THE ROTTERDAM DEMAND MODEL
AND ITS APPLICATION IN MARKETING

by

Kenneth W. Clements

and

E. Antony Selvanathan

The University of Western Australia

Discussion Paper 87.16

December 1987

ISSN 0811-6067

ISBN 0-86422-603-9

THE ROTTERDAM DEMAND MODEL

AND ITS APPLICATION

IN MARKETING*

by

Kenneth W. Clements
Economic Research Centre
Department of Economics
The University of Western Australia

and

E. Antony Selvanathan
University of Jaffna and
Economic Research Centre
Department of Economics
The University of Western Australia

*Part of this research was carried out while the first author was McKethan-Matherly Research Fellow at the University of Florida, Gainesville, and he is grateful for the partial financial support of the McKethan-Matherly Eminent Scholar Chair. We would like to acknowledge the assistance of Peter Goldschmidt; and the helpful comments of Subrata Sen, the Area Editor and three referees of this journal.

Abstract

This paper shows how the system-wide approach to demand analysis can be utilized in marketing. In the context of the Rotterdam model, we describe how the approach can be applied to narrowly defined groups of goods (such as beer, wine and spirits) to estimate income and price elasticities of demand. The paper also provides extensions to deal with advertising and introduces a new way of identifying market structure.

1. INTRODUCTION

The theory of the utility-maximizing consumer enjoys widespread acceptance and use in economics. The objective of this theory is to describe the way in which consumption of a good depends on the consumer's income and the prices he faces. In recent years much progress has been made in combining this theory with the empirical analysis of consumption data. The data are used to test the theory, and the theory is used to provide structure to the empirical analysis. This work is known as the system-wide approach in which demand equations for all n goods are estimated simultaneously. For surveys of these developments, see Barten (1977), Brown and Deaton (1972), Philips (1974), Powell (1974), Theil (1975/76, 1980a, 1980b) and Theil and Clements (1987).

A leading example of a system of demand equations is the Rotterdam model, due to Barten (1964) and Theil (1965); the name "Rotterdam" comes from the location of Barten and Theil in the 1960s. This model has strong links with the economic theory of the consumer and its elegant simplicity has contributed to its popularity and influential role in the development of the system-wide approach.

The objective of this paper is to give a self-contained account of the Rotterdam model so that it is accessible to nonspecialists and to indicate how it may be of use in marketing research. The most frequent use of the model involves the demand for broad commodity groups such as food, clothing, housing and so on. For marketing purposes, however, these groups are much too broad.

Fortunately, under certain conditions the model can be reformulated so that it can be applied to narrower groups of goods such as beer, wine and spirits or different brands of a certain product. But even with reformulation, to make the model of further use in marketing requires extensions to deal with advertising and other elements of the marketing mix. In this paper we show how all this can be done as well as how the model provides a new way of identifying market structure.

Sections 2 and 3 of the paper set out the economic theory of the consumer and the Rotterdam model. In Section 4 we use the convergence approach to aggregation to show how the microeconomic theory of the consumer can be applied with macro data. Sections 5 and 6 show how the model can be used to analyse the demand for narrowly defined goods, with the consumption of beer, wine and spirits used as an example. In Section 7 we introduce a new way of identifying market structure which is based on consumption theory. We then use the Rotterdam model to illustrate the application of this procedure. Sections 8 and 9 deal with advertising and alternatives to the Rotterdam model. Concluding comments are given in Section 10. Throughout the paper we draw on Theil (1975/76, 1980a) and Theil and Clements (1987).

2. DIFFERENTIAL DEMAND EQUATIONS

We write p_i, q_i for the price and quantity of good i ($i=1, \dots, n$). The consumer behaves as if he chooses $q_1, \dots, q_n$ to maximize his utility function, $u(q_1, \dots, q_n)$, subject to the budget

constraint $M = \sum_{i=1}^n p_i q_i$, M being income. The optimal quantities obviously depend on income and prices, so we write

$$(2.1) \quad q_i = q_i(M, p_1, \dots, p_n) \quad i=1, \dots, n.$$

These are the demand functions.

The differential of (2.1) can be expressed as (Theil and Clements, 1987)

$$(2.2) \quad w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^n v_{ij} d\left[\log \frac{p_j}{P'}\right],$$

where $w_i = p_i q_i / M$ is the budget share of i ; $\theta_i = \partial(p_i q_i) / \partial M$ is the i^{th} marginal share; $d(\log Q) = \sum_{i=1}^n w_i d(\log q_i)$ is the Divisia volume index of the change in the consumer's real income; $d[\log (p_j / P')]$ is the change in the deflated price of j with $d(\log P') = \sum_{i=1}^n \theta_i d(\log p_i)$ the Frisch price index; and

$$(2.3) \quad v_{ij} = \frac{\lambda}{M} p_i u^{ij} p_j,$$

is the $(i, j)^{\text{th}}$ price coefficient, with λ the marginal utility of income and u^{ij} the $(i, j)^{\text{th}}$ element of the inverse of the Hessian of the utility function, $U = [\partial^2 u / \partial q_i \partial q_j]$. These v_{ij} 's satisfy

$$(2.4) \quad \sum_{j=1}^n v_{ij} = \phi \theta_i \quad i=1, \dots, n,$$

where $\phi = [\partial(\log \lambda) / \partial(\log M)]^{-1}$ is the income flexibility, the inverse of the income elasticity of the marginal utility of income.

Equation (2.2) is formulated in terms of relative (or deflated) prices. The corresponding absolute price version is

$$(2.5) \quad w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^n \pi_{ij} d(\log p_j) ,$$

where $\pi_{ij} = v_{ij} - \phi \theta_i \theta_j$ is the $(i,j)^{\text{th}}$ Slutsky coefficient. These coefficients satisfy demand homogeneity

$$(2.6) \quad \sum_{j=1}^n \pi_{ij} = 0 \quad i=1, \dots, n;$$

and Slutsky symmetry

$$(2.7) \quad \pi_{ij} = \pi_{ji} \quad i, j=1, \dots, n.$$

The $n \times n$ matrix $[\pi_{ij}]$ is negative semidefinite with rank $(n-1)$.

Demand homogeneity implies that an equiproportionate change in all prices has no effect on consumption, while Slutsky symmetry implies that the substitution effects of price changes are symmetric. Dividing both sides of (2.5) by w_i , we obtain θ_i/w_i as the income elasticity of demand for i ; and π_{ij}/w_i as the $(i,j)^{\text{th}}$ price elasticity.

3. THE ROTTERDAM MODEL

The differential demand equations presented in the last section are in terms of infinitesimal changes. The Rotterdam model is

simply a finite-change version of these demand equations. To obtain the relative price version of the Rotterdam model we make the following adjustments to (2.2). The budget share w_i is replaced with the arithmetic average of the budget share in $t-1$ and t , $\bar{w}_{it} = \frac{1}{2}(w_{it} + w_{i,t-1})$; $d(\log q_i)$ is replaced with the change in the logarithm of q_i from period $t-1$ to t , $Dq_{it} = \log q_{it} - \log q_{i,t-1}$; $d(\log Q)$ is replaced with $DQ_t = \sum_{i=1}^n \bar{w}_{it} Dq_{it}$; $d(\log p_j)$ is replaced with $Dp_{jt} = \log p_{jt} - \log p_{j,t-1}$; and $d(\log P')$ with $DP'_t = \sum_{i=1}^n \theta_i Dp_{it}$.

