HOW SIMILAR ARE OECD CONSUMERS?

by

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1. INTRODUCTION

The OECD countries are all high-income industrialized countries. Although in this sense they are similar, there are obvious differences of language, culture and geography. Are these differences of economic importance? In this paper we answer this question by reference to consumer behaviour in 15 OECD countries.

We commence by allowing OECD consumers to be idiosyncratic and estimate 15 systems of demand equations (one for each country) in which the parameters take different values for each country. We then specify that consumers in different countries are the same and take the coefficients of the demand equations to be the same across countries. This involves estimating a common demand system for all countries and pooling the data. An analysis of the budget share predictions from the demand equations shows that the pooled model performs quite well. This finding points in the direction of there being more similarities than differences in consumption patterns across countries.

In addition, the paper analyses the extent to which Theil's (1987) cross-country demand system for 30 countries can explain the OECD time-series data. We also use our estimates to test Frisch's (1959) famous conjecture about the income dependence of the income elasticity of the marginal utility of income. In the next section of the paper we describe the OECD database.

2. THE DATA

In our database for 15 OECD countries goods and services are classified into 10 commodity groups, namely, food, beverages, clothing, housing, durables, medical care, transport, recreation, education and miscellaneous. See Selvanathan (1987) for details of the data source.

Table 2.1 summarizes the general characteristics of the database. Column 2 presents the sample period for each country and column 3 gives the sample size. In column 4 we present the per capita gross domestic product (GDP) expressed in 1975 international dollars. The 15 countries are listed in the order of declining per capita GDP. As can be seen, the U.S. has the highest per capita GDP, while Italy has the lowest. Column 5 gives the population in 1975 for the 15 countries.

Table 2.2 presents the budget (or expenditure) shares at sample means for each commodity in the 15 countries. Looking at the fifth row we see that, on average, Australians spend 19 percent of their total expenditure on food, 9 percent on beverages, 9 percent on clothing, 16 percent on housing and so on. The last row of the table presents the budget share of each commodity averaged over the 15 countries. As can be seen, the average OECD consumer spends 23 percent of his total expenditure on food.

Columns 2-11 of Table 2.3 present the mean log-changes in per capita consumption of each commodity and column 12 presents the Divisia volume index for each country, which measures the growth in per capita consumption as a whole. This volume index is a budget-share-weighted average of the log-changes in per capita consumption. The last row of the table presents the mean quantity log-changes and Divisia volume

TABLE 2.1
CHARACTERISTICS OF THE DATABASE

Country	Sample period	Sample size	Per capita GDP in 1975 international dollars	Mid-year population in 1975 (millions)
(1)	(2)	(3)	(4)	(5)
1. United States	1960-1981	21	7132	215.97
2. Canada	1960-1981	21	6788	22.73
3. Sweden	1964-1981	17	6749	8.19
4. Denmark	1966-1981	15	5969	5.10
5. Australia	1960-1981	21	5919	13.89
6. France	1964-1981	17	5864	52.71
7. Belgium	1960-1981	21	5554	9.80
8. Norway	1964-1981	17	5419	4.01
9. Netherlands	1952-1977	25	5321	13.65
10. Iceland	1960-1973	13	5201	.22
11. Finland	1960-1977	17	5192	4.71
12. Austria	1964-1981	17	4994	7.52
13. United Kingdom	1964-1981	17	4601	55.89
14. Spain	1964-1977	13	4032	35.60
15. Italy	1964-1981	17	3870	55.83

Note: Sample size is after lagging.

indexes averaged over all countries. As can be seen, transport and recreation have the highest average growth rates of 5.0 and 4.7 percent per annum, respectively, while food has the lowest growth rate of 1.4 percent per annum. Per capita consumption as a whole, as measured by the Divisia volume index, on average increases by 3.1 percent per annum.

