S^C

Country (1)	Food β_1^C (2)	Beverages β_2^C (3)	Clothing β_3^C (4)	Housing β_4^C (5)	Durables β_5^C (6)	Medical care β_6^C (7)	Transport β_7^C (8)	Recreation β_8^C (9)	Education β_9^C (10)	Miscellaneous β_{10}^C (11)	Income flexibility ϕ^C (12)	Mean (13)
<u>S^C unknown for each trial</u>												
1. U.S.	.57	.44	.50	.54	.58	.49	.54	.59	.56	.56	.43	.53
2. Canada	.51	.59	.53	.62	.61	.53	.53	.51	.61	.56	.48	.55
3. Switzerland	.68	.64	.69	.63	.68	.68	.63	.69		.63	.53	.65
4. Australia	.63	.53	.57	.57	.56	.58	.51	.53	.49	.59	.40	.54
5. Germany	.73		.74	.80	.77	.58	.65	.63		.68	.56	.68
6. Belgium	.45	.41	.47	.43	.40	.43	.55	.55	.46	.48	.41	.46
7. Netherlands	.60	.67	.66	.59	.64	.60	.63	.55	.74	.68	.59	.63
8. Mean	.60	.55	.59	.60	.61	.56	.58	.58	.57	.60	.49	.58
<u>S^C known for each trial</u>												
9. U.S.	1.04	.88	1.02	.98	.91	1.07	.93	1.02	1.10	1.08	.86	.99
10. Canada	.99	1.01	1.11	1.03	1.07	1.14	1.04	.92	1.08	1.08	1.00	1.04
11. Switzerland	.98	1.02	.96	1.07	.99	1.04	.96	1.05		.88	1.04	1.00
12. Australia	1.10	.87	.87	1.04	.96	1.01	.92	.98	.88	1.06	.92	.96
13. Germany	.97		.93	1.03	1.05	.98	.97	.90		1.01	.99	.98
14. Belgium	1.06	.94	1.10	.99	1.01	.97	1.13	.99	.92	1.05	.93	1.01
15. Netherlands	1.00	1.02	.96	1.00	.91	1.02	1.12	1.04	1.09	1.02	1.07	1.02
16. Mean	1.02	.96	.99	1.02	.99	1.03	1.01	.99	1.01	1.02	.97	1.00

TABLE 7.3

RMSASE/RMSE RATIOS FOR THE CONSTANT TERMS
FOR 10 COMMODITIES IN 7 COUNTRIES: 100 TRIALS WITH S^C

Country (1)	Food α_1^C (2)	Beverages α_2^C (3)	Clothing α_3^C (4)	Housing α_4^C (5)	Durables α_5^C (6)	Medical care α_6^C (7)	Transport α_7^C (8)	Recreation α_8^C (9)	Education α_9^C (10)	Miscellaneous α_{10}^C (11)	Mean (12)
<u>S^C unknown for each trial</u>											
1. U.S.	.86	.73	.66	.72	.76	.82	.78	.79	.79	.77	.77
2. Canada	.88	.75	.70	.80	.74	.69	.76	.62	1.18	.70	.78
3. Switzerland	.82	.85	.79	.81	.73	.95	.76	.77		.82	.81
4. Australia	.80	.63	.69	.73	.62	.81	.77	.69	.66	.75	.72
5. Germany	.85		.83	.92	.85	.81	.83	.71		.80	.83
6. Belgium	.70	.57	.62	.85	.65	.86	.65	.65	.72	.90	.72
7. Netherlands	.73	.90	.81	.73	.76	.90	.75	.83	.87	.78	.81
8. Mean	.81	.74	.73	.79	.73	.83	.76	.72	.85	.79	.78
<u>S^C known for each trial</u>											
9. U.S.	1.13	.90	1.09	.98	.94	1.05	.99	1.05	.91	1.09	1.01
10. Canada	1.07	1.00	1.09	.98	1.06	1.04	1.07	.91	1.18	.92	1.03
11. Switzerland	1.00	1.04	.96	.88	.97	1.03	1.00	1.04		.98	.99
12. Australia	1.02	.87	.98	.97	.96	1.07	1.02	1.01	.97	1.04	.99
13. Germany	1.04		.98	1.02	1.05	1.00	1.17	.93		1.14	1.04
14. Belgium	1.14	.97	1.13	1.02	.94	1.01	1.00	1.02	1.08	1.05	1.04
15. Netherlands	1.07	1.14	1.03	.98	1.05	1.01	1.01	1.11	1.07	.99	1.05
16. Mean	1.07	.99	1.04	.98	.99	1.03	1.04	1.01	1.04	1.03	1.02

TABLE 8.1

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, U.S.

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.055	-.056	.019	.018	.978	-.056	.019	.019	1.030
2. Beverages	-.031	-.027	.015	.013	.873	-.027	.015	.014	.919
3. Clothing	.027	.024	.021	.020	.954	.024	.021	.021	1.003
4. Housing	-.115	-.116	.026	.025	.978	-.116	.026	.026	1.028
5. Durables	.054	.051	.016	.016	.976	.051	.016	.017	1.026
6. Medical care	-.059	-.060	.019	.020	1.040	-.060	.019	.021	1.093
7. Transport	.209	.211	.022	.023	1.040	.211	.022	.024	1.094
8. Recreation	.014	.013	.018	.017	.944	.013	.018	.018	.992
9. Education	-.003	-.003	.009	.008	.980	-.003	.009	.009	1.031
10. Miscellaneous	-.041	-.038	.021	.022	1.027	-.038	.021	.023	1.080
<u>Income flexibility ϕ^c</u>									
11.	-.384	-.384	.063	.063	.996	-.384	.063	.066	1.047
<u>Constant terms α_i^c (x100)</u>									
12. Food	-.121	-.121	.053	.057	1.077	-.121	.053	.060	1.133
13. Beverages	.033	.025	.044	.037	.850	.025	.044	.039	.894
14. Clothing	-.131	-.120	.066	.065	.984	-.120	.066	.069	1.034
15. Housing	.384	.384	.078	.074	.945	.384	.078	.078	.994
16. Durables	-.203	-.197	.052	.050	.958	-.197	.052	.052	1.008
17. Medical care	.351	.351	.052	.050	.966	.351	.052	.053	1.016
18. Transport	-.418	-.425	.064	.070	1.091	-.425	.064	.074	1.147
19. Recreation	.040	.042	.053	.056	1.039	.042	.053	.058	1.092
20. Education	.028	.027	.022	.022	.991	.027	.022	.023	1.042
21. Miscellaneous	.037	.033	.062	.060	.973	.033	.062	.063	1.024
<u>Proportionality constant λ^{c2} (x10⁴)</u>									
22.	.269	.243	.038						
23. Mean					.984				1.035

estimates are also quite close to the corresponding true values in column 2. Using the test statistic (7.1), we conclude that the ML estimates are unbiased. Comparing the RMSEs given in column 4 with the RMSASEs in column 5 we see that the latter are in general slightly lower. Row 23 presents the mean of the RMSASE/RMSE ratios. This mean shows that the RMSASEs are about 1½ percent less than the RMSEs on average. Therefore, we conclude that while the asymptotic standard errors still tend to understate the sampling variability of the estimates, the first peril is not nearly such a problem when the unknown covariance matrix Σ^c is specified as Σ^{*c} . (Compare column 6 of Table 8.1 with the same column of Table 7.1.)

Columns 7-10 of the Table 8.1 present the results when Σ^{*c} is treated as known. As λ^{c2} is the only unknown parameter in Σ^{*c} [see equation (4.1)], this amounts to fixing this one parameter at its data-based ML value. Comparing column 10 with column 6, we see that the RMSASE/RMSE ratios have now slightly increased. However, the ratios are all close to unity in both cases. It can also be seen that the RMSEs in column 4 are virtually equal to the RMSEs in column 8. This indicates that the second peril is also not a problem when we use Σ^{*c} as the estimator of Σ^c . We present the results for the remaining 17 countries in Appendix A2.

Tables 8.2 and 8.3 present the RMSASE/RMSE ratios for all 18 countries with $\hat{\Sigma}^c = \Sigma^{*c}$. The upper half of the tables present the results when Σ^{*c} (i.e. λ^{c2}) is re-estimated in each trial and the lower half presents the results when this matrix is treated as known. Rows 19 and 38 of the tables present the mean ratios over countries. As can be seen from row 19 the ratios are now only a bit below unity when Σ^{*c} is treated as unknown. The ratios move even closer to unity when Σ^{*c} is specified as being known in each trial, as can be seen from the lower halves of Tables 8.2 and 8.3.