The marginal share and price coefficients in (2.2) are not necessarily constant. In the Rotterdam model, as a simplification these are treated as constants. After making the above adjustments to (2.2) we obtain

$$(3.1) \quad \bar{w}_{it} Dq_{it} = \theta_i DQ_t + \sum_{j=1}^n v_{ij} (Dp_{jt} - DP'_t),$$

which is the i^{th} demand equation of the relative price version of the Rotterdam model. The complete model is made up of n equations of the form (3.1), one for each good. It follows from the definition of the price coefficient v_{ij} given in (2.3) that this coefficient gives information about the extent to which commodities i and j interact in the utility function. It will often be the case that when the number of goods (n) is large, it is reasonable to postulate that many of the v_{ij} 's are zero; more on this in Section 5. It also follows from the definition of v_{ij} that the $n \times n$ matrix $[v_{ij}]$ is symmetric negative definite.

The i^{th} demand equation of the absolute price version of the Rotterdam model is the finite-change version of (2.5),

$$(3.2) \quad \bar{w}_{it} Dq_{it} = \theta_i DQ_t + \sum_{j=1}^n \pi_{ij} Dp_{jt} .$$

The Slutsky coefficients in (3.2) are subject to the homogeneity and symmetry constraints (2.6) and (2.7). As can be seen, (3.2) and the constraints are all linear in the parameters which makes estimation and hypothesis testing straightforward.

4. AGGREGATION OVER INDIVIDUAL CONSUMERS

The demand equations derived above are based on the utility-maximizing behaviour of the individual consumer. As data in economics are usually available only in aggregate form, it is natural to raise the question, to what extent do the previous results carry over to the aggregate or market demand functions? In this section, which is mainly based on Barnett (1979), Selvanathan (1987) and Theil (1975/76), we use the convergence approach to aggregation to analyse this problem. This approach is a statistical tool which is used to analyse the market demand functions when the size of the consumer population becomes very large.

The Micro and Macro Demand Equations

Consider equation (2.5), the absolute price version of the differential demand equation for good i . As that equation refers to an individual consumer we add a consumer subscript c ,

$$(4.1) \quad w_{ic} d(\log q_{ic}) = \theta_{ic} d(\log Q_c) + \sum_{j=1}^n \pi_{ijc} d(\log p_j),$$

$c=1, \dots, N,$

where we have made the assumption that all consumers pay the same prices; and N is the number of consumers. The micro coefficients θ_{ic} and π_{ijc} of the micro demand equation (4.1) are functions of income and prices and satisfy the homogeneity and symmetry restrictions (2.6) and (2.7).

We define the per capita variables which we shall refer to as the macro variables.

$$\bar{q}_i = \frac{1}{N} \sum_{c=1}^N q_{ic}, \quad \bar{M} = \frac{1}{N} \sum_{c=1}^N M_c = \sum_{i=1}^n p_i \bar{q}_i, \quad \bar{w}_i = \frac{p_i \bar{q}_i}{\bar{M}}.$$

Our objective is to develop a macro demand equation in terms of macro variables and macro coefficients which is analogous to (4.1). To obtain this we first multiply both sides of that equation by $M_c/N\bar{M}$ and then take the summation over $c=1, \dots, N$,

$$(4.2) \quad \sum_{c=1}^N \frac{M_c}{N\bar{M}} w_{ic} d(\log q_{ic}) = \sum_{c=1}^N \frac{M_c}{N\bar{M}} \theta_{ic} d(\log Q_c) + \sum_{c=1}^N \sum_{j=1}^n \frac{M_c}{N\bar{M}} \pi_{ijc} d(\log p_j).$$

The Left-hand Variable of (4.2)

The left-hand term of (4.2) is

$$\begin{aligned}
 \sum_{c=1}^N \frac{M_c}{NM} w_{ic} d(\log q_{ic}) &= \sum_{c=1}^N \frac{M_c}{NM} \frac{p_i q_{ic}}{M_c} \frac{dq_{ic}}{q_{ic}} = \frac{p_i}{NM} \sum_{c=1}^N dq_{ic} \\
 (4.3) \qquad \qquad \qquad &= \frac{p_i d\bar{q}_i}{\bar{M}} = \bar{w}_i d(\log \bar{q}_i) .
 \end{aligned}$$

The last member of (4.3) is the macro analogue of the left-hand variable of equation (4.1).

Assumptions

We assume that $M_1, \dots, M_N$ are identically and independently distributed. We also assume that the macro coefficients, defined as

$$\bar{\theta}_i = \frac{E[M_c \theta_{ic}]}{E[M_c]} , \quad \bar{\pi}_{ij} = \frac{E[M_c \pi_{ijc}]}{E[M_c]} , \quad i, j=1, \dots, n,$$

exist and are finite. These macro coefficients are population versions of weighted-average micro coefficients with weights proportional to income M_c . It can easily be shown that the definition of the macro Slutsky coefficients implies that they satisfy homogeneity and symmetry restrictions.

$$\sum_{j=1}^n \bar{\pi}_{ij} = 0, \quad i=1, \dots, n, \quad \bar{\pi}_{ij} = \bar{\pi}_{ji}, \quad i, j=1, \dots, n;$$

that $\sum_{i=1}^n \bar{\theta}_i = 1$; and that the matrix $[\bar{\pi}_{ij}]$ is negative semidefinite with rank $(n - 1)$.

The Right-hand Variables of (4.2)

Applying the convergence approach under the above assumptions, it can be shown that

$$(4.4) \quad \sum_{c=1}^N \frac{M_c}{NM} \theta_{ic} d(\log Q_c) = \bar{\theta}_i d(\log \bar{Q}) + \frac{\text{Cov}[K_{ic}, V_c]}{E[M_c]} + o_p(1),$$

where $d(\log \bar{Q})$ is the change in per capita real income; $K_{ic} = M_c(\theta_{ic} - \bar{\theta}_i)$; $V_c = d(\log Q_c)$; and $o_p(1)$ denotes a random variable that converges in probability to zero as N (the number of consumers) tends to infinity. It can also be shown that

$$(4.5) \quad \sum_{c=1}^N \sum_{j=1}^n \frac{M_c}{NM} \pi_{ijc} d(\log p_j) = \sum_{j=1}^n \bar{\pi}_{ij} d(\log p_j) + o_p(1).$$

For derivations of these results, which are based on Khinchine's theorem, see Barnett (1979) or Selvanathan (1987).

The Final Form of the Macro Demand Equations

Substituting (4.3), (4.4) and (4.5) in (4.2) we obtain

$$\bar{w}_i d(\log \bar{q}_i) = \bar{\theta}_i d(\log \bar{Q}) + \sum_{j=1}^n \bar{\pi}_{ij} d(\log p_j) + \frac{\text{Cov}[K_{ic}, V_c]}{E[M_c]} + o_p(1) .$$

In Appendix A1 we argue that $\text{Cov}[K_{ic}, V_c]/E[M_c]$ is likely to be negligibly small. Therefore, for sufficiently large N , we can write the above equation as

$$(4.6) \quad \bar{w}_i d(\log \bar{q}_i) = \bar{\theta}_i d(\log \bar{Q}) + \sum_{j=1}^n \bar{\pi}_{ij} d(\log p_j) ,$$

where we have also used $\text{plim}_{N \rightarrow \infty} o_p(1) = 0$.

Equation (4.6) is the macro analogue of the micro demand equation (4.1). The coefficients of (4.6) satisfy the homogeneity and symmetry restrictions (2.6) and (2.7). Consequently, the convergence approach reveals that all of the restrictions at the individual consumer level carry over to the aggregate equations. This is the justification for applying the Rotterdam model to aggregate data. In order to keep the notation as simple as possible, in what follows we shall not distinguish between micro coefficients and variables and their macro counterparts.