TABLE 2.2

BUDGET SHARES OF 10 COMMODITIES FOR 15 COUNTRIES

(Means $\times 100$)

Country	Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1. U.S.	14.38	4.24	8.03	19.29	7.28	9.46	15.93	6.25	1.81	13.33
2. Canada	16.42	6.21	8.41	18.85	8.73	3.84	15.02	5.96	2.33	14.22
3. Sweden	20.72	7.96	8.41	22.79	7.66	2.27	13.65	9.18	.18	7.18
4. Denmark	19.18	9.09	6.97	20.92	8.98	1.92	15.04	7.88	.83	9.19
5. Australia	19.48	8.96	9.13	15.77	8.19	6.04	15.13	5.71	.69	10.89
6. France	21.20	4.69	8.22	14.41	10.23	10.50	11.99	6.05	.29	12.42
7. Belgium	23.11	6.95	7.53	15.72	14.14	6.74	10.67	3.98	.23	10.93
8. Norway	23.45	7.56	9.97	14.54	8.86	4.20	13.31	7.68	.54	9.89
9. Netherlands	26.20	6.38	14.78	10.81	12.48	6.67	6.67	3.41	3.10	9.50
10. Iceland	23.50	7.86	10.66	19.06	10.00	6.07	12.13	5.24	.44	5.05
11. Finland	28.24	8.62	8.26	14.36	7.51	2.54	13.60	5.92	1.44	9.49
12. Austria	22.06	7.29	11.37	11.99	8.89	3.49	13.36	5.67	.35	15.54
13. U.K.	18.92	6.26	8.40	18.20	7.78	.85	13.13	7.88	1.98	16.50
14. Spain	38.53	3.51	10.58	13.99	8.55	4.62	9.50	4.28	2.02	8.87
15. Italy	30.85	5.69	9.41	12.94	6.79	3.73	10.48	7.05	.43	12.62
Mean	23.08	6.75	9.34	16.24	9.07	4.86	12.64	6.14	1.11	11.04

TABLE 2.3
PER CAPITA QUANTITIES CONSUMED OF 10 COMMODITIES
AND DIVISIA VOLUME INDEX FOR 15 COUNTRIES
(Mean Log-Changes $\times$ 100)

Country	Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous	Divisia volume index
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. U.S.	.60	1.41	2.33	2.97	1.64	4.49	2.80	4.10	3.33	1.77	2.43
2. Canada	.91	2.57	3.12	3.17	3.02	.22	4.17	6.27	4.20	2.79	2.87
3. Sweden	.50	.80	2.66	2.35	1.18	3.02	2.06	4.53	3.17	-.95	1.71
4. Denmark	.17	1.43	.59	3.23	-1.01	1.50	1.49	3.34	8.53	.75	1.42
5. Australia	1.35	1.03	.71	3.54	4.45	1.23	3.49	4.33	-2.40	1.95	2.37
6. France	1.62	1.37	1.71	4.89	3.31	6.68	4.65	5.37	2.43	3.20	3.57
7. Belgium	1.53	2.58	2.76	2.80	4.20	5.13	4.52	4.04	1.98	2.98	3.11
8. Norway	1.35	2.08	1.40	3.43	3.21	2.14	4.06	5.56	1.26	2.17	2.68
9. Netherlands	2.22	4.39	3.19	3.93	6.91	5.53	7.52	4.07	5.15	5.20	4.34
10. Iceland	1.64	5.11	5.65	2.54	8.12	7.20	8.58	5.92	4.54	8.56	5.07
11. Finland	1.77	4.89	1.62	3.60	5.41	6.43	6.48	6.18	-1.60	5.58	3.86
12. Austria	1.90	2.34	4.26	4.29	3.61	2.62	5.58	4.35	1.94	1.98	3.27
13. U.K.	.15	1.48	2.52	1.87	1.58	.90	2.81	3.65	2.10	1.70	1.76
14. Spain	2.82	4.01	3.00	3.79	3.44	9.24	10.66	5.31	4.24	5.72	4.40
15. Italy	2.20	3.15	2.81	2.80	3.90	6.82	5.71	3.98	1.61	3.65	3.33
Mean	1.38	2.58	2.56	3.28	3.53	4.21	4.97	4.73	2.70	3.14	3.08

Tables 2.2 and 2.3 show that there are substantial differences in consumption patterns across countries. Looking at column 2 of Table 2.3 for example, we see that per capita food consumption grows by .6 percent per annum in the US while in Spain its growth rate is more than four times larger (2.8 percent per annum). Many of the differences in consumption patterns are attributable to differences in income and prices across countries. Consequently, in the next four sections we present demand equations which explain these differences in terms of income and prices. Then, in Section 7, we analyse the extent to which OECD consumers differ by examining cross-country differences in the demand equations.