TABLE 8.2

RMSASE/RMSE RATIOS FOR INCOME COEFFICIENTS FOR 10 COMMODITIES AND
INCOME FLEXIBILITY IN 18 COUNTRIES: 100 TRIALS WITH Σ^{*c}

Country	Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous	Income flexibility	Mean
(1)	β_1^c	β_2^c	β_3^c	β_4^c	β_5^c	β_6^c	β_7^c	β_8^c	β_9^c	β_{10}^c	ϕ^c	(13)
<u>Σ^{*c} unknown for each trial</u>												
1. U.S.	.98	.87	.95	.98	.98	1.04	1.04	.94	.98	1.03	1.00	.98
2. Canada	1.00	.90	.98	.89	1.02	1.01	.89	.88	.95	.97	.93	.95
3. Sweden	.98	.93	.86	.87	.90	.95	.83	1.02	1.00	.98	.84	.92
4. Switzerland	.94	.98	.90	.96	.97	.95	.82	.92		.90	.88	.92
5. Denmark	.93	.92	1.02	1.03	.91	.95	.88	.99	.86	.94	.87	.94
6. Australia	1.04	.89	.83	.92	.92	.97	.96	.88	.88	1.00	.88	.93
7. France	.90	.93	.89	.92	1.00	.90	.85	.96	.77	.97	.85	.90
8. Germany	.94		.89	.94	.97	.96	.97	.92		1.05	1.02	.96
9. Belgium	.93	.88	1.05	1.00	.90	1.01	1.03	.95	1.00	1.09	1.01	.99
10. Norway	.92	.86	.92	.93	.95	1.01	.93	1.01	.91	.94	.99	.94
11. Netherlands	.98	.88	.97	.90	.89	1.06	.98	1.00	1.00	.93	.97	.95
12. Iceland	1.02	.84	.85	.74	.88	.88	.85	.81	.80	.96	.88	.86
13. Finland	.95	.86	.94	.90	.90	.88	1.00	1.04	.85	1.07	.85	.93
14. Austria	.98	.87	.96	.91	.89	1.10	.91	1.00	.86	.93	.81	.93
15. Japan	.80		.89	.93	.88	.81	.87	.95		1.03	.82	.89
16. U.K.	.86	.97	.97	.97	.94	.91	.84	.99	1.00	1.03	.89	.94
17. Spain	.83	.78	.92	.87	.86	.75	.89	.81	.92	1.04	.84	.86
18. Italy	.90	.81	1.01	.81	.96	.97	.98	1.04	.91	1.02	1.00	.95
19. Mean	.93	.89	.93	.91	.93	.95	.92	.95	.91	.99	.91	.93
<u>Σ^{*c} known for each trial</u>												
20. U.S.	1.03	.92	1.00	1.03	1.03	1.09	1.09	.99	1.03	1.08	1.05	1.03
21. Canada	1.05	.95	1.03	.94	1.08	1.07	.94	.93	1.00	1.02	.98	1.00
22. Sweden	1.04	.99	.92	.92	.96	1.01	.89	1.08	1.06	1.04	.89	.98
23. Switzerland	1.00	1.04	.95	1.01	1.02	1.01	.87	.98		.95	.93	.98
24. Denmark	.99	.98	1.10	1.11	.97	1.02	.94	1.05	.92	1.01	.93	1.00
25. Australia	1.10	.94	.88	.97	.97	1.02	1.01	.93	.93	1.06	.93	.98
26. France	.95	.98	.94	.98	1.06	.96	.90	1.03	.82	1.03	.90	.96
27. Germany	.99		.93	.99	1.02	1.01	1.02	.97		1.10	1.07	1.01
28. Belgium	.98	.92	1.10	1.05	.94	1.07	1.09	1.00	1.05	1.15	1.06	1.04
29. Norway	.98	.92	.97	.99	1.01	1.07	.99	1.08	.97	.99	1.05	1.00
30. Netherlands	.92	.92	1.01	.94	.93	1.11	1.02	1.05	1.05	.97	1.01	.99
31. Iceland	1.11	.91	.93	.80	.96	.96	.92	.89	.87	1.05	.96	.94
32. Finland	1.01	.92	1.00	.95	.96	.94	1.06	1.11	.91	1.14	.90	.99
33. Austria	1.04	.92	1.02	.96	.95	1.17	.97	1.06	.91	.99	.86	.99
34. Japan	.90		.99	1.04	.98	.91	.97	1.06		1.15	.92	.99
35. U.K.	.92	1.03	1.03	1.03	1.00	.97	.89	1.05	1.06	1.10	.95	1.00
36. Spain	.91	.85	.99	.94	.94	.81	.97	.88	1.00	1.13	.91	.94
37. Italy	.96	.87	1.08	.86	1.02	1.03	1.04	1.10	.97	1.08	1.06	1.01
38. Mean	.99	.94	.99	.97	.99	1.01	.98	1.01	.97	1.06	.96	.99

TABLE 8.3

RMSASE/RMSE RATIOS FOR THE CONSTANT TERMS FOR 10 COMMODITIES
IN 18 COUNTRIES: 100 TRIALS WITH Σ^{*c}

Country	Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous	Mean
(1)	α_{1c}	α_{2c}	α_{3c}	α_{4c}	α_{5c}	α_{6c}	α_{7c}	α_{8c}	α_{9c}	α_{10c}	(12)
<u>Σ^{*c} unknown for each trial</u>											
1. U.S.	1.08	.85	.98	.94	.96	.97	1.09	1.04	.99	.97	.99
2. Canada	1.03	.91	.98	.87	1.04	.94	.89	.90	1.04	.99	.96
3. Sweden	1.03	1.04	.84	.90	.88	.98	.89	.98	1.00	.84	.94
4. Switzerland	.96	1.00	.92	.85	1.01	.95	.98	.96		.96	.95
5. Denmark	.92	.98	.99	.91	.88	1.00	.97	1.03	1.03	.86	.96
6. Australia	.98	.88	.93	.84	.94	1.07	1.00	.89	.95	.97	.95
7. France	.93	.92	.91	.88	1.03	.88	.84	.96	.84	.94	.91
8. Germany	1.00		.94	.99	.97	1.05	1.08	.88		1.10	1.00
9. Belgium	1.03	.89	1.03	.94	.92	.98	1.03	.94	1.02	1.04	.98
10. Norway	.94	.90	.89	.94	.98	1.09	1.00	.96	.93	.93	.96
11. Netherlands	.95	.93	1.00	.92	1.03	1.07	.95	.92	1.01	.90	.97
12. Iceland	.97	.97	.76	.86	.95	.97	.91	.88	.89	.88	.91
13. Finland	1.00	.92	.89	.96	.90	1.00	1.02	1.04	.88	1.02	.96
14. Austria	.91	.93	.95	.84	.94	1.06	.88	.96	.82	.94	.92
15. Japan	.81	.00	.85	.88	.90	.88	.85	.93	.00	.87	.87
16. U.K.	.91	1.14	.99	.90	.87	.87	.88	.91	1.03	.88	.94
17. Spain	.86	.79	.95	.88	.84	.79	.96	.85	.90	1.04	.88
18. Italy	.91	.85	1.01	.82	1.01	.99	1.02	1.00	.94	.97	.95
19. Mean	.96	.93	.93	.89	.95	.97	.96	.95	.95	.95	.94
<u>Σ^{*c} known for each trial</u>											
20. U.S.	1.13	.89	1.03	.99	1.01	1.02	1.15	1.09	1.04	1.02	1.04
21. Canada	1.09	.96	1.03	.92	1.10	.99	.94	.95	1.10	1.04	1.01
22. Sweden	1.10	1.11	.89	.95	.93	1.04	.95	1.04	1.06	.90	1.00
23. Switzerland	1.01	1.06	.97	.90	1.07	1.00	1.03	1.02	.00	1.02	1.01
24. Denmark	.99	1.05	1.06	.97	.95	1.07	1.04	1.11	1.10	.92	1.02
25. Australia	1.03	.93	.98	.89	.99	1.13	1.06	.94	1.01	1.02	1.00
26. France	.99	.98	.97	.94	1.10	.94	.89	1.02	.89	.99	.97
27. Germany	1.05	.00	.99	1.03	1.01	1.10	1.13	.92	.00	1.16	1.05
28. Belgium	1.09	.93	1.08	.99	.96	1.03	1.08	.99	1.08	1.09	1.03
29. Norway	1.00	.96	.94	.99	1.04	1.16	1.06	1.02	.99	.99	1.02
30. Netherlands	1.00	.97	1.05	.96	1.08	1.12	1.00	.97	1.06	.94	1.01
31. Iceland	1.06	1.06	.83	.94	1.04	1.06	.99	.96	.97	.96	.99
32. Finland	1.07	.98	.95	1.02	.96	1.06	1.09	1.11	.94	1.09	1.03
33. Austria	.97	.99	1.01	.89	1.00	1.13	.93	1.02	.87	.99	.98
34. Japan	.91	.00	.95	.98	1.00	.99	.95	1.03	.00	.98	.97
35. U.K.	.96	1.22	1.05	.95	.93	.92	.94	.97	1.10	.94	1.00
36. Spain	.93	.85	1.03	.95	.91	.86	1.04	.93	.98	1.13	.96
37. Italy	.96	.90	1.07	.87	1.07	1.05	1.09	1.06	.99	1.03	1.01
38. Mean	1.02	.99	.99	.95	1.01	1.04	1.02	1.01	1.01	1.01	1.01

In a typical application, the error covariance matrix Σ^C will be unknown and has to be estimated. To compare the effects of using the alternative estimator of Σ^C rather than the usual ML estimator, we should consequently focus on the contrast between upper halves of Tables 8.2 and 8.3, on the one hand, and Tables 7.2 and 7.3, on the other. This contrast is stark indeed. The alternative estimator of Σ^C leads to a spectacular improvement in the results in all cases.