The above results are very appealing as they imply that the microeconomic theory of the consumer is directly applicable with aggregate data. The homogeneity and symmetry constraints play the role of reducing the number of free parameters to be estimated; and more often than not prices are highly collinear making it impossible to estimate parameters precisely without these constraints. It should nevertheless be acknowledged that the use of aggregate data can yield information about demand behaviour at the aggregate level only, not at the individual level. While some marketing applications no doubt require details of the heterogeneity at the individual level, for others aggregate level inferences are sufficient. An example of the latter is the case of alcoholic beverages in Australia where one of the major marketing decisions faced by brewers in recent years is whether or not beer and wine are closely competing for the drinker's dollar. This example will be taken up below.

In concluding this section, we should note a caveat regarding the convergence approach. Equations (4.4) and (4.5) contain a $o_p(1)$ term with $\text{plim}_{N \rightarrow \infty} o_p(1) = 0$. Like all probability statements, this convergence result cannot of course guarantee that the actual value of this term is zero. It should also be pointed out that not all applications of the Rotterdam model have been with aggregate data. The use of micro data to estimate the model requires that prices vary across individuals, as is the case with panel data. A recent example of this approach is Kiefer (1984). These micro applications are still quite rare due to the lack of databases containing cross-sectional price variability.

5. THE DEMAND FOR GROUPS OF GOODS AND CONDITIONAL DEMAND EQUATIONS

The analysis thus far has dealt with the demand for all n goods. We now show how to analyse the demand for a sub-set or group of goods.

Block-independent Preferences

Let the n goods now be divided into G groups, $S_1, \dots, S_G$, such that each good belongs to only one group. Further, let the consumer's preferences be such that the utility function is the sum of G subutility functions, each involving the quantities of only one group,

$$(5.1) \quad u(\mathbf{q}) = \sum_{g=1}^G u_g(\mathbf{q}_g),$$

where $\mathbf{q} = (q_1, \dots, q_n)'$ and $\mathbf{q}_g$ is the vector of the q_i 's that fall under S_g . Under (5.1), the marginal utility of a good depends only on the consumption of goods belonging to the same group. When the goods are numbered appropriately, the Hessian U and its inverse become block-diagonal. Accordingly, specification (5.1) is known as block-independent preferences.

Under block independence, $v_{ij} = 0$ for i and j in different groups. Thus (2.4) for $i \in S_g$ becomes

$$(5.2) \quad \sum_{j \in S_g} v_{ij} = \phi \theta_i \quad g=1, \dots, G.$$

The demand equation (3.1) for $i \in S_g$ becomes

$$(5.3) \quad \bar{w}_{it} Dq_{it} = \theta_i DQ_t + \sum_{j \in S_g} v_{ij} (Dp_{jt} - DP'_t),$$

so that the only deflated prices which appear are those of goods belonging to the same group as i .

Composite Demand Equations

We write

$$\bar{W}_{gt} = \sum_{i \in S_g} \bar{w}_{it}, \quad \theta_g = \sum_{i \in S_g} \theta_i$$

for the budget and marginal shares of group S_g , and define the group volume and Frisch price indexes as

$$DQ_{gt} = \sum_{i \in S_g} \frac{\bar{w}_{it}}{\bar{W}_{gt}} Dq_{it}, \quad DP'_{gt} = \sum_{i \in S_g} \frac{\theta_i}{\theta_g} Dp_{it}.$$

If we then add (5.3) over $i \in S_g$ and use (5.2) as well as the symmetry of the matrix $[v_{ij}]$, we obtain the composite demand equation for S_g as a group (Theil and Clements, 1987),

$$(5.4) \quad \bar{w}_{gt} DQ_{gt} = \theta_g DQ_t + \phi \theta_g (DP'_{gt} - DP'_t) .$$

Thus under block-independence, only the deflated price of the group, $DP'_{gt} - DP'_t$, and income affect the demand for the group as a whole. The income and own-price elasticities for the group are $\theta_g / \bar{w}_{gt}$ and $\phi \theta_g / \bar{w}_{gt}$, respectively.

Conditional Demand Equations

Combining (5.3) and (5.4) to eliminate DQ_t , we obtain (Theil and Clements, 1987)

$$(5.5) \quad \bar{w}_{it} Dq_{it} = \frac{\theta_i}{\theta_g} \bar{w}_{gt} DQ_{gt} + \sum_{j \in S_g} v_{ij} (Dp_{jt} - DP'_{gt}) .$$

This is the demand equation for $i \in S_g$, given the demand for the group as a whole $\bar{w}_{gt} DQ_{gt}$. As the variables on the right of this equation are exclusively concerned with the group S_g to which the i^{th} commodity belongs, it is known as a conditional demand equation. The term θ_i / θ_g is the conditional marginal share of i within the group S_g , with $\sum_{i \in S_g} \theta_i / \theta_g = 1$. The conditional marginal share answers the question, if income increases by \$1, resulting in a certain additional amount spent on the group S_g , what is the proportion of this additional amount that is allocated to commodity i ?

Equation (5.5) can also be formulated in terms of absolute prices by using the definition of DP'_{gt} to write the substitution term as

$$(5.6) \quad \left\{ \begin{aligned} & \sum_{j \in S_g} v_{ij} \left[Dp_{jt} - \sum_{k \in S_g} \frac{\theta_k}{\theta_g} Dp_{kt} \right] \\ & = \sum_{j \in S_g} v_{ij} Dp_{jt} - \phi \theta_i \sum_{k \in S_g} \frac{\theta_k}{\theta_g} Dp_{kt} \\ & = \sum_{j \in S_g} (v_{ij} - \phi \theta_g \theta'_i \theta'_j) Dp_{jt} = \sum_{j \in S_g} \pi_{ij}^g Dp_{jt} . \end{aligned} \right.$$

where the first step is based on (5.2); $\theta'_i = \theta_i / \theta_g$; and

$$(5.7) \quad \pi_{ij}^g = v_{ij} - \phi \theta_g \theta'_i \theta'_j \quad i, j \in S_g$$

is the $(i, j)^{th}$ conditional Slutsky coefficient. Substitution of the fourth member of (5.6) into (5.5) gives the absolute price version of the conditional demand equation for $i \in S_g$.

$$(5.8) \quad \bar{w}_{it} Dq_{it} = \theta'_i \bar{w}_{gt} DQ_{gt} + \sum_{j \in S_g} \pi_{ij}^g Dp_{jt} .$$

Our previous assumptions imply that θ'_i and π_{ij}^g in this equation are constant coefficients.

By dividing both sides of (5.8) by $\bar{w}_{it}$, we find that $\frac{\theta'_i \bar{w}_{gt}}{\bar{w}_{it}} = \frac{\theta_i / \bar{w}_{it}}{\theta_g / \bar{w}_{gt}}$ is the ratio of the income elasticity of i to that of the group to which the good belongs. We shall refer to this ratio as the conditional income elasticity of i . We also find that $\pi_{ij}^g / \bar{w}_{it}$ is the conditional price elasticity, i.e. the elasticity of q_i with respect to the absolute price p_j ($i, j \in S_g$). These conditional elasticities hold constant total consumption of the group, as measured by the Divisia volume index DQ_{gt} .