3. MODEL I: A DEMAND SYSTEM FOR EACH OF THE 15 COUNTRIES

We commence by using the data to estimate 15 systems of demand equations, one for each country. As the parameters of each system take different values for each country, we shall refer to Model I as the individual country model.

Let p_{it}^c and q_{it}^c be the price and quantity demanded of good i ($=1, \dots, 10$) in country c ($=1, \dots, 15$) in year t ($=1, \dots, T_c$). Then $M_t^c = \sum_{i=1}^{10} p_{it}^c q_{it}^c$ is total expenditure ('income' for short) and $w_{it}^c = p_{it}^c q_{it}^c / M_t^c$ is the budget share of i . We shall use Working's (1943) model under preference independence; see Clements (1987) for details. For commodity i and country c , this model takes the form

$$(3.1) \quad y_{it}^c = \alpha_i^c + \beta_i^c DQ_t^c + \phi^c z_{it}^c + \epsilon_{it}^c,$$

where $y_{it}^c = \bar{w}_{it}^c (Dq_{it}^c - DQ_t^c)$; $\bar{w}_{it}^c = \frac{1}{2}(w_{it}^c + w_{i,t-1}^c)$ is the arithmetic average of the budget share of commodity i in years $t-1$ and t ; $Dq_{it}^c = \log q_{it}^c - \log q_{i,t-1}^c$ is the quantity log-change of commodity i ; $DQ_t^c = \sum_{i=1}^{10} \bar{w}_{it}^c Dq_{it}^c$ is the Divisia volume index;

$$z_{it}^c = (\beta_i^c + \bar{w}_{it}^c) \left[Dp_{it}^{*c} - \sum_{j=1}^9 (\beta_j^c + \bar{w}_{jt}^c) Dp_{jt}^{*c} \right],$$

$Dp_{it}^{*c} = Dp_{it}^c - Dp_{10,t}^c$; $Dp_{it}^c = \log p_{it}^c - \log p_{i,t-1}^c$ is the price log-change of commodity i ; α_i^c is a constant term of commodity i ; β_i^c is the income coefficient of commodity i ; ϕ^c is the income flexibility; and ϵ_{it}^c is a disturbance term. Here and elsewhere log refers to the natural logarithm.

Selvanathan (1987) estimates model (3.1) for $i=1, \dots, n_c$ (n_c is the number of commodities in country c) for 18 OECD countries by maximum likelihood (ML). All except 3 countries have 10 commodities and we omit those 3 countries in the present study. Selvanathan also tests the restrictions implicit in (3.1) and finds that, in the main, they are not incompatible with the data for most countries.

Table 3.1 presents the estimates of the income coefficients and the income flexibilities. The commodity subscript i on the income coefficient β_i^c takes values between 1 to 10, where 1=food, 2=beverages, 3=clothing, 4=housing, 5=durables, 6=medical care, 7=transport, 8=recreation, 9=education and 10=miscellaneous. The last row of the table presents the arithmetic means of the parameters across countries. We present the estimates of the constants in Appendix A1. We will evaluate the performance of this model in Section 7.

TABLE 3.1
INCOME COEFFICIENTS FOR 10 COMMODITIES
AND INCOME FLEXIBILITY FOR 15 COUNTRIES

(Asymptotic standard errors are in parentheses)