9. THE SIMULATION RESULTS FOR THE POOLED MODEL

Table 9.1 presents the summary results of the simulations for the pooled model with $\hat{\Sigma}^C = \Sigma^{*C}$. Column 2 reproduces the data-based ML estimates from Table 5.1. Columns 3-6 present the results for 100 trials when the covariance matrix is unknown. As before, the estimates are unbiased. As can be seen from column 6, the RMSASEs are in general slightly less than the RMSEs, but the differences are very small.

Columns 7-10 of Table 9.1 present the results when the covariance matrix is treated as known. In general, here the RMSEs fall while the RMSASEs increase, so that the ratios move closer to unity. However, the changes are again very small, as can be seen from the means of the RMSASE/RMSE ratios: .983 (Σ^{*C} unknown) and .998 (Σ^{*C} known). (The estimation procedure for the pooled model uses the data deviated from means and hence calculates the constant terms from the income coefficient and income flexibility estimates. To reduce what would otherwise be an enormous amount of computing time, the simulated values of the constant terms were not calculated.)

The above results clearly show that the two perils are not a problem when the pooled demand model is estimated with the specified covariance matrix. Therefore, we conclude that the ML estimates of this model and the standard errors presented in Table 5.1 are reliable.

TABLE 9.1

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS,

POOLED MODEL FOR 15 COUNTRIES

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i</u>									
1. Food	-.097	-.097	.0841	.0836	.993	-.097	.0863	.0843	.977
2. Beverages	-.011	-.011	.0461	.0450	.977	-.011	.0462	.0455	.985
3. Clothing	.040	.039	.0654	.0628	.961	.039	.0642	.0634	.987
4. Housing	-.114	-.114	.0707	.0747	1.056	-.114	.0703	.0754	1.072
5. Durables	.054	.053	.0494	.0568	1.150	.053	.0492	.0573	1.166
6. Medical care	-.012	-.012	.0287	.0291	1.015	-.012	.0285	.0294	1.034
7. Transport	.122	.122	.0761	.0658	.864	.121	.0753	.0663	.881
8. Recreation	.017	.018	.0560	.0526	.940	.018	.0551	.0531	.964
9. Education	-.002	-.002	.0109	.0105	.965	-.002	.0108	.0106	.983
10. Miscellaneous	.004	.004	.0602	.0596	.990	.004	.0598	.0601	1.004
<u>Income flexibility ϕ</u>									
11.	-.449	-.452	.2381	.2147	.901	-.451	.2352	.2167	.921
12. Mean					.983				.998

All entries in columns 4, 5, 8 and 9 are to be divided by 10.

10. CONCLUDING COMMENTS

Theil (1987) demonstrates that there are problems with the usual ML estimator when applied to large demand systems. In particular, when the error covariance matrix is replaced by its usual ML-estimator S , the matrix of mean squares and cross products of the residuals, the estimates suffer from two problems. First, the asymptotic standard errors severely understate the sampling variability of the estimates, so that the precision of the estimates is overstated. Second, the efficiency of the coefficient estimates is greatly impaired. Theil refers to these problems as the 'two perils' of estimating large systems.

If the estimator is working satisfactorily, the root-mean-squared asymptotic standard errors (RMSASEs) of the parameters from a Monte Carlo simulation should be approximately equal to the root-mean-squared errors (RMSEs). That is, the RMSASE/RMSE ratios should be close to unity. However, if we used S in a large system the first peril implies that these ratios will be substantially below unity. In this paper we initially used S to estimate large systems for 7 countries. The results of simulations for these countries are illustrated by column 2 of Table 10.1. This column presents the RMSASE/RMSE ratios averaged over all parameters for the 7 countries and their mean over countries. As can be seen, the ratios are all well below unity, confirming that the estimates suffer from the first peril.

Column 3 of Table 10.1 presents the ratios when S is treated as known in each trial of the simulation. Comparing columns 2 and 3, we see that, on average, knowledge of the S matrix causes the ratios to increase from .67 to 1.01. The satisfactory performance of the estimates when S is known indicates that the cause of the first peril is not knowing the true

TABLE 10.1
ML ESTIMATION: SUMMARY RESULTS OF MONTE CARLO SIMULATIONS
FOR 18 COUNTRIES

Country	Average RMSASE/RMSE ratio with error covariance matrix specified as			
	S^c		Σ^{*c}	
	S^c unknown in each trial	S^c known in each trial	Σ^{*c} unknown in each trial	Σ^{*c} known in each trial
(1)	(2)	(3)	(4)	(5)
1. U.S.	.64	1.00	.98	1.04
2. Canada	.66	1.04	.95	1.01
3. Sweden			.93	.99
4. Switzerland	.73	.99	.94	.99
5. Denmark			.95	1.01
6. Australia	.62	.98	.94	.99
7. France			.91	.97
8. Germany	.75	1.01	.98	1.03
9. Belgium	.58	1.02	.98	1.04
10. Norway			.95	1.01
11. Netherlands	.72	1.03	.96	1.00
12. Iceland			.89	.96
13. Finland			.95	1.01
14. Austria			.93	.98
15. Japan			.88	.98
16. U.K.			.94	1.00
17. Spain			.87	.95
18. Italy			.95	1.01
19. Mean	.67	1.01	.94	1.00
20. Pooled			.98	1.00

S^c is the usual ML estimator of the error covariance matrix for country c ; and Σ^{*c} is the alternative specification of the error covariance matrix for country c . This table draws on Tables 7.1, A1.1-A1.6, 8.1, A2.1-A2.17 and 9.1.

value of S . Other results in this paper also clearly showed the existence of the second peril when S was used.

We then analysed whether the two perils can be avoided by using an alternative specification for the error covariance matrix. This matrix, denoted by Σ^* , contains only one unknown parameter. To analyse the performance of Σ^* , again we used Monte Carlo simulations, but now with data from 18 OECD countries. To illustrate these results, columns 4-5 of Table 10.1 present the average RMSASE/RMSE ratios. The ratios in column 4 are now all slightly less than unity. The ratios increase a bit when Σ^* is treated as known for each trial; however, they are close to unity in both cases.

It will usually be the case that the error covariance matrix is unknown and has to be estimated. To compare the effects of using the alternative estimator Σ^* rather than the usual ML estimator S , we should thus focus on the contrast between columns 4 and 2 of Table 10.1. This contrast shows very clearly that Σ^* leads to a marked improvement in all cases. In other words, the first peril can be virtually avoided by using Σ^* . Our other results indicate that Σ^* avoids the second peril also.

Finally, we used a similar procedure with the pooled model, where the parameters are the same across countries. Row 20 of Table 10.1 presents the mean ratios from this model. These ratios are approximately equal to unity when the covariance matrix is both unknown and known, again indicating the satisfactory performance of Σ^* .

Based on these results, we conclude that the ML estimates and their asymptotic standard errors present a fairly realistic picture for the 18 OECD countries when Σ^* is used for the error covariance matrix. Consequently, the results of this paper clearly demonstrates that the alternative specification Σ^* is a viable way to avoid the two perils.

APPENDICES

A1. SIMULATION RESULTS FOR SIX COUNTRIES
WITH THE USUAL COVARIANCE MATRIX

Section 7 of the text presents the detailed simulation results for the U.S with $\hat{\Sigma}^c = S^c$, the usual ML estimator. Tables A1.1-A1.6 of this appendix give the detailed results for the remaining 6 countries where S^c is non-singular.

A2. SIMULATION RESULTS FOR SEVENTEEN COUNTRIES WITH
THE ALTERNATIVE COVARIANCE MATRIX

Section 8 of the text presents the detailed simulation results for the U.S. with $\hat{\Sigma}^c = \Sigma^{*c}$, the alternative estimator defined in (4.1). Tables A2.1-A2.17 of this appendix present the detailed results for the remaining 17 countries.