As $\sum_{j \in S_g} \theta'_j = 1$, it follows from (5.2) and (5.7) that

$$(5.9) \quad \sum_{j \in S_g} \pi_{ij}^g = 0 \quad i \in S_g.$$

This reflects the homogeneity proposition that a proportionate change in all prices in the group does not affect the demand for any good in the group, total consumption of the group remaining unchanged. If S_g consists of n_g commodities, the $n_g \times n_g$ matrix of price coefficients referring to S_g is a principal submatrix of the $n \times n$ price coefficient matrix $[v_{ij}]$. As the latter matrix is symmetric, so is the former. It then follows from (5.7) that the conditional Slutsky matrix $[\pi_{ij}^g]$ is also symmetric,

$$(5.10) \quad \pi_{ij}^g = \pi_{ji}^g \quad i, j = 1, \dots, n_g.$$

The negative definiteness of $[v_{ij}]$ together with (5.7) and (5.9) imply that $[\pi_{ij}^g]$ is negative semidefinite with rank $(n_g - 1)$.

A Two-level Decision Hierarchy

It is to be noted that the variables of (5.8) deal only with the group to which the good belongs. Consequently, conditional demand functions allow us to ignore the prices of the goods outside the group in question. As will be seen in the next section, this allows us to analyse the demand for quite narrowly defined goods.

Block-independent preferences imply that the consumer's problem can be solved in two steps. The first decision involves the allocation of income to the G groups, as described by the G group demand equations (5.4). Each of these demand equations contains real income and the relative price of the group in question, but not the prices of the individual goods. Then in the second decision, for each of the groups, expenditure is allocated to the goods within the group. The conditional demand equations (5.8) describe this allocation and they contain total consumption of the group, as determined by the previous decision, and relative prices within the group. Accordingly, there is a two-level decision hierarchy under block independence. In many applications such a separation into levels of the consumer's decision problem makes good intuitive sense. This is illustrated below with the demand for alcoholic beverages. Under block independence, the consumer determines expenditure on total alcohol in the first step; and then in the second step that expenditure is allocated to beer, wine and spirits as a function of the relative prices of the beverages.

6. AN EXAMPLE: BEER, WINE AND SPIRITS

Clements and Johnson (1983) used annual per capita Australian data, 1956-1977, to estimate the conditional Rotterdam model (5.8) for beer, wine and spirits. At means, beer accounts for 72 percent of total alcohol expenditure, wine 13 percent and spirits 15 percent. The maximum likelihood estimates are presented in Table 1 (constant terms are included to take account of trend-like changes in

tastes, etc., but these are not reported in the table). These estimates are constrained by the homogeneity and symmetry restrictions (5.9) and (5.10). For the details of the estimation procedure see Appendix A3, available on request.

The conditional marginal shares given in Table 1 indicate that when alcohol expenditure increases by \$1, expenditure on beer rises by \$0.59, wine by \$0.10, with the remaining \$0.31 being spent on spirits. We give only the upper triangle of the matrix of conditional Slutsky coefficients as it is symmetric. All the diagonal elements of this matrix are negative, as they should be, and the off-diagonal elements are all positive. The characteristic roots of the conditional Slutsky matrix are all nonpositive, which verifies that this matrix is

TABLE 1
CONDITIONAL DEMAND EQUATIONS FOR ALCOHOLIC BEVERAGES:
AUSTRALIA, 1956-1977

$$\bar{w}_{it} Dq_{it} = \theta_i' \bar{w}_{gt} DQ_{gt} + \sum_{j=1}^3 \pi_{ij}^g Dp_{jt}$$

(Asymptotic standard errors are in parentheses)

Beverage	Conditional marginal share θ_i'	Conditional Slutsky coefficients		
		π_{i1}^g	π_{i2}^g	π_{i3}^g
		$\times 100$	$\times 100$	$\times 100$
Beer	.590 (.045)	-.390 (.145)	.150 (.114)	.240 (.097)
Wine	.099 (.035)		-.293 (.124)	.144 (.072)
Spirits	.311 (.042)			-.384 (.102)

negative semidefinite. In Appendix A2 we show that the data are not inconsistent with the assumption that the alcoholic beverages are block independent.

The upper part of Table 2 gives the conditional demand elasticities implied by the Australian estimates contained in Table 1. These are conditional in that total expenditure on alcohol is held constant, so that they deal with inter-beverage competition. The conditional income elasticities are .8, .8 and 2.0 for beer, wine and spirits, respectively. These indicate that, within alcohol, beer and wine are necessities (having elasticities less than one), while spirits is a strong luxury. The high elasticity for spirits agrees well with the notion that in Australia the more affluent tend to be spirits drinkers. All the conditional price elasticities are less than one in absolute value, the own-price elasticities being $-.1$, $-.4$ and $-.4$ for beer, wine and spirits. The conditional cross-price elasticities are all positive, indicating that the three beverages are pairwise substitutes. As expected, there is only a moderate amount of substitutability between the three beverages. The corresponding unconditional elasticities are

	<u>Income</u>	<u>Own-price</u>
Beer	.80	-.36
Wine	.75	-.43
Spirits	1.91	-.74

These elasticities hold real income constant and allow total alcohol expenditure to vary.

TABLE 2
 CONDITIONAL DEMAND ELASTICITIES FOR ALCOHOLIC BEVERAGES
 IN THREE COUNTRIES

(Asymptotic standard errors are in parentheses)

Beverage	Conditional income elasticity	Conditional price elasticities		
		Beer	Wine	Spirits
<u>Australia, 1956-1977</u>				
Beer	.83 (.06)	-.09 (.03)	.04 (.03)	.06 (.02)
Wine	.78 (.28)	.20 (.15)	-.39 (.16)	.19 (.10)
Spirits	1.97 (.27)	.26 (.10)	.15 (.08)	-.41 (.11)
<u>United Kingdom, 1955-1975</u>				
Beer	.41 (.08)	-.21 (.10)	.05 (.06)	.16 (.07)
Wine	1.93 (.19)	.20 (.24)	-.13 (.19)	-.07 (.16)
Spirits	1.80 (.14)	.34 (.15)	-.04 (.08)	-.30 (.13)
<u>United States, 1949-1982</u>				
Beer	.76 (.11)	-.09 (.08)	-.00 (.03)	.09 (.09)
Wine	.66 (.27)	-.02 (.18)	-.05 (.13)	.07 (.22)
Spirits	1.31 (.12)	.09 (.09)	.01 (.04)	-.10 (.10)

The middle and lower parts of Table 2 present the same elasticities for the U.K. and the U.S. These are from Clements and Selvanathan (1987) who estimated alcohol demand equations similar to (5.8) with British and American data. A comparison of the income elasticities across countries reveals that they are fairly similar. In particular, beer is always a necessity and spirits a luxury. Although the price elasticities for the U.S. are insignificant, there is still a broadly similar pattern of substitutability in the three countries. Clements and Selvanathan (1987) introduce ways of improving the precision of the price elasticities estimates.

7. A MARKETING APPLICATION: IDENTIFYING MARKET STRUCTURE

The analysis of market structure in marketing is concerned with identifying closely-competing brands of the same product. This analysis refers to those factors determining the probability of the consumer choosing one brand of a certain product, rather than another. These factors include the degree of consumer loyalty, which determines the pattern of repeat-buying; the nature of competition from other brands, which in part relates to the probability of switching brands; the evolution of consumer perception of the various brands; and so on. For examples of this literature, see Grover and Dillon (1985), Kalwani and Morrion (1977), Rao and Sabavala (1981), Robinson et al. (1980), Srivastava et al. (1981), Srivastava et al. (1984) and Urban et al. (1984).