Country	β_1^C	β_2^C	β_3^C	β_4^C	β_5^C	β_6^C	β_7^C	β_8^C	β_9^C	β_{10}^C	ϕ^C
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. U.S.	-.055 (.019)	-.031 (.014)	.027 (.021)	-.115 (.026)	.054 (.017)	-.059 (.021)	.209 (.024)	.014 (.018)	-.003 (.009)	-.041 (.023)	-.384 (.066)
2. Canada	-.007 (.037)	-.025 (.022)	-.015 (.029)	-.183 (.041)	.050 (.034)	.055 (.020)	.084 (.036)	.051 (.027)	.001 (.014)	-.011 (.044)	-.554 (.120)
3. Sweden	-.094 (.025)	.008 (.015)	.038 (.022)	-.190 (.032)	.067 (.021)	-.022 (.005)	.138 (.026)	.054 (.021)	-.001 (.001)	.002 (.021)	-.580 (.088)
4. Denmark	-.123 (.032)	-.028 (.018)	.044 (.016)	-.130 (.024)	.053 (.017)	-.010 (.008)	.188 (.023)	.019 (.018)	-.009 (.005)	-.005 (.018)	-.465 (.084)
5. Australia	-.144 (.037)	-.015 (.025)	.027 (.030)	-.059 (.033)	.100 (.031)	-.018 (.025)	.074 (.040)	.072 (.024)	.013 (.012)	-.050 (.027)	-.463 (.077)
6. France	-.115 (.034)	-.024 (.014)	.024 (.023)	-.081 (.024)	.059 (.022)	-.047 (.033)	.146 (.025)	.001 (.020)	-.001 (.003)	.039 (.024)	-.527 (.077)
7. Belgium	-.119 (.044)	-.001 (.025)	.011 (.028)	-.066 (.033)	.070 (.039)	-.028 (.025)	-.012 (.030)	-.002 (.021)	-.002 (.005)	.150 (.041)	-.127 (.069)
8. Norway	-.180 (.029)	.010 (.015)	.021 (.021)	-.145 (.024)	.026 (.020)	-.009 (.014)	.293 (.024)	.002 (.022)	-.003 (.006)	-.016 (.022)	-.484 (.088)
9. Netherlands	-.132 (.040)	-.024 (.013)	.157 (.028)	-.046 (.019)	.051 (.029)	-.027 (.014)	.051 (.016)	-.002 (.017)	-.006 (.012)	-.023 (.025)	-.842 (.107)
10. Iceland	-.129 (.025)	-.020 (.016)	.037 (.018)	-.198 (.022)	.089 (.018)	-.015 (.017)	.181 (.020)	.023 (.017)	-.004 (.006)	.036 (.014)	-.695 (.089)
11. Finland	-.126 (.037)	.024 (.025)	.055 (.027)	-.122 (.032)	.042 (.023)	-.009 (.015)	.109 (.029)	.053 (.021)	.002 (.010)	-.028 (.027)	-.378 (.081)
12. Austria	-.175 (.041)	-.036 (.023)	.090 (.031)	-.099 (.030)	.085 (.028)	-.060 (.018)	.282 (.034)	-.011 (.021)	-.005 (.005)	-.071 (.036)	-.154 (.093)
13. U.K.	-.127 (.027)	.002 (.011)	.017 (.020)	-.118 (.023)	.088 (.018)	-.003 (.005)	.087 (.020)	.027 (.018)	.000 (.009)	.027 (.023)	-.396 (.071)
14. Spain	-.049 (.061)	-.003 (.024)	.031 (.036)	-.114 (.045)	.039 (.030)	-.002 (.024)	.130 (.032)	.015 (.020)	-.012 (.012)	-.034 (.028)	-.291 (.073)
15. Italy	-.044 (.029)	-.017 (.016)	.077 (.018)	-.067 (.023)	.036 (.015)	.001 (.012)	.044 (.019)	-.024 (.016)	-.002 (.004)	-.006 (.021)	-.184 (.074)
Mean	-.108 (.009)	-.012 (.005)	.043 (.007)	-.116 (.008)	.061 (.006)	-.017 (.005)	.134 (.007)	.019 (.005)	-.002 (.002)	-.002 (.007)	-.435 (.022)

4. MODEL II: COMMON DEMAND PARAMETERS

Next, we analyse informally the extent to which the estimates of the individual country models differ across countries. This will provide some preliminary evidence regarding the similarity (or lack thereof) of consumers.

Let $\beta^c = [\beta_1^c \dots \beta_{10}^c]'$ be the vector of estimates of the 10 income coefficients for country c given in Table 3.1. Consider the matrix-weighted mean of $\beta^1, \dots, \beta^{15}$,

$$(4.1) \quad \tilde{\beta} = \sum_{c=1}^{15} \Lambda^c \beta^c.$$

In Appendix A2 we show that (4.1) defines an optimal combination of the β^c 's when the weight matrices $\Lambda^1, \dots, \Lambda^{15}$ are related to the inverses of the relevant covariance matrices. In Appendix A2 we also show how to deal with the constraint $\sum_{i=1}^{10} \beta_i^c = 0$.