TABLE A1.1

ML ESTIMATION WITH S^C : SUMMARY RESULTS FOR 100 TRIALS, CANADA

Commodity	True value	S^C unknown				S^C known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^C</u>									
1. Food	-.011	-.012	.018	.009	.514	-.011	.013	.013	.994
2. Beverages	-.027	-.027	.008	.005	.594	-.026	.006	.006	1.008
3. Clothing	.006	.004	.019	.010	.532	.005	.013	.014	1.108
4. Housing	-.172	-.174	.017	.011	.616	-.174	.014	.014	1.027
5. Durables	.078	.075	.018	.011	.614	.075	.013	.014	1.068
6. Medical care	.025	.024	.051	.027	.530	.024	.032	.037	1.143
7. Transport	.032	.034	.028	.015	.530	.032	.019	.020	1.042
8. Recreation	.069	.070	.019	.010	.509	.071	.014	.013	.918
9. Education	-.004	-.003	.008	.005	.611	-.004	.006	.007	1.076
10. Miscellaneous	.005	.009	.024	.014	.565	.009	.018	.019	1.076
<u>Income flexibility ϕ^C</u>									
11.	-.904	-.902	.099	.048	.482	-.898	.069	.069	1.003
<u>Constant terms α_i^C ($\times 100$)</u>									
12. Food	-.153	-.154	.064	.056	.878	-.156	.058	.062	1.072
13. Beverages	.048	.045	.034	.025	.750	.044	.029	.029	.998
14. Clothing	-.098	-.081	.085	.060	.703	-.085	.065	.071	1.091
15. Housing	.552	.558	.071	.057	.801	.557	.064	.063	.980
16. Durables	-.299	-.290	.063	.047	.742	-.289	.052	.055	1.057
17. Medical care	-.213	-.213	.184	.126	.687	-.212	.144	.149	1.037
18. Transport	.039	.027	.107	.082	.761	.034	.087	.093	1.072
19. Recreation	-.143	-.148	.081	.050	.622	-.148	.066	.061	.914
20. Education	.077	.075	.023	.027	1.183	.075	.023	.027	1.184
21. Miscellaneous	.190	.180	.073	.051	.697	.181	.065	.059	.917
22. Mean				.663					1.037

TABLE A1.2

ML ESTIMATION WITH S^C : SUMMARY RESULTS FOR 100 TRIALS, SWITZERLAND

Commodity	True value	S^C unknown				S^C known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^C</u>									
1. Food	-.020	-.018	.023	.016	.680	-.018	.020	.019	.977
2. Beverages	.045	.045	.019	.012	.638	.045	.015	.015	1.018
3. Clothing	.058	.056	.018	.012	.686	.058	.015	.015	.963
4. Housing	-.181	-.181	.026	.017	.633	-.181	.020	.022	1.065
5. Durables	.087	.087	.014	.009	.678	.088	.011	.011	.992
6. Medical care	-.042	-.043	.012	.008	.682	-.043	.009	.010	1.037
7. Transport	.083	.083	.038	.024	.631	.083	.031	.030	.958
8. Recreation	.000	.000	.016	.011	.685	.000	.013	.013	1.048
9. Education									
10. Miscellaneous	-.030	-.029	.019	.012	.632	-.030	.018	.015	.877
<u>Income flexibility ϕ^C</u>									
11.	-.589	-.597	.068	.036	.534	-.593	.048	.050	1.040
<u>Constant terms α_i^C ($\times 100$)</u>									
12. Food	-.070	-.074	.066	.055	.822	-.074	.060	.060	1.000
13. Beverages	-.148	-.146	.050	.042	.852	-.145	.045	.047	1.045
14. Clothing	-.267	-.261	.052	.041	.785	-.264	.048	.046	.965
15. Housing	.381	.372	.080	.064	.808	.373	.077	.068	.883
16. Durables	-.329	-.329	.038	.027	.729	-.330	.032	.031	.970
17. Medical care	.168	.166	.022	.021	.946	.166	.021	.022	1.030
18. Transport	.030	.036	.093	.071	.762	.037	.080	.080	.999
19. Recreation	.077	.080	.041	.032	.775	.080	.035	.036	1.037
20. Education									
21. Miscellaneous	.159	.155	.036	.030	.824	.156	.034	.033	.979
22. Mean					.725				.994

TABLE A1.3

ML ESTIMATION WITH S^C : SUMMARY RESULTS FOR 100 TRIALS, AUSTRALIA

Commodity	True value	S ^C unknown				S ^C known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^C</u>									
1. Food	-.139	-.141	.022	.014	.633	-.140	.017	.019	1.101
2. Beverages	-.009	-.009	.014	.008	.527	-.007	.012	.010	.872
3. Clothing	.016	.019	.027	.015	.568	.018	.023	.020	.867
4. Housing	-.005	-.009	.032	.018	.570	-.003	.023	.024	1.043
5. Durables	.052	.052	.058	.032	.557	.048	.043	.041	.958
6. Medical care	.053	.052	.031	.018	.585	.054	.025	.026	1.007
7. Transport	-.013	-.011	.050	.025	.510	-.016	.038	.035	.920
8. Recreation	.074	.075	.038	.020	.530	.074	.028	.028	.979
9. Education	.002	.003	.011	.006	.489	.003	.009	.008	.883
10. Miscellaneous	-.031	-.032	.018	.011	.586	-.032	.014	.015	1.061
<u>Income flexibility ϕ^C</u>									
11.	-.404	-.417	.077	.031	.404	-.405	.049	.045	.921
<u>Constant terms α_i^C ($\times 100$)</u>									
12. Food	.104	.109	.073	.059	.798	.106	.067	.068	1.019
13. Beverages	-.099	-.101	.044	.028	.630	-.106	.038	.033	.870
14. Clothing	-.213	-.217	.077	.053	.687	-.216	.064	.063	.978
15. Housing	.249	.254	.088	.064	.733	.241	.076	.074	.973
16. Durables	-.077	-.082	.205	.128	.621	-.064	.160	.153	.956
17. Medical care	-.111	-.107	.074	.060	.814	-.114	.065	.070	1.073
18. Transport	.177	.171	.134	.103	.767	.180	.116	.119	1.025
19. Recreation	-.057	-.056	.096	.066	.690	-.055	.080	.080	1.007
20. Education	-.028	-.030	.022	.015	.659	-.030	.018	.018	.969
21. Miscellaneous	.055	.058	.050	.038	.755	.058	.043	.044	1.036
22. Mean					.624				.977

TABLE A1.4

ML ESTIMATION WITH S^C : SUMMARY RESULTS FOR 100 TRIALS, GERMANY

Commodity	True value	S^C unknown				S^C known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_1^C</u>									
1. Food	-.101	-.099	.029	.021	.732	-.101	.026	.025	.973
2. Beverages									
3. Clothing	.056	.055	.017	.012	.741	.056	.015	.014	.931
4. Housing	-.131	-.133	.022	.018	.795	-.131	.021	.021	1.030
5. Durables	.049	.050	.017	.013	.770	.050	.015	.016	1.046
6. Medical care	-.009	-.010	.008	.004	.584	-.009	.006	.005	.976
7. Transport	.162	.164	.036	.023	.652	.163	.029	.028	.968
8. Recreation	-.001	-.003	.015	.009	.627	-.003	.012	.011	.903
9. Education									
10. Miscellaneous	-.024	-.024	.010	.007	.678	-.025	.008	.008	1.010
<u>Income flexibility ϕ^C</u>									
11.	-.627	-.640	.111	.062	.558	-.630	.083	.082	.990
<u>Constant terms α_1^C ($\times 100$)</u>									
12. Food	-.086	-.089	.103	.088	.848	-.082	.096	.099	1.036
13. Beverages									
14. Clothing	-.264	-.256	.057	.047	.834	-.260	.053	.053	.984
15. Housing	.451	.455	.066	.061	.915	.452	.064	.066	1.022
16. Durables	-.186	-.192	.064	.055	.853	-.190	.059	.062	1.054
17. Medical care	.004	.005	.020	.016	.805	.004	.018	.018	1.004
18. Transport	-.140	-.152	.118	.098	.833	-.151	.095	.110	1.166
19. Recreation	.060	.066	.052	.037	.715	.064	.046	.042	.931
20. Education									
21. Miscellaneous	.162	.163	.031	.025	.801	.164	.025	.028	1.142
22. Mean					.749				1.010

TABLE A1.5

ML ESTIMATION WITH S^C : SUMMARY RESULTS FOR 100 TRIALS, BELGIUM

Commodity	True value	S^C unknown				S^C known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^C</u>									
1. Food	-.092	-.093	.029	.013	.448	-.092	.020	.021	1.062
2. Beverages	-.001	-.002	.021	.009	.414	.000	.014	.013	.941
3. Clothing	.042	.042	.020	.009	.467	.041	.013	.014	1.096
4. Housing	-.104	-.105	.015	.007	.430	-.105	.010	.010	.994
5. Durables	.194	.191	.040	.016	.403	.190	.025	.026	1.014
6. Medical care	-.046	-.047	.014	.006	.433	-.046	.010	.010	.971
7. Transport	-.020	-.018	.028	.016	.545	-.017	.019	.022	1.127
8. Recreation	.005	.007	.013	.007	.551	.006	.010	.010	.986
9. Education	-.002	-.002	.000	.000	.461	-.002	.000	.000	.921
10. Miscellaneous	.025	.028	.025	.012	.481	.026	.017	.017	1.051
<u>Income flexibility ϕ^C</u>									
11.	-.607	-.612	.074	.030	.409	-.610	.054	.050	.933
<u>Constant terms α_i^C ($\times 100$)</u>									
12. Food	-.103	-.100	.115	.080	.697	-.104	.085	.098	1.144
13. Beverages	-.043	-.043	.079	.045	.573	-.047	.056	.054	.971
14. Clothing	-.229	-.222	.077	.048	.624	-.219	.054	.061	1.133
15. Housing	.308	.304	.080	.067	.845	.304	.069	.071	1.020
16. Durables	-.561	-.546	.163	.106	.652	-.543	.135	.126	.935
17. Medical care	.300	.297	.055	.047	.857	.294	.051	.052	1.006
18. Transport	.227	.221	.101	.066	.655	.218	.081	.081	.998
19. Recreation	.027	.020	.045	.029	.650	.023	.035	.036	1.017
20. Education	.004	.004	.001	.001	.721	.004	.001	.001	1.079
21. Miscellaneous	.071	.066	.124	.112	.899	.069	.111	.117	1.054
23. Mean					.582				1.022