Consumption theory provides an alternative way of identifying market structure. The approach is to define brands i and j as being closely related if additional purchases of j cause the marginal utility of i to change significantly. If the marginal utility of i falls with increased consumption of j , then it could be said that these two brands both satisfy a common want of the consumer; i and j are then competitive or substitutes for each other. This was basically the procedure used in Section 6 where we partitioned the n goods into $G = 2$ groups, (i) alcoholic beverages and (ii) all other goods. The marginal utility of each of the alcoholic beverages, $\partial u / \partial q_i$, $i \in S_g$, depends only on the consumption of those beverages and not on that of goods outside the group. This means that

$$\frac{\partial^2 u}{\partial q_i \partial q_j} = \begin{cases} \neq 0 & i, j \in S_g \\ 0 & i \in S_g, j \notin S_g \end{cases}$$

which implies that the price coefficient $v_{ij} = 0$ for $i \in S_g$, $j \notin S_g$. This blocking of goods amounts to postulating that the alcoholic beverages, on the one hand, and all other goods, on the other, are not closely competitive.

The same approach can also be used to test whether goods are closely related. We can analyse the existence of partitions within the alcoholic beverages group by asking, how does the marginal utility of one beverage change with increased consumption of the others? The most convenient way to do this is via the v_{ij} 's ($i, j \in S_g$) whose definition we repeat here:

$$v_{ij} = \frac{\lambda p_i p_j}{M} u^{ij},$$

where u^{ij} is the $(i,j)^{th}$ element of the inverse of the Hessian matrix of the utility function. If, for example, the marginal utility of wine is independent of the consumption of beer and spirits, $\partial^2 u / \partial q_i \partial q_j = 0$ for $i = \text{wine}$ and $j = \text{beer and spirits}$; and it follows from the above definition that the corresponding v_{ij} 's are also zero. We would then say that wine is not closely competitive with the other two beverages, so that the market structure so identified partitions alcoholic beverages into two groups (i) wine and (ii) beer and spirits.

It turns out that the Australian data are in approximate agreement with the idea that wine is in a class by itself. If we use in (5.7) the estimates of the π_{ij}^g 's and the θ_i' 's contained in Table 1, together with the estimates $\phi = -.595$ and $\theta_g = .0576$ (see Appendix A3, available on request), we obtain the following estimates ($\times 100$) of the v_{ij} 's for alcohol:

	Beer	Wine	Spirits
Beer	-1.583 (.315)	-.050 (.127)	-.389 (.151)
Wine		-.327 (.133)	.038 (.082)
Spirits			-.716 (.134)

Here we have only given the upper triangle of the $[v_{ij}]$ matrix as it is symmetric. The numbers in parentheses are the asymptotic standard errors. As can be seen, the two off-diagonal elements for wine, v_{12} and v_{23} , are both very small in comparison with the other elements. Moreover, these two elements are not significantly different from zero so that we cannot reject the hypothesis that wine is independent of the

other two beverages. In other words, the marginal utility of wine is independent of beer and spirits consumption.

A similar analysis of the British data reveals the following structure of the $[v_{ij}]$ matrix:

	Beer	Wine	Spirits
Beer	Significant	Insignificant	Insignificant
Wine		Significant	Significant
Spirits			Significant

This indicates that beer is in a class by itself, so that the British drinker survives by beer alone. For the U.S., this matrix takes the form

	Beer	Wine	Spirits
Beer	Significant	Insignificant	Significant
Wine		Significant	Insignificant
Spirits			Significant

which has the same structure as the Australian matrix, i.e. wine is in a class by itself. These results for the U.K. and the U.S. are from Clements and Selvanathan (1987).

As can be seen, using the degree of utility interaction of the beverages (as measured by the v_{ij} 's) is a very appealing way to analyse the degree of competition between beverages i and j . In contrast to the approaches in the marketing literature, it is a theory-based procedure for identifying market structure.

8. EXTENDING THE ROTTERDAM MODEL

TO DEAL WITH ADVERTISING

In the previous sections we used the Rotterdam model to analyse the effect of income and prices on the demand for the alcoholic beverages. The model can be extended to incorporate the role of other elements of the marketing mix (e.g. advertising, product quality, distribution) in determining consumption. In this section we restrict ourselves to advertising. As our objective is to illustrate how the Rotterdam model can be extended to deal with advertising, we mainly consider only a very simple case and just mention more general formulations.

Advertising and Utility

We postulate that a one percent increase in advertising of good i changes the logarithm of the marginal utility of that good, $\log(\partial u / \partial q_i)$, by a positive constant μ ,

$$(8.1) \quad \frac{\partial \left[\log \frac{\partial u}{\partial q_i} \right]}{\partial (\log a_i)} = \mu ,$$

where a_i is a measure of the volume of advertising for good i . The coefficient μ is interpreted as the elasticity of the marginal utility of good i with respect to its advertising.

Conditional Demand Equations With Advertising

The conditional demand equation for $i \in S_g$ under block independence is given by (5.5). When the model is extended to deal with advertising by using (8.1), it can be shown (Theil, 1980b, pp. 42-4, 126-9) that this demand equation becomes

$$(8.2) \quad \bar{w}_{it} Dq_{it} = \frac{\theta_i}{\theta_g} \bar{w}_{gt} DQ_{gt} + \sum_{j \in S_g} v_{ij} [(Dp_{jt} - DP'_{gt}) - \mu(Da_{jt} - DA'_{gt})] ,$$

where Da_{jt} is the log-change in a_j and $DA'_{gt} = \sum_{k \in S_g} (\theta_k / \theta_g) Da_{kt}$ is the Frisch index of advertising for the group S_g . This index is a weighted average of the change in advertising of each member of the group, the weights being the conditional marginal shares.

Equation (8.2) is a simple, one-parameter extension of the Rotterdam model. The deflation of each Da_{jt} by DA'_{gt} means that an equiproportional increase in advertising of each member of the group has no effect on consumption; it is only relative advertising that matters. Consider the situation when all prices change proportionately, so that $Dp_{jt} - DP'_{gt} = 0$ in (8.2). As long as $Da_{jt} - DA'_{gt} \neq 0$, it still may be possible to estimate the v_{ij} 's of (8.2). This is likely to be important in marketing research where the prices of different brands tend to move in tandem.

Using (5.7) in (8.2) we can write the absolute price version of the conditional demand equation for $i \in S_g$ as

$$(8.3) \quad \bar{w}_{it} Dq_{it} = \theta'_i \bar{w}_{gt} DQ_{gt} + \sum_{j \in S_g} \pi_{ij}^g (Dp_{jt} - \mu Da_{jt})$$

where $\theta'_i = \theta_i / \theta_g$ is the conditional marginal share of i . Equation (8.3) is the extended version of (5.8). By dividing both sides of (8.3) by $\bar{w}_{it}$, we find that $-\mu \pi_{ij}^g / \bar{w}_{it}$ is the conditional advertising elasticity of i , i.e. $\partial(\log q_i) / \partial(\log a_j)$ for $i, j \in S_g$. This elasticity is proportional to the corresponding conditional price elasticity with $-\mu$ the factor of proportionality.

Application to U.K. Alcohol Consumption

We use annual per capita data for the U.K., 1955-1975, to estimate (8.3) for beer, wine and spirits. These data are from McGuinness (1979, App. 2). The maximum likelihood estimates are presented in Table 3. As before, these estimates are constrained by the homogeneity and symmetry restrictions.