Next, consider the 15 estimates presented in Table 3.1 of the income flexibility, $\phi^1, \dots, \phi^{15}$. We proceed as before and define the weighted mean,

$$(4.2) \quad \tilde{\phi} = \sum_{c=1}^{15} \omega^c \phi^c,$$

where

$$\omega^c = \frac{(1/\text{var } \phi^c)}{\sum_{d=1}^{15} (1/\text{var } \phi^d)}$$

is the weight for country c . See Appendix A2 for details.

The first row of Table 4.1 presents the weighted means of the income coefficients and the income flexibility and the second row gives their standard errors (defined in Appendix A2). Note that these values are not too different from their unweighted counterparts given at the bottom of Table 3.1. Consequently, the weights do not seem to play a vital role.

The root-mean-square errors (RMSE) of the β_i^c 's ($c=1, \dots, 15$) around the weighted mean $\tilde{\beta}_i$ and corresponding measure for the ϕ^c 's are

$$\left[\frac{1}{15} \sum_{c=1}^{15} (\beta_i^c - \tilde{\beta}_i)^2 \right]^{1/2} \quad \text{and} \quad \left[\frac{1}{15} \sum_{c=1}^{15} (\phi^c - \tilde{\phi})^2 \right]^{1/2}.$$

The third row of Table 4.1 presents these RMSEs.

TABLE 4.1
WEIGHTED MEANS OF INCOME COEFFICIENTS FOR 10 COMMODITIES
AND INCOME FLEXIBILITY, AND THE ROOT-MEAN-SQUARE ERRORS

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	ϕ
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. Weighted means	-.102	-.013	.042	-.110	.058	-.013	.121	.017	-.001	.001	-.404
2. SEs	.008	.004	.006	.007	.005	.003	.006	.005	.001	.006	.021
3. RMSEs	.048	.017	.039	.047	.021	.027	.084	.027	.006	.051	.193

Figures 4.1-4.10 present the income coefficients for the 15 countries and their weighted means. The figures also give the two standard error bands using the RMSEs given in row 3 of Table 4.1. Figure 4.11 presents the analogous plot for the income flexibility. As can be seen, among the 165 points in the 11 plots, all except 8 lie within the two standard error bands. That is, more than 95 percent of the estimates lie within the band. This evidence points in the direction of similarity of consumer responses (as measured by the coefficients β_i and ϕ) across countries.

5. MODEL III: A FORMALLY POOLED DEMAND SYSTEM

In this section we specify that the coefficients of the demand equations are the same across countries and pool the data to estimate a common demand system for all 15 countries. Again, we use Working's model under preference independence.

We now assume that $\beta_i^c = \beta_i$, $\phi^c = \phi$, $i=1, \dots, 10$; $c=1, \dots, 15$. We allow the constant term α_i^c to vary over countries. Thus the estimating form of the model is

$$(5.1) \quad y_{it}^c = \alpha_i^c + \beta_i DQ_t^c + \phi z_{it}^c + \epsilon_{it}^c, \quad \begin{array}{l} i=1, \dots, 10; \\ t=1, \dots, T_c; \\ c=1, \dots, 15, \end{array}$$

where

$$z_{it}^c = (\beta_i + \bar{w}_{it}^c) \left[Dp_{it}^{*c} - \sum_{j=1}^9 (\beta_j + \bar{w}_{jt}^c) Dp_{jt}^{*c} \right];$$

INCOME COEFFICIENTS FOR FOOD FOR 15 COUNTRIES

Figure 4.1

INCOME COEFFICIENTS FOR BEVERAGES FOR 15 COUNTRIES

Figure 4.2

INCOME COEFFICIENTS FOR CLOTHING FOR 15 COUNTRIES

Figure 4.3

INCOME COEFFICIENTS FOR HOUSING FOR 15 COUNTRIES

Figure 4.4

INCOME COEFFICIENTS FOR DURABLES FOR 15 COUNTRIES

Figure 4.5

INCOME COEFFICIENTS FOR MEDICAL CARE FOR 15 COUNTRIES

Figure 4.6

Figure 4.10

Figure 4.11

and all other notation as before. We shall refer model (5.1) as the pooled model.