TABLE A1.6

ML ESTIMATION WITH S^C : SUMMARY RESULTS FOR 100 TRIALS, NETHERLANDS

Commodity	True value	S^C unknown				S^C known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^C</u>									
1. Food	-.138	-.143	.040	.024	.603	-.139	.031	.031	.997
2. Beverages	-.017	-.017	.011	.008	.674	-.017	.009	.010	1.016
3. Clothing	.132	.132	.034	.022	.661	.134	.028	.027	.964
4. Housing	-.046	-.046	.020	.012	.588	-.048	.015	.015	1.002
5. Durables	.058	.061	.047	.030	.644	.060	.040	.037	.909
6. Medical care	-.010	-.011	.012	.007	.600	-.011	.010	.010	1.019
7. Transport	.051	.052	.020	.012	.627	.051	.014	.015	1.119
8. Recreation	.011	.010	.013	.007	.552	.009	.009	.010	1.042
9. Education	-.016	-.017	.008	.006	.744	-.017	.006	.007	1.092
10. Miscellaneous	-.023	-.021	.022	.015	.681	-.021	.018	.018	1.020
<u>Income flexibility ϕ^C</u>									
11.	-.782	-.782	.103	.061	.591	-.785	.078	.084	1.074
<u>Constant terms α_i^C (x100)</u>									
12. Food	.067	.088	.186	.136	.733	.072	.152	.163	1.072
13. Beverages	.046	.046	.067	.061	.900	.046	.059	.068	1.140
14. Clothing	-.786	-.784	.168	.135	.806	-.795	.148	.153	1.029
15. Housing	.223	.221	.096	.070	.732	.228	.080	.078	.975
16. Durables	-.157	-.169	.249	.189	.759	-.164	.213	.223	1.048
17. Medical care	.285	.284	.057	.052	.905	.286	.056	.056	1.011
18. Transport	-.045	-.046	.114	.085	.745	-.039	.095	.096	1.012
19. Recreation	.005	.006	.047	.039	.831	.010	.040	.044	1.114
20. Education	.118	.120	.031	.027	.872	.119	.027	.029	1.066
21. Miscellaneous	.245	.234	.094	.073	.779	.236	.084	.083	.991
22. Mean					.716				1.034

TABLE A2.1

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, CANADA

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.007	-.008	.034	.034	1.000	-.008	.034	.036	1.055
2. Beverages	-.025	-.022	.023	.021	.904	-.022	.023	.022	.953
3. Clothing	-.015	-.017	.028	.028	.983	-.017	.028	.029	1.035
4. Housing	-.183	-.183	.043	.039	.892	-.183	.043	.041	.940
5. Durables	.050	.041	.031	.032	1.025	.041	.031	.034	1.078
6. Medical care	.055	.053	.019	.019	1.015	.053	.019	.020	1.069
7. Transport	.084	.083	.038	.034	.892	.083	.038	.036	.940
8. Recreation	.051	.055	.029	.026	.884	.055	.029	.027	.930
9. Education	.001	.002	.014	.014	.952	.002	.014	.014	1.004
10. Miscellaneous	-.011	-.004	.043	.042	.967	-.004	.043	.044	1.017
<u>Income flexibility ϕ^c</u>									
11.	-.554	-.560	.120	.112	.928	-.560	.120	.117	.976
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	-.218	-.220	.114	.117	1.029	-.220	.114	.123	1.085
13. Beverages	.046	.035	.089	.081	.908	.035	.089	.086	.957
14. Clothing	.014	.029	.116	.114	.980	.029	.116	.119	1.032
15. Housing	.581	.577	.157	.136	.868	.577	.157	.143	.915
16. Durables	-.177	-.153	.114	.119	1.041	-.153	.114	.125	1.095
17. Medical care	-.300	-.295	.073	.069	.943	-.295	.073	.072	.992
18. Transport	-.103	-.104	.141	.126	.893	-.104	.141	.133	.940
19. Recreation	-.026	-.038	.114	.103	.901	-.038	.114	.108	.948
20. Education	.047	.044	.036	.037	1.039	.044	.036	.039	1.095
21. Miscellaneous	.136	.124	.115	.114	.989	.124	.115	.120	1.042
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.978	.883	.139						
23. Mean					.954				1.005

TABLE A2.2

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, SWEDEN

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.094	-.095	.024	.023	.981	-.095	.024	.025	1.044
2. Beverages	.008	.011	.015	.014	.931	.011	.015	.015	.991
3. Clothing	.038	.040	.024	.021	.861	.040	.024	.022	.916
4. Housing	-.190	-.189	.035	.030	.866	-.189	.035	.032	.921
5. Durables	.067	.067	.022	.020	.905	.067	.022	.021	.962
6. Medical care	-.022	-.022	.005	.004	.947	-.022	.005	.005	1.008
7. Transport	.138	.135	.030	.025	.834	.135	.030	.026	.887
8. Recreation	.054	.055	.019	.019	1.016	.055	.019	.021	1.080
9. Education	-.001	-.001	.001	.001	.997	-.001	.001	.001	1.061
10. Miscellaneous	.002	-.001	.020	.019	.977	-.001	.020	.021	1.038
<u>Income flexibility ϕ^c</u>									
11.	-.580	-.572	.098	.082	.842	-.572	.098	.088	.895
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	-.099	-.096	.065	.067	1.033	-.096	.065	.071	1.098
13. Beverages	-.073	-.077	.042	.043	1.042	-.077	.042	.046	1.108
14. Clothing	-.156	-.155	.090	.076	.839	-.155	.090	.080	.892
15. Housing	.507	.498	.071	.063	.896	.498	.071	.067	.953
16. Durables	-.142	-.140	.055	.048	.876	-.140	.055	.051	.931
17. Medical care	.067	.068	.023	.022	.975	.068	.023	.024	1.037
18. Transport	-.108	-.104	.068	.061	.895	-.104	.068	.064	.952
19. Recreation	.110	.109	.056	.055	.981	.109	.056	.058	1.043
20. Education	.004	.004	.006	.006	1.000	.004	.006	.006	1.064
21. Miscellaneous	-.110	-.108	.046	.039	.843	-.108	.046	.041	.897
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.343	.303	.058						
23. Mean					.930				.989

TABLE A2.3

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, SWITZERLAND

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.007	-.003	.025	.023	.941	-.003	.025	.025	.995
2. Beverages	.033	.034	.017	.016	.982	.034	.017	.017	1.039
3. Clothing	.055	.055	.017	.015	.900	.055	.017	.016	.952
4. Housing	-.153	-.154	.020	.019	.959	-.154	.020	.020	1.014
5. Durables	.086	.086	.018	.018	.967	.086	.018	.019	1.024
6. Medical care	-.046	-.048	.014	.013	.953	-.048	.014	.014	1.009
7. Transport	.063	.063	.024	.019	.823	.063	.024	.020	.870
8. Recreation	.001	.000	.023	.022	.924	.000	.023	.023	.978
9. Education									
10. Miscellaneous	-.033	-.033	.023	.021	.898	-.033	.023	.022	.950
<u>Income flexibility ϕ^c</u>									
11.	-.551	-.558	.080	.070	.880	-.558	.080	.075	.932
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	-.100	-.107	.070	.067	.959	-.107	.070	.071	1.015
13. Beverages	-.122	-.122	.047	.048	1.001	-.122	.047	.050	1.060
14. Clothing	-.258	-.255	.052	.047	.917	-.255	.052	.050	.971
15. Housing	.342	.338	.060	.050	.847	.338	.060	.053	.896
16. Durables	-.327	-.330	.049	.050	1.009	-.330	.049	.052	1.068
17. Medical care	.169	.168	.031	.030	.946	.168	.031	.031	1.001
18. Transport	.063	.067	.052	.051	.977	.067	.052	.054	1.033
19. Recreation	.074	.080	.058	.056	.964	.080	.058	.060	1.021
20. Education									
21. Miscellaneous	.159	.160	.044	.042	.961	.160	.044	.045	1.017
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.280	.250	.042						
23. Mean					.937				.992