The estimate of μ given in Table 3 is significantly positive. Its value indicates that when advertising on beverage i is increased by ten percent then the marginal utility of that beverage increases by 5.6 percent, all other variables remaining unchanged. Table 4 explores the implications of this estimate by presenting the conditional advertising elasticities. Consider the first row of this table, that for beer. The first entry is .10; this indicates that a ten percent increase in the advertising of beer raises beer sales by one percent (all other variables held constant). The second entry of -.06 implies that a ten percent rise in wine advertising depresses

beer sales by .6 percent, so that beer and wine are competitive, as expected. The last entry of $-.04$ indicates that beer sales fall by .4 percent as a result of a 10 percent increase in spirits advertising. Note that the sum of these effects is zero ($1 - .6 - .4 = 0$), reflecting the fact that an equiproportional increase in advertising of all beverages has a cancelling effect. In this application it turns out that the corresponding unconditional elasticities are approximately equal to their conditional counterparts.

TABLE 3

CONDITIONAL DEMAND EQUATIONS FOR ALCOHOLIC BEVERAGES:

UNITED KINGDOM, 1955-1975

$$\bar{w}_{it} Dq_{it} = \theta'_i \bar{w}_{gt} DQ_{gt} + \sum_{j=1}^3 \pi_{ij}^g (Dp_{jt} - \mu Da_{jt})$$

(Asymptotic standard errors are in parentheses)

Beverage	Conditional marginal share θ'_i	Conditional Slutsky coefficients		
		π_{i1}^g × 100	π_{i2}^g × 100	π_{i3}^g × 100
Beer	.231 (.041)	-.695 (.292)	.398 (.160)	.297 (.175)
Wine	.271 (.020)		-.248 (.116)	-.149 (.106)
Spirits	.498 (.042)			-.148 (.151)
Advertising elasticity of the marginal utility of consumption			$\mu = .556$ (.242)	

TABLE 4
 CONDITIONAL ADVERTISING ELASTICITIES FOR ALCOHOLIC BEVERAGES:
 UNITED KINGDOM, 1955-1975
 (Asymptotic standard errors are in parentheses)

Beverage	Beer	Wine	Spirits
Beer	.10 (.02)	-.06 (.02)	-.04 (.02)
Wine	-.24 (.07)	.15 (.07)	.09 (.05)
Spirits	-.09 (.04)	.04 (.02)	.04 (.04)

Further Extensions

There are two generalizations of the above extension of the Rotterdam model. First, equation (8.1) specifies that a one percent increase in advertising of good i changes the logarithm of that good's marginal utility by μ , which is the same for all goods. As some goods are more sensitive to advertising than others, this specification can be generalized by replacing μ with one parameter for each good in the group, $\mu_1, \dots, \mu_n$. This can be extended still further by allowing advertising of good j to directly affect the marginal utility of i , so that (8.1) becomes

$$\frac{\partial \left(\log \frac{\partial u}{\partial q_i} \right)}{\partial (\log a_j)} = \mu_{ij} , \quad i, j = 1, \dots, n_g.$$

For the corresponding demand model, see Selvanathan (1987).

Second, it is reasonable to postulate that in addition to current advertising, past advertising also affects current sales. This can be handled by interpreting the variable a_i as the stock of accumulated advertising, rather than the current flow of advertising expenditure. This approach is used by Selvanathan (1987). Using the U.K. data, he is, however, unable to reject the hypothesis that the effects last for one year only.

9. ALTERNATIVES TO THE ROTTERDAM MODEL

In addition to the Rotterdam model there are a number of other system-wide demand models, the most popular being the linear expenditure system (LES), the translog model and the almost ideal demand system (AIDS); see Theil and Clements (1987) for a survey. While these share some features with the Rotterdam model, there are a number of differences.

The utility function underlying the LES is the sum of n subutility functions, one for each good. This form of the utility function implies that each marginal utility is independent of the consumption of the other goods, a characteristic known as preference independence. As most narrowly defined goods could not be expected to exhibit this property, the LES is probably too rigid for marketing applications.

The translog model and AIDS are more general than the LES as they are capable of dealing with the interaction of goods in the utility function. Like the Rotterdam model, these models can be used to test the hypotheses of homogeneity and symmetry. The translog and AIDS are both based on the algebraic specification of the form of the utility function (strictly speaking, the indirect utility function and the cost function), a property shared by the LES. By contrast, the Rotterdam model is a first-order approximation to completely general demand equations and is thus approximately consistent with a wide variety of forms of the underlying utility function. In this sense, the model is more attractive than its competitors. However, until more comparative studies are available, one cannot be dogmatic and state definitely that one model will always dominate the others.

The idea of the Rotterdam model as an approximation to the true demand equations leads to a wider class of model, known as differential demand equations (Theil, 1980a). Within this class the "coefficients" are not necessarily constant and can vary with income and prices. For example, the marginal share θ_i in the Rotterdam model is specified as a constant; as an alternative, we could specify that this share equals a constant plus the corresponding budget share to give another member of the class, known as Working's (1943) model which has been recently used by Theil et al. (1981). It is possible to devise numerous other members of the class which may be useful in marketing research. For further details, see Theil (1979).

10. CONCLUDING COMMENTS

In this paper we have presented a self-contained exposition of the Rotterdam model which is a leading example of a system of consumer demand equations. The model has strong links with the economic theory of the consumer and is attractive for its elegant simplicity. We showed that although the model is based on the theory of the individual consumer, it can nonetheless be applied to aggregate data.

To illustrate how the model could be employed in marketing research, we used the consumption of beer, wine and spirits. When the alcoholic beverages are block independent in the consumer's utility function, the analyst can confine his attention to variables pertaining to those goods only. One of the features of the Rotterdam model is that the hypothesis of block independence can be tested in a straightforward way. The alcohol application included the marketing topics of identifying market structure and advertising. Our hope is that this paper will inspire marketing researchers to use the Rotterdam model in their work. (For other recent examples of the use of the Rotterdam model in marketing, see Lee, 1987, and Vilcassim, 1987.)

APPENDIX A1

The Covariance Term in the Macro Demand Equations

In Section 4 we showed that the macro demand equations contained the term $\text{Cov}[K_{ic}, V_c]/E[M_c]$, where $K_{ic} = M_c(\theta_{ic} - \bar{\theta}_i)$; $\bar{\theta}_i = E[M_c \theta_{ic}]/E[M_c]$; and $V_c = d(\log Q_c)$. In this appendix we use three different approaches to show that this term is likely to be negligibly small.

The First Approach

As $E[K_{ic}] = 0$, we have $\text{Cov}[K_{ic}, V_c] = E[K_{ic} V_c]$. If we assume M_c , $(\theta_{ic} - \bar{\theta}_i)$ and V_c are uncorrelated, we obtain

$$E[K_{ic} V_c] = E[M_c] E[\theta_{ic} - \bar{\theta}_i] E[V_c] \text{ and } E[\theta_{ic}] = \bar{\theta}_i.$$

Therefore $E[\theta_{ic} - \bar{\theta}_i] = 0$, which implies that $E[K_{ic} V_c] = 0$. Hence $\text{Cov}[K_{ic}, V_c] = 0$. It is to be noted that the assumptions required for this approach are quite strong. In the next two sections we relax these assumptions.

The Second Approach

To derive equation (4.4) of the text, we have used Khinchine's theorem in the form

$$\frac{1}{N} \sum_{c=1}^N K_{ic} V_c = \text{Cov}[K_{ic}, V_c] + o_p(1),$$

where we have also used $E[K_{ic}] = 0$. We write the left-hand side of the above equation as

$$\frac{1}{N} \sum_{c=1}^N M_c(\theta_{ic} - \bar{\theta}_i) d(\log Q_c),$$

which we denote by A . If we assume $M_c(\theta_{ic} - \bar{\theta}_i)$ and $d(\log Q_c)$ are uncorrelated, then

$$\begin{aligned} E[A] &= \frac{1}{N} \sum_{c=1}^N E[M_c(\theta_{ic} - \bar{\theta}_i)] E[d(\log Q_c)] \\ &= 0, \end{aligned}$$

where we have used $E[M_c(\theta_{ic} - \bar{\theta}_i)] = 0$.