The first two rows of Table 5.1 present the pooled estimates and their asymptotic standard errors. See Appendix A3 for details of the ML estimation procedure and the estimates of the constants. For comparison, in the next two rows we reproduce from Table 4.1 the weighted means of the individual country estimates and their standard errors. As can be seen, the two sets of estimates are very similar. A likelihood ratio test of the difference between the two sets of estimates yields a χ^2 value of 6.8, less than the critical value of $\chi^2_{.05}(10)$ of 18.3. Consequently, the test confirms the similarity of the pooled estimates and the weighted means.

Recall the result from the previous section that more than 95 percent of the individual country parameter estimates lie within the two standard error bands. As the pooled estimates are so close to the weighted means, we can conclude that the pooled estimates are not too different from those of the individual country model. Since the estimates of Model II and Model III are not significantly different from each other, in what follows we shall concentrate on Model III.

We test the restrictions of the pooled model against the unrestricted individual country model. Using a likelihood ratio test we find that the restrictions may not be accepted. See Appendix A3 for full details. In light of our previous findings regarding the cross-country similarities of parameters, this result is puzzling. It may be that the asymptotic test is not performing satisfactorily, as has been found to be the case in other applications (Laitinen, 1978; Meisner, 1979). Accordingly, in what follows we shall keep an open mind about the status of the restrictions of the pooled model.

TABLE 5.1
 POOLED ESTIMATES AND WEIGHTED MEANS OF INCOME COEFFICIENTS
 FOR 10 COMMODITIES AND INCOME FLEXIBILITY

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	ϕ
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. Pooled estimates	-.097	-.011	.040	-.114	.054	-.012	.122	.017	-.002	.004	-.449
2. ASEs of pooled estimates	.008	.005	.006	.008	.006	.003	.007	.005	.001	.006	.022
3. Weighted means	-.102	-.013	.042	-.110	.058	-.013	.121	.017	-.001	.001	-.404
4. SEs of weighted means	.008	.004	.006	.007	.005	.003	.006	.005	.001	.006	.021

6. MODEL IV: THEIL'S CROSS-COUNTRY DEMAND SYSTEM

The application of model (5.1) involves the use of time-series data. Theil (1987), however, develops another version of this model in which countries play the role of time periods. He estimates this model with data on 30 countries from Kravis et al. (1982). Theil uses the same 10 commodity groups as we do. Although the unit of observation is now different (countries rather than years), the parameters of the cross-country model have exactly the same interpretation as those of the time-series version. In this section we present Theil's estimates and in Section 7 analyse how these estimates perform in explaining the OECD time-series data.

Theil's ML estimates are presented in column 2 of Table 6.1. For comparison, in column 3 we reproduce from Table 5.1 the pooled estimates and their asymptotic standard errors. As can be seen, the estimates are not too similar. It should be noted that among the 15 OECD countries considered in this paper only 9 are included in the 30 countries used by Theil.

TABLE 6.1

THREE SETS OF INCOME COEFFICIENTS AND INCOME FLEXIBILITY

(Asymptotic standard errors are in parentheses)

Commodity	Income coefficient β_i		
	Model IV: Theil's	Model III: Pooled model	
		With constant	No constant
(1)	(2)	(3)	(4)
1. Food	-.154 (.010)	-.097 (.008)	-.109 (.006)
2. Beverages	.001 (.005)	-.011 (.005)	-.013 (.003)
3. Clothing	-.005 (.005)	.040 (.006)	-.005 (.004)
4. Housing	.032 (.006)	-.114 (.008)	-.021 (.005)
5. Durables	.025 (.004)	.054 (.006)	.021 (.004)
6. Medical Care	.024 (.003)	-.012 (.003)	.006 (.003)
7. Transport	.030 (.006)	.122 (.007)	.086 (.004)
8. Recreation	.019 (.003)	.017 (.005)	.023 (.003)
9. Education	-.004 (.005)	-.002 (.001)	-.001 (.001)
10. Miscellaneous	.033 (.006)	.004 (.006)	.012 (.005)
Income flexibility	-.526 (.037)	-.449 (.022)	-.382 (.022)

It may be objected that the two models are not strictly comparable as Model III has constant terms which allow for trend-like changes in tastes etc.; Theil's model contains no similar terms. To put the two models on more equal footing, we re-estimate Model III with the constant terms suppressed. Column 4 of Table 6.1 contains the results. While the new estimates are a bit 'closer' to Theil's, differences still remain. In what follows we use Model III with constants since more than 75 percent of the constant terms of model (5.1) are significant (see Table A3.1) and since models with constants are more compatible with the data than those without (see Selvanathan, 1987).