TABLE A2.4

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, DENMARK

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.123	-.123	.032	.030	.927	-.123	.032	.032	.992
2. Beverages	-.028	-.029	.018	.017	.916	-.029	.018	.018	.981
3. Clothing	.044	.045	.015	.015	1.025	.045	.015	.016	1.098
4. Housing	-.130	-.128	.022	.022	1.032	-.128	.022	.024	1.105
5. Durables	.053	.055	.017	.015	.908	.055	.017	.017	.972
6. Medical care	-.010	-.010	.007	.007	.952	-.010	.007	.008	1.020
7. Transport	.188	.189	.024	.021	.877	.189	.024	.022	.939
8. Recreation	.019	.015	.017	.016	.985	.015	.017	.018	1.055
9. Education	-.009	-.009	.006	.005	.860	-.009	.006	.005	.921
10. Miscellaneous	-.005	-.005	.018	.017	.943	-.005	.018	.018	1.010
<u>Income flexibility ϕ^c</u>									
11.	-.465	-.478	.089	.077	.869	-.478	.089	.083	.930
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	-.065	-.063	.081	.075	.920	-.063	.081	.080	.985
13. Beverages	-.010	-.011	.061	.060	.977	-.011	.061	.064	1.046
14. Clothing	-.228	-.228	.056	.055	.986	-.228	.056	.059	1.056
15. Housing	.649	.641	.075	.068	.906	.641	.075	.073	.970
16. Durables	-.286	-.285	.058	.051	.884	-.285	.058	.055	.947
17. Medical care	.017	.015	.024	.024	.996	.015	.024	.025	1.067
18. Transport	-.209	-.210	.068	.066	.968	-.210	.068	.071	1.037
19. Recreation	.084	.090	.051	.053	1.034	.090	.051	.057	1.107
20. Education	.070	.069	.015	.015	1.027	.069	.015	.016	1.100
21. Miscellaneous	-.022	-.018	.058	.050	.859	-.018	.058	.054	.920
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.444	.387	.076						
23. Mean					.945				1.012

TABLE A2.5

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, AUSTRALIA

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.144	-.141	.033	.035	1.045	-.141	.033	.036	1.102
2. Beverages	-.015	-.011	.027	.024	.895	-.011	.027	.025	.943
3. Clothing	.027	.026	.034	.028	.834	.026	.034	.030	.880
4. Housing	-.059	-.058	.034	.031	.916	-.058	.034	.033	.966
5. Durables	.100	.097	.032	.029	.919	.097	.032	.031	.969
6. Medical care	-.018	-.018	.024	.023	.972	-.018	.024	.024	1.025
7. Transport	.075	.071	.039	.037	.962	.071	.039	.040	1.014
8. Recreation	.072	.072	.026	.023	.878	.072	.026	.024	.925
9. Education	.013	.014	.012	.011	.878	.014	.012	.011	.926
10. Miscellaneous	-.050	-.051	.025	.025	1.002	-.051	.025	.027	1.057
<u>Income flexibility ϕ^c</u>									
11.	-.463	-.471	.080	.071	.883	-.471	.080	.075	.932
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	.122	.116	.108	.106	.979	.116	.108	.112	1.032
13. Beverages	-.074	-.085	.079	.070	.885	-.085	.079	.074	.933
14. Clothing	-.227	-.219	.088	.082	.930	-.219	.088	.087	.981
15. Housing	.377	.373	.098	.083	.843	.373	.098	.088	.889
16. Durables	-.234	-.229	.111	.104	.943	-.229	.111	.110	.994
17. Medical care	.017	.015	.047	.051	1.068	.015	.047	.053	1.126
18. Transport	-.015	-.009	.105	.105	1.002	-.009	.105	.111	1.057
19. Recreation	-.031	-.030	.070	.062	.891	-.030	.070	.065	.939
20. Education	-.041	-.042	.022	.021	.955	-.042	.022	.022	1.007
21. Miscellaneous	.106	.109	.070	.068	.966	.109	.070	.072	1.018
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.569	.513	.081						
23. Mean					.936				.986

TABLE A2.6

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, FRANCE

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.115	-.118	.036	.032	.898	-.118	.036	.034	.955
2. Beverages	-.024	-.022	.015	.014	.925	-.022	.015	.014	.982
3. Clothing	.024	.021	.024	.021	.888	.021	.024	.023	.945
4. Housing	-.081	-.077	.025	.023	.924	-.077	.025	.024	.982
5. Durables	.059	.057	.021	.021	1.000	.057	.021	.022	1.064
6. Medical care	-.047	-.047	.034	.031	.904	-.047	.034	.033	.963
7. Transport	.146	.146	.028	.024	.849	.146	.028	.025	.904
8. Recreation	.001	.003	.020	.020	.964	.003	.020	.021	1.026
9. Education	-.001	-.001	.004	.003	.772	-.001	.004	.003	.820
10. Miscellaneous	.039	.038	.023	.022	.970	.038	.023	.024	1.032
<u>Income flexibility ϕ^c</u>									
11.	-.527	-.521	.083	.070	.848	-.521	.083	.075	.904
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	-.011	.000	.135	.125	.926	.000	.135	.133	.985
13. Beverages	-.027	-.036	.064	.059	.924	-.036	.064	.063	.983
14. Clothing	-.280	-.264	.097	.088	.908	-.264	.097	.094	.967
15. Housing	.510	.491	.087	.076	.881	.491	.087	.081	.936
16. Durables	-.299	-.291	.084	.087	1.035	-.291	.084	.093	1.102
17. Medical care	.475	.476	.138	.122	.883	.476	.138	.130	.941
18. Transport	-.311	-.315	.108	.091	.841	-.315	.108	.097	.895
19. Recreation	.060	.055	.091	.087	.956	.055	.091	.092	1.018
20. Education	.001	.000	.012	.010	.840	.000	.012	.011	.891
21. Miscellaneous	-.118	-.117	.088	.082	.935	-.117	.088	.087	.994
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.184	.162	.031						
23. Mean					.908				.966

TABLE A2.7

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, GERMANY

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.113	-.111	.031	.030	.941	-.111	.031	.031	.987
2. Beverages									
3. Clothing	.052	.052	.022	.019	.888	.052	.022	.020	.932
4. Housing	-.133	-.134	.020	.019	.945	-.134	.020	.020	.991
5. Durables	.053	.054	.020	.019	.971	.054	.020	.020	1.019
6. Medical care	-.005	-.004	.010	.009	.960	-.004	.010	.010	1.006
7. Transport	.171	.172	.021	.020	.970	.172	.021	.021	1.018
8. Recreation	.000	-.002	.018	.016	.922	-.002	.018	.017	.967
9. Education									
10. Miscellaneous	-.025	-.026	.015	.016	1.048	-.026	.015	.016	1.099
<u>Income flexibility ϕ^c</u>									
11.	-.602	-.605	.093	.095	1.018	-.605	.093	.099	1.069
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	-.043	-.045	.114	.114	1.000	-.045	.114	.120	1.049
13. Beverages									
14. Clothing	-.251	-.248	.075	.071	.945	-.248	.075	.074	.991
15. Housing	.455	.459	.058	.058	.987	.459	.058	.060	1.035
16. Durables	-.200	-.206	.079	.077	.966	-.206	.079	.080	1.014
17. Medical care	-.007	-.008	.026	.027	1.047	-.008	.026	.028	1.098
18. Transport	-.172	-.178	.071	.077	1.080	-.178	.071	.081	1.132
19. Recreation	.055	.058	.068	.060	.878	.058	.068	.062	.921
20. Education									
21. Miscellaneous	.161	.168	.048	.053	1.102	.168	.048	.055	1.156
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.292	.266	.041						
23. Mean					.980				1.028

TABLE A2.8

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, BELGIUM

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.119	-.119	.045	.042	.932	-.119	.045	.044	.980
2. Beverages	-.001	.002	.027	.024	.879	.002	.027	.025	.924
3. Clothing	.011	.009	.025	.027	1.048	.009	.025	.028	1.102
4. Housing	-.066	-.068	.032	.031	.997	-.068	.032	.033	1.047
5. Durables	.070	.056	.041	.037	.897	.056	.041	.039	.943
6. Medical care	-.028	-.026	.023	.024	1.013	-.026	.023	.025	1.065
7. Transport	-.012	-.008	.028	.029	1.034	-.008	.028	.030	1.087
8. Recreation	-.002	.002	.021	.020	.950	.002	.021	.021	1.000
9. Education	-.002	-.002	.004	.004	.998	-.002	.004	.004	1.049
10. Miscellaneous	.150	.155	.035	.038	1.094	.155	.035	.040	1.149
<u>Income flexibility ϕ^c</u>									
11.	-.127	-.129	.064	.064	1.008	-.129	.064	.067	1.061
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	-.008	-.008	.148	.153	1.034	-.008	.148	.161	1.087
13. Beverages	-.036	-.046	.098	.087	.887	-.046	.098	.092	.932
14. Clothing	-.073	-.061	.097	.100	1.027	-.061	.097	.105	1.080
15. Housing	.164	.169	.122	.115	.943	.169	.122	.121	.991
16. Durables	-.087	-.044	.150	.137	.915	-.044	.150	.145	.962
17. Medical care	.238	.230	.085	.083	.978	.230	.085	.088	1.028
18. Transport	.179	.166	.102	.104	1.026	.166	.102	.110	1.079
19. Recreation	.045	.031	.074	.070	.945	.031	.074	.074	.994
20. Education	.004	.003	.016	.016	1.024	.003	.016	.017	1.077
21. Miscellaneous	-.426	-.441	.139	.145	1.038	-.441	.139	.152	1.091
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.671	.607	.094						
23. Mean					.984				1.035