Consider the conditional variance of A , conditioned by $d(\log Q_c)$.

$$\begin{aligned} \text{var}[A | d(\log Q_c), c=1, \dots, N] &= \frac{1}{N^2} \text{var}[M_c(\theta_{ic} - \bar{\theta}_i)] \sum_{c=1}^N [d(\log Q_c)]^2 \\ &= \frac{1}{N} \text{var}[M_c(\theta_{ic} - \bar{\theta}_i)] \frac{1}{N} \sum_{c=1}^N [d(\log Q_c)]^2. \end{aligned}$$

It is reasonable to assume that $(1/N) \sum_{c=1}^N [d(\log Q_c)]^2$ does not increase systematically when N increases. Therefore $\text{var}[A | d(\log Q_c), c=1, \dots, N] \rightarrow 0$ as $N \rightarrow \infty$. This implies that $A \rightarrow 0$ as $N \rightarrow \infty$. For more details of this approach, see Theil (1971).

The Third Approach

By writing $d(\log Q_c) = d(\log M_c) - \sum_{j=1}^n w_{jc} d(\log p_j)$, we obtain

$$\begin{aligned} \frac{\text{Cov}[K_{ic}, V_c]}{E[M_c]} &= \frac{\text{Cov}[K_{ic}, d(\log M_c)]}{E[M_c]} - \frac{\text{Cov}[K_{ic}, \sum_{j=1}^n w_{jc} d(\log p_j)]}{E[M_c]} \\ &= \frac{\rho_i \{\text{var}[K_{ic}] \text{var}[d(\log M_c)]\}^{1/2}}{E[M_c]} \\ &\quad - \sum_{j=1}^n \frac{\rho_{ij} \{\text{var}[K_{ic}] \text{var}[w_{jc}]\}^{1/2} d(\log p_j)}{E[M_c]} \end{aligned}$$

where ρ_i = correlation $[K_{ic}, d(\log M_c)]$; and ρ_{ij} = correlation $[K_{ic}, w_{jc}]$.

For a developed economy it is reasonable to assume that (a) $\theta_{ic} \approx \bar{\theta}_i$, $c=1, \dots, N$, so that $\text{var}[K_{ic}] \approx 0$; (b) the variation in the change in income across consumers is small in comparison with their average income, so that $\text{var}[d(\log M_c)]/E[M_c] \approx 0$; and (c) the variation in budget shares across consumers is small in comparison with their average income, so that $\text{var}[w_{jc}]/E[M_c] \approx 0$. It is therefore likely that $\text{Cov}[K_{ic}, V_c]/E[M_c]$ is negligibly small. For more details of this approach, see Barnett (1979).

APPENDIX A2

Testing Block Independence

In the text the alcoholic beverages group was taken to be block independent of all other goods. We now test this assumption with the Australian data. The i^{th} (unconditional) demand equation of the relative price version of the Rotterdam model is given by (3.1) which we repeat here,

$$(A2.1) \quad \bar{w}_{it} Dq_{it} = \theta_i DQ_t + \sum_{j=1}^n v_{ij} (Dp_{jt} - DP'_t).$$

The price coefficients v_{ij} are symmetric and satisfy (2.4). Let there be $n = 4$ goods, beer ($i = 1$), wine ($i = 2$), spirits ($i = 3$) and all other goods ($i = 4$). When alcohol is block independent the off-diagonal v_{ij} 's involving all other goods are zero,

$$(A2.2) \quad v_{14} = v_{24} = v_{34} = 0.$$

Our objective is to estimate (A2.1) to test restriction (A2.2).

To estimate (A2.1) we add constant terms. To identify the v_{ij} 's we set the income flexibility ϕ equal to -0.6 (see Appendix A3, available on request). This ϕ appears in constraint (2.4). The ML estimates ($\times 100$) of the v_{ij} 's are as follows (asymptotic standard errors are in parentheses):

	Beer	Wine	Spirits	All other
Beer	-1.444(.365)	.047(.140)	-.483(.154)	-.246 (.585)
Wine		-.307(.131)	.054(.080)	-.232 (.216)
Spirits			-.805(.127)	.235 (.296)
All other				-56.193(1.308)

As can be seen, the off-diagonal v_{ij} 's involving all other goods are insignificant. A likelihood ratio test of (A2.2) yields a χ^2 value of 1.46 which is not significant. Accordingly, we are unable to reject block independence.

REFERENCES

- Barnett, W.A. (1979). "Theoretical Foundations for the Rotterdam Model." *Review of Economic Studies* 46: 109-30.
- Barten, A.P. (1964). "Consumer Demand Functions Under Conditions of Almost Additive Preferences." *Econometrica* 32: 1-38.
- (1977). "The Systems of Consumer Demand Functions Approach: A Review." *Econometrica* 45: 23-51.
- Brown, A. and A. Deaton (1972). "Surveys in Applied Economics: Models of Consumer Behavior." *Economic Journal* 82: 1145-1236.
- Clements, K.W. and L.W. Johnson (1983). "The Demand for Beer, Wine and Spirits: A System-wide Analysis." *Journal of Business* 56: 273-304.
- Clements, K.W. and E.A. Selvanathan (1987). "Alcohol Consumption." Chapter 5 in H. Theil and K.W. Clements *Applied Demand Analysis: Results from System-Wide Approaches*, Cambridge, Mass: Ballinger Publishing Company. Pp. 185-264.
- Grover, R. and W.R. Dillon (1985). "A Probabilistic Model for Testing Hypothesized Hierarchical Market Structures." Marketing Science 4: 312-35..
- Kalwani, M.U. and D.G. Morrison (1977). "A Parsimonious Description of the Hendry System." *Management Science* 23: 467-77.
- Kiefer, N.M. (1984). "Microeconomic Evidence on the Neoclassical Model of Demand." *Journal of Econometrics* 25: 285-302.
- Lee, J.C. (1987). "Estimation of Demand for Toll Services by Time of Day." Bell Communications Research, Inc., Morristown, NJ 07960. Unpublished manuscript.

- McGuinness, T. (1979). "An Econometric Analysis of the Total Demand for Alcoholic Beverages in the U.K., 1956/75." Scottish Health Education Unit Report. Published without the data listing in *Journal of Industrial Economics* 29: 85-109.
- Phlips, L. (1974). *Applied Consumption Analysis*. Amsterdam: North-Holland Publishing Company.
- Powell, A.A. (1974). *Empirical Analytics of Demand Systems*. Lexington, MA: D.C. Heath
- Rao, V.R. and D.J. Sabavala (1981). "Inference of Hierarchical Choice Processes from Panel Data." *Journal of Consumer Research* 8: 85-96.
- Rubinson, J.R., W.R. Vanhonacker and F.M. Bass (1980). "Note on 'A Parsimonious Description of the Hendry System'." *Management Science* 26: 215-26.
- Selvanathan, E.A. (1987). *Explorations in Consumer Demand*. Ph.d thesis, Department of Economics, Murdoch University.
- Srivastava, R.K., M.I. Alpert and A.D. Shocker (1984). "A Customer-oriented Approach for Determining Market Structure." *Journal of Marketing* 48: 32-45.
- Srivastava, R.K., R.P. Leone and A.D. Shocker (1981). "Market Structure Analysis: Hierarchical Clustering of Products Based on Substitution-in-use." *Journal of Marketing* 45: 38-48.
- Theil, H. (1965). "The Information Approach to Demand Analysis." *Econometrica* 33: 67-87.
- (1971). *Principles of Econometrics*. New York: John Wiley and Sons, Inc., and Amsterdam: North-Holland Publishing Company.
- (1975/76). *Theory and Measurement of Consumer Demand*. Two volumes. Amsterdam: North-Holland Publishing Company.