TABLE A2.9

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, NORWAY

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.180	-.183	.029	.027	.919	-.183	.029	.028	.975
2. Beverages	.010	.011	.017	.014	.861	.011	.017	.015	.915
3. Clothing	.021	.020	.021	.020	.916	.020	.021	.021	.972
4. Housing	-.145	-.145	.024	.022	.927	-.145	.024	.024	.985
5. Durables	.026	.026	.019	.019	.955	.026	.019	.020	1.015
6. Medical care	-.009	-.010	.013	.013	1.007	-.010	.013	.014	1.070
7. Transport	.293	.291	.025	.023	.933	.291	.025	.024	.991
8. Recreation	.002	.005	.020	.020	1.015	.005	.020	.022	1.078
9. Education	-.003	-.002	.006	.006	.914	-.002	.006	.006	.970
10. Miscellaneous	-.016	-.013	.022	.020	.936	-.013	.022	.021	.994
<u>Income flexibility ϕ^c</u>									
11.	-.485	-.496	.083	.082	.989	-.496	.083	.087	1.050
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	.160	.169	.102	.096	.945	.169	.102	.102	1.003
13. Beverages	-.073	-.076	.061	.055	.900	-.076	.061	.059	.956
14. Clothing	-.204	-.196	.080	.071	.888	-.196	.080	.075	.943
15. Housing	.502	.495	.084	.078	.935	.495	.084	.083	.992
16. Durables	-.026	-.027	.067	.066	.981	-.027	.067	.070	1.042
17. Medical care	.018	.021	.037	.041	1.094	.021	.037	.044	1.161
18. Transport	-.550	-.545	.081	.080	.997	-.545	.081	.085	1.059
19. Recreation	.148	.139	.086	.083	.964	.139	.086	.088	1.024
20. Education	.000	-.001	.018	.017	.934	-.001	.018	.018	.992
21. Miscellaneous	.024	.020	.069	.064	.931	.020	.069	.068	.989
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.413	.367	.070						
23. Mean					.950				1.008

TABLE A2.10

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, NETHERLANDS

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.132	-.137	.043	.038	.884	-.137	.043	.040	.924
2. Beverages	-.024	-.024	.014	.012	.877	-.024	.014	.013	.917
3. Clothing	.157	.161	.029	.028	.968	.161	.029	.029	1.012
4. Housing	-.046	-.048	.020	.018	.897	-.048	.020	.019	.938
5. Durables	.051	.052	.031	.028	.887	.052	.031	.029	.927
6. Medical care	-.027	-.026	.013	.014	1.064	-.026	.013	.014	1.113
7. Transport	.051	.052	.016	.015	.976	.052	.016	.016	1.020
8. Recreation	-.002	-.003	.016	.016	1.003	-.003	.016	.017	1.049
9. Education	-.006	-.008	.012	.012	1.004	-.008	.012	.012	1.050
10. Miscellaneous	-.023	-.018	.025	.024	.928	-.018	.025	.025	.970
<u>Income flexibility ϕ^c</u>									
11.	-.842	-.840	.104	.101	.967	-.840	.104	.106	1.011
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	.049	.069	.198	.188	.952	.069	.198	.197	.995
13. Beverages	.085	.084	.085	.078	.926	.084	.085	.082	.968
14. Clothing	-.896	-.913	.141	.142	1.005	-.913	.141	.148	1.050
15. Housing	.228	.236	.095	.087	.916	.236	.095	.091	.958
16. Durables	-.129	-.128	.160	.165	1.033	-.128	.160	.173	1.079
17. Medical care	.316	.316	.052	.055	1.067	.316	.052	.058	1.116
18. Transport	-.045	-.044	.090	.085	.952	-.044	.090	.089	.995
19. Recreation	.046	.050	.064	.059	.924	.050	.064	.062	.967
20. Education	.095	.100	.046	.046	1.011	.100	.046	.048	1.057
21. Miscellaneous	.250	.229	.112	.101	.900	.229	.112	.106	.941
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	1.068	.977	.134						
23. Mean					.959				1.003

TABLE A2.11

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, ICELAND

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.129	-.128	.022	.022	1.017	-.128	.022	.024	1.106
2. Beverages	-.020	-.019	.018	.015	.838	-.019	.018	.016	.911
3. Clothing	.037	.036	.019	.016	.853	.036	.019	.018	.927
4. Housing	-.198	-.197	.027	.020	.736	-.197	.027	.022	.800
5. Durables	.089	.090	.018	.016	.880	.090	.018	.018	.957
6. Medical care	-.015	-.015	.018	.016	.884	-.015	.018	.017	.961
7. Transport	.181	.179	.022	.018	.846	.179	.022	.020	.919
8. Recreation	.023	.022	.019	.015	.815	.022	.019	.017	.886
9. Education	-.004	-.003	.006	.005	.796	-.003	.006	.005	.866
10. Miscellaneous	.036	.036	.014	.013	.964	.036	.014	.014	1.048
<u>Income flexibility ϕ^c</u>									
11.	-.695	-.711	.091	.080	.881	-.711	.091	.087	.958
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	.033	.047	.176	.172	.973	.047	.176	.187	1.058
13. Beverages	.106	.095	.126	.122	.972	.095	.126	.133	1.057
14. Clothing	-.262	-.248	.192	.146	.764	-.248	.192	.159	.831
15. Housing	.536	.511	.197	.170	.861	.511	.197	.185	.936
16. Durables	-.272	-.273	.146	.140	.955	-.273	.146	.152	1.038
17. Medical care	.289	.290	.102	.099	.974	.290	.102	.108	1.059
18. Transport	-.472	-.465	.162	.147	.909	-.465	.162	.160	.988
19. Recreation	.015	.018	.115	.102	.885	.018	.115	.111	.963
20. Education	.020	.012	.035	.031	.894	.012	.035	.034	.971
21. Miscellaneous	.006	.013	.114	.101	.882	.013	.114	.110	.959
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	1.966	1.663	.372						
23. Mean					.885				.962

TABLE A2.12

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, FINLAND

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.126	-.124	.037	.035	.952	-.124	.037	.037	1.012
2. Beverages	.024	.022	.027	.023	.864	.022	.027	.025	.918
3. Clothing	.055	.058	.027	.025	.938	.058	.027	.027	.997
4. Housing	-.122	-.124	.034	.030	.896	-.124	.034	.032	.952
5. Durables	.042	.045	.024	.022	.904	.045	.024	.023	.960
6. Medical care	-.009	-.011	.016	.014	.880	-.011	.016	.015	.935
7. Transport	.109	.105	.027	.027	.996	.105	.027	.029	1.059
8. Recreation	.053	.056	.019	.020	1.040	.056	.019	.021	1.105
9. Education	.002	.002	.011	.010	.853	.002	.011	.010	.907
10. Miscellaneous	-.028	-.029	.024	.026	1.069	-.029	.024	.027	1.136
<u>Income flexibility ϕ^c</u>									
11.	-.378	-.391	.089	.075	.847	-.391	.089	.080	.901
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	-.065	-.071	.171	.172	1.005	-.071	.171	.183	1.068
13. Beverages	-.008	-.001	.127	.117	.920	-.001	.127	.124	.978
14. Clothing	-.441	-.446	.142	.127	.891	-.446	.142	.135	.947
15. Housing	.419	.415	.164	.158	.963	.415	.164	.168	1.024
16. Durables	-.077	-.087	.124	.112	.902	-.087	.124	.119	.959
17. Medical care	.095	.101	.073	.073	.998	.101	.073	.077	1.060
18. Transport	-.075	-.057	.135	.138	1.021	-.057	.135	.147	1.085
19. Recreation	-.090	-.102	.098	.102	1.044	-.102	.098	.109	1.109
20. Education	-.058	-.059	.048	.042	.881	-.059	.048	.045	.936
21. Miscellaneous	.300	.307	.114	.116	1.021	.307	.114	.123	1.085
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	1.302	1.153	.221						
23. Mean					.947				1.006

TABLE A2.13

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, AUSTRIA

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.175	-.172	.039	.038	.979	-.172	.039	.041	1.040
2. Beverages	-.036	-.038	.025	.022	.870	-.038	.025	.023	.922
3. Clothing	.090	.086	.030	.029	.958	.086	.030	.031	1.017
4. Housing	-.099	-.095	.031	.028	.906	-.095	.031	.030	.962
5. Durables	.085	.077	.030	.027	.891	.077	.030	.028	.946
6. Medical care	-.060	-.057	.016	.017	1.103	-.057	.016	.018	1.171
7. Transport	.282	.278	.035	.032	.913	.278	.035	.034	.970
8. Recreation	-.011	-.009	.020	.020	1.001	-.009	.020	.021	1.063
9. Education	-.005	-.005	.005	.004	.861	-.005	.005	.005	.913
10. Miscellaneous	-.071	-.066	.036	.034	.929	-.066	.036	.036	.986
<u>Income flexibility ϕ^c</u>									
11.	-.154	-.146	.103	.083	.807	-.146	.103	.088	.857
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	.247	.238	.159	.145	.912	.238	.159	.154	.968
13. Beverages	.042	.048	.095	.088	.928	.048	.095	.093	.985
14. Clothing	-.222	-.200	.122	.117	.955	-.200	.122	.124	1.014
15. Housing	.458	.437	.119	.099	.837	.437	.119	.105	.888
16. Durables	-.270	-.243	.114	.107	.941	-.243	.114	.114	.999
17. Medical care	.158	.151	.052	.056	1.064	.151	.052	.059	1.129
18. Transport	-.569	-.562	.154	.135	.877	-.562	.154	.143	.931
19. Recreation	.092	.085	.080	.078	.965	.085	.080	.082	1.024
20. Education	.010	.009	.021	.018	.820	.009	.021	.019	.869
21. Miscellaneous	.053	.037	.127	.119	.936	.037	.127	.126	.993
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.524	.465	.087						
23. Mean					.926				.983