- Theil, H. (1979). "Can Economists Contribute to Marketing Research?" *Sloan Management Review* 20: 19-29.
- _____ (1980a). *The System-wide Approach to Microeconomics*. Chicago: The University of Chicago Press.
- _____ (1980b). *System-wide Explorations in International Economics, Input-output Analysis and Marketing Research*. Amsterdam: North-Holland Publishing Company.
- Theil, H. and K.W. Clements (1987). *Applied Demand Analysis: Results from System-wide Approaches*. Cambridge, Mass: Ballinger Publishing Co.
- Theil, H., F.E. Suhm and J.F. Meisner (1981). *International Consumption Comparisons: A System-wide Approach*. Amsterdam: North-Holland Publishing Company.
- Urban, G.L., P.L. Johnson and J.R. Hauser (1984). "Testing Competitive Market Structures." *Marketing Science* 3: 83-112.
- Vilcassim, N.J. (1987). "A System-Wide Approach to Demand Estimation and Competitive Structure Analysis of Product Markets." J.L. Kellogg Graduate School of Management, Northwestern University, Evanston, Ill. 60602. Unpublished manuscript.
- Working, H. (1943). "Statistical Laws of Family Expenditure." *Journal of the American Statistical Association* 38: 43-56.

Note to the Editor: This Appendix is not for publication; it will be made available to readers on request.

APPENDIX A3

Estimation Procedures

In this appendix we describe the procedures used to obtain the estimates presented in Table 1. We start by taking total alcohol consumption $\bar{w}_{gt}^{DQ}$ to be a predetermined variable and then extend the analysis to the case when it is endogenous.

Consider equation (5.8) for $t=1, \dots, T$ observations with S_g the alcoholic beverages group consisting of $n_g = 3$ members. We add to that equation a constant (α_i) to take account of trend-like changes in tastes and an error term (ϵ_{it}).

$$(A3.1) \quad \bar{w}_{it}^{DQ} = \alpha_i + \theta_i \bar{w}_{gt}^{DQ} + \sum_{j=1}^3 \pi_{ij}^g D_{p_{jt}} + \epsilon_{it},$$

$i=1,2,3; t=1, \dots, T.$

As (A3.1) is an allocation model, the errors satisfy $\sum_{i=1}^3 \epsilon_{it} = 0$ and hence are linearly dependent. This implies that one equation is redundant and Barten (1969) shows that it does not matter which equation is deleted. We delete the third.

The homogeneity restriction (5.9) takes the form $\sum_{j=1}^3 \pi_{ij}^g = 0$, $i=1,2,3$. To impose this, we replace π_{i3}^g in (A3.1) for $i=1,2$ with $-\pi_{i1}^g - \pi_{i2}^g$. We then obtain

$$(A3.2) \quad \bar{w}_{it} Dq_{it} = \alpha_i + \theta_i' \bar{w}_{gt} DQ_{gt} + \sum_{j=1}^2 \pi_{ij}^g (Dp_{jt} - Dp_{3t}) + \epsilon_{it} .$$

$i=1,2; t=1,\dots,T.$

We write (A3.2) in vector form as

$$(A3.3) \quad y = (I \otimes X)\gamma + \epsilon ,$$

where y and ϵ are vectors consisting of two subvectors of T elements, viz., $y_i = [\bar{w}_{it} Dq_{it}]$ and $\epsilon_i = [\epsilon_{it}]$ for $t=1,\dots,T$; I is the 2×2 identity matrix; X is a $T \times 4$ matrix whose t^{th} row equals $(1, \bar{w}_{gt} DQ_{gt}, Dp_{1t} - Dp_{3t}, Dp_{2t} - Dp_{3t})$; and $\gamma = [\gamma_1' \gamma_2']'$ with $\gamma_i = [\alpha_i \ \theta_i' \ \pi_{i1}^g \ \pi_{i2}^g]'$. Given homogeneity, the symmetry restriction (5.10) takes the form $\pi_{12}^g = \pi_{21}^g$, which we write as

$$(A3.4) \quad R\gamma = 0 ,$$

where $R = [0 \ 0 \ 0 \ 1 \ 0 \ 0 \ -1 \ 0]$.

Let $\hat{\gamma}$ be the unrestricted least squares estimator of γ consisting of two subvectors of the form $\hat{\gamma}_i = (X'X)^{-1}X'y_i$. The generalized least squares estimator of γ constrained by (A3.4) is (Theil, 1975/76, p. 194)

$$(A3.5) \quad \tilde{\gamma} = \hat{\gamma} - [\Sigma \otimes (X'X)^{-1}]R' \{R[\Sigma \otimes (X'X)^{-1}]R'\}^{-1}R\hat{\gamma} ,$$

where $\Sigma = \text{var}(\epsilon_t)$ and $\epsilon_t = (\epsilon_{1t} \ \epsilon_{2t})'$. When the errors are normally distributed and Σ is known it can be easily shown that (A3.5) is equivalent to the maximum likelihood estimator.

Now consider the composite demand equation (5.4) with S_g the alcoholic beverages group. We add to that equation a constant term (β) and an error term (E_{gt}).

$$(A3.6) \quad \bar{W}_{gt} DQ_{gt} = \beta + \theta_g DQ_t + \phi \theta_g (DP'_{gt} - DP'_t) + E_{gt}, \quad t=1, \dots, T.$$

When we treat total alcohol consumption $\bar{W}_{gt} DQ_{gt}$ as an endogenous variable, we estimate equations (A3.2) and (A3.6) jointly by assuming the disturbances E_{gt} and ϵ_{it} are correlated. As this simultaneous system is nonlinear in the parameters, it is convenient to estimate by maximum likelihood under the assumptions of normal errors and that DQ_t and the price changes are predetermined variables. Table 1 presents the ML estimates of the conditional demand equations with the symmetry restriction (A3.4) imposed.

Using the Australian data, the ML estimates of the coefficients of (A3.6) are (asymptotic standard errors are in parentheses)

$$(A3.7) \quad \begin{array}{ll} \theta_g = & .0576 \\ & (.0095) \end{array} \quad \begin{array}{ll} \phi = & -.595 \\ & (.130) \end{array}$$

These estimates imply that a \$1 rise in income leads to a \$0.06 increase in alcohol expenditure; and a 10 percent increase in income causes the marginal utility of income to fall by $10/.595 \approx 17$ percent. At sample means, the income and price elasticities implied by (A3.7) are

$$\frac{\theta_g}{\bar{w}_{gt}} = .97$$

$$\frac{\phi\theta_g}{\bar{w}_{gt}} = -.58.$$

On the basis of a likelihood ratio test, we are unable to reject the homogeneity and symmetry restrictions. For further details, see Clements and Johnson (1983).

REFERENCES

- Barten, A.P. (1969). "Maximum Likelihood Estimation of a Complete System of Demand Equations." *European Economic Review* 1: 7-73.
- Clements, K.W. and L.W. Johnson (1983). "The Demand for Beer, Wine and Spirits: A System-wide Analysis." *Journal of Business* 56: 273-304.
- Theil, H. (1975/76). *Theory and Measurement of Consumer Demand*. Two volumes. Amsterdam: North-Holland Publishing Company.