TABLE A2.14

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, JAPAN

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.105	-.104	.043	.034	.803	-.104	.043	.038	.897
2. Beverages									
3. Clothing	.069	.066	.031	.027	.887	.066	.031	.031	.991
4. Housing	-.130	-.134	.036	.034	.930	-.134	.036	.038	1.039
5. Durables	.073	.071	.022	.019	.877	.071	.022	.022	.981
6. Medical care	-.020	-.015	.026	.021	.813	-.015	.026	.023	.909
7. Transport	.031	.032	.026	.023	.870	.032	.026	.026	.972
8. Recreation	.013	.009	.025	.023	.947	.009	.025	.026	1.058
9. Education									
10. Miscellaneous	.070	.074	.030	.031	1.029	.074	.030	.035	1.150
<u>Income flexibility ϕ^c</u>									
11.	-.370	-.389	.108	.089	.821	-.389	.108	.099	.916
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	-.140	-.130	.191	.155	.812	-.130	.191	.173	.907
13. Beverages									
14. Clothing	-.335	-.331	.126	.107	.851	-.331	.126	.120	.951
15. Housing	.711	.725	.162	.142	.878	.725	.162	.159	.980
16. Durables	-.403	-.402	.099	.088	.898	-.402	.099	.099	1.003
17. Medical care	.309	.288	.110	.097	.883	.288	.110	.108	.986
18. Transport	.015	.013	.113	.096	.854	.013	.113	.108	.953
19. Recreation	-.052	-.047	.113	.104	.926	-.047	.113	.117	1.034
20. Education									
21. Miscellaneous	-.106	-.115	.140	.122	.874	-.115	.140	.136	.976
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.795	.637	.190						
23. Mean					.880				.983

TABLE A2.15

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, U.K.

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.127	-.125	.029	.025	.863	-.125	.029	.027	.916
2. Beverages	.002	.003	.011	.010	.971	.003	.011	.011	1.029
3. Clothing	.017	.013	.020	.019	.974	.013	.020	.020	1.034
4. Housing	-.118	-.113	.022	.022	.968	-.113	.022	.023	1.027
5. Durables	.088	.084	.019	.017	.937	.084	.019	.018	.995
6. Medical care	-.003	-.003	.005	.005	.913	-.003	.005	.005	.969
7. Transport	.087	.085	.023	.019	.840	.085	.023	.020	.892
8. Recreation	.027	.027	.017	.017	.987	.027	.017	.018	1.048
9. Education	.000	.002	.009	.009	1.001	.002	.009	.009	1.063
10. Miscellaneous	.027	.027	.021	.022	1.035	.027	.021	.023	1.099
<u>Income flexibility ϕ^c</u>									
11.	-.396	-.405	.075	.066	.890	-.405	.075	.071	.946
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	-.090	-.093	.076	.069	.906	-.093	.076	.074	.962
13. Beverages	-.046	-.050	.032	.037	1.145	-.050	.032	.039	1.215
14. Clothing	-.066	-.055	.065	.064	.993	-.055	.065	.068	1.055
15. Housing	.280	.268	.061	.055	.899	.268	.061	.058	.954
16. Durables	-.216	-.208	.057	.050	.871	-.208	.057	.053	.925
17. Medical care	-.002	-.003	.017	.014	.870	-.003	.017	.015	.924
18. Transport	.055	.057	.062	.055	.885	.057	.062	.059	.940
19. Recreation	.080	.078	.052	.048	.911	.078	.052	.050	.968
20. Education	.015	.011	.020	.021	1.031	.011	.020	.022	1.095
21. Miscellaneous	-.010	-.008	.067	.059	.885	-.008	.067	.063	.940
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.306	.271	.051						
23. Mean					.942				1.000

TABLE A2.16

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, SPAIN

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.049	-.048	.066	.055	.835	-.048	.066	.060	.908
2. Beverages	-.003	-.005	.027	.021	.778	-.005	.027	.023	.846
3. Clothing	.031	.027	.036	.033	.915	.027	.036	.036	.994
4. Housing	-.114	-.110	.048	.041	.868	-.110	.048	.045	.943
5. Durables	.039	.037	.032	.027	.860	.037	.032	.030	.935
6. Medical care	-.002	-.001	.030	.022	.748	-.001	.030	.024	.813
7. Transport	.130	.129	.033	.029	.891	.129	.033	.032	.969
8. Recreation	.015	.014	.022	.018	.808	.014	.022	.020	.878
9. Education	-.012	-.011	.012	.011	.917	-.011	.012	.012	.997
10. Miscellaneous	-.034	-.031	.024	.025	1.043	-.031	.024	.028	1.134
<u>Income flexibility ϕ^c</u>									
11.	-.291	-.298	.078	.066	.836	-.298	.078	.071	.907
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	-.349	-.347	.307	.262	.856	-.347	.307	.285	.930
13. Beverages	-.016	-.007	.144	.113	.785	-.007	.144	.123	.853
14. Clothing	-.235	-.214	.153	.146	.949	-.214	.153	.158	1.032
15. Housing	.406	.378	.239	.209	.875	.378	.239	.227	.951
16. Durables	-.216	-.205	.149	.125	.839	-.205	.149	.136	.911
17. Medical care	.220	.213	.122	.096	.786	.213	.122	.104	.855
18. Transport	-.155	-.154	.149	.143	.959	-.154	.149	.155	1.043
19. Recreation	.009	.015	.092	.078	.851	.015	.092	.085	.925
20. Education	.054	.047	.052	.047	.903	.047	.052	.051	.982
21. Miscellaneous	.282	.275	.113	.118	1.038	.275	.113	.128	1.128
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.499	.423	.094						
23. Mean					.873				.949

TABLE A2.17

ML ESTIMATION WITH Σ^{*c} : SUMMARY RESULTS FOR 100 TRIALS, ITALY

Commodity	True value	Σ^{*c} unknown				Σ^{*c} known			
		Mean	RMSE	RMSASE	(5)/(4)	Mean	RMSE	RMSASE	(9)/(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<u>Income coefficients β_i^c</u>									
1. Food	-.044	-.042	.031	.028	.905	-.042	.031	.029	.961
2. Beverages	-.017	-.016	.019	.015	.815	-.016	.019	.016	.865
3. Clothing	.077	.073	.017	.017	1.014	.073	.017	.018	1.077
4. Housing	-.067	-.063	.027	.022	.810	-.063	.027	.023	.861
5. Durables	.036	.034	.015	.014	.958	.034	.015	.015	1.016
6. Medical care	.001	.001	.011	.011	.972	.001	.011	.012	1.032
7. Transport	.044	.044	.018	.018	.979	.044	.018	.019	1.039
8. Recreation	-.024	-.024	.014	.015	1.039	-.024	.014	.016	1.103
9. Education	-.002	-.002	.004	.004	.913	-.002	.004	.004	.969
10. Miscellaneous	-.006	-.005	.019	.019	1.016	-.005	.019	.021	1.079
<u>Income flexibility ϕ^c</u>									
11.	-.184	-.183	.069	.069	1.002	-.183	.069	.073	1.064
<u>Constant terms α_i^c ($\times 100$)</u>									
12. Food	-.225	-.230	.126	.114	.907	-.230	.126	.121	.963
13. Beverages	.023	.019	.081	.068	.845	.019	.081	.072	.897
14. Clothing	-.301	-.285	.068	.069	1.007	-.285	.068	.073	1.069
15. Housing	.161	.145	.102	.084	.818	.145	.102	.089	.869
16. Durables	-.083	-.076	.057	.058	1.011	-.076	.057	.062	1.073
17. Medical care	.114	.115	.047	.047	.994	.115	.047	.050	1.055
18. Transport	.109	.109	.070	.071	1.021	.109	.070	.076	1.085
19. Recreation	.114	.116	.063	.062	.995	.116	.063	.066	1.057
20. Education	-.002	-.002	.017	.016	.935	-.002	.017	.017	.993
21. Miscellaneous	.090	.089	.079	.076	.967	.089	.079	.081	1.027
<u>Proportionality constant λ^{c2} ($\times 10^4$)</u>									
22.	.344	.305	.057						
23. Mean					.949				1.007

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