7. COMPARISON OF THE MODELS

Models III and IV assume that consumers in different countries are more or less the same, while Model I allows different consumers to be idiosyncratic. Thus a comparison of models allows us to analyse the extent to which OECD consumers are similar across countries. We compare the models on the basis of a goodness-of-fit criterion, the information inaccuracy. This criterion measures the quantity of the implied budget share predictions; see Appendix A4 for technical details. The information inaccuracy is zero when the fit of the model is perfect and increases as it becomes worse.

Columns 4-6 of Table 7.1 present the average information inaccuracies of the three models. Column 7 gives the information inaccuracy for the naive model of no-change extrapolation of the budget shares, whereby the prediction of w_{it} is specified as $w_{i,t-1}$. For a given row, as we move across columns 4 to 7, in general, the inaccuracy

TABLE 7.1
GDP AND AVERAGE INFORMATION INACCURACIES
IN 15 COUNTRIES FOR FOUR DEMAND MODELS

Country (1)	GDP in 1975 international dollars ('000 millions) (2)	GDP weights × 100 (3)	Average information inaccuracy			
			Model I: Individual country model (4)	Model III: Pooled model (5)	Model IV: Theil's estimates (6)	Naive model (7)
1. U.S.	1540.30	51.34	1.36	1.48	2.33*	3.18
2. Canada	154.27	5.14	4.99	5.51	6.01	5.45
3. Sweden	55.30	1.84	1.76	1.82	3.30	4.05
4. Denmark	30.45	1.01	2.33	2.34	4.16*	7.48
5. Australia	82.22	2.74	2.90	3.02	3.30	5.15
6. France	309.06	10.30	.96	.96	1.19*	3.52
7. Belgium	54.44	1.81	3.35	3.75	4.04*	6.16
8. Norway	21.71	.72	2.15	2.76	4.06	4.98
9. Netherlands	72.65	2.42	5.13	6.40	6.61*	8.23
10. Iceland	1.14	.04	11.03	12.47	21.69	27.73
11. Finland	24.46	.82	6.77	6.72	8.15	10.65
12. Austria	37.56	1.25	2.74	3.12	4.28*	7.09
13. U.K.	257.15	8.57	1.56	1.61	2.31*	4.05
14. Spain	143.52	4.78	2.71	2.62	3.22*	7.40
15. Italy	216.06	7.20	1.77	2.26	2.43*	4.21
16. Unweighted mean	200.02		3.43	3.79	5.14	7.29
17. GDP weighted mean			1.87	2.04	2.73	4.10

Column 2 is obtained by multiplying per capita GDP given in column 4 of Table 2.1 by the corresponding population given in column 5 of the same table. The GDP weights given in column 3 are obtained by dividing the entries in column 2 by their total. All entries in columns 4-7 are to be divided by 10^4 . The information inaccuracies are all corrected for degrees of freedom (see Appendix A4). A * denotes that the country is included in Theil's (1987) sample.

increases. Rows 16 and 17 of columns 4-7 present the unweighted and the GDP-weighted-means across countries of the average information inaccuracies. As can be seen from these rows, overall the individual country model performs best and the naive model worst. However, the GDP-weighted-mean ($\times 10^4$) for Models I and III are 1.87 and 2.04 which are not too different. A similar result also holds for the unweighted means. That is, the pooled model is almost as good as the individual country models. Thus we conclude that the pooled model is a reasonable alternative to the individual country models.

In Table 7.1, looking at rows 1-15 of columns 4, 5 and 6, we see that the three sets of inaccuracies display similarities across countries. For example, in all models France always has the best fit and Iceland the worst. The conclusion is that the same general patterns emerge whether or not we specify the demand parameters to be the same across countries. Again, this points in the direction of cross-country similarities.

Looking at columns 6 and 7 we see that the inaccuracies of Theil's model and the naive model are higher than Models I and III. Interestingly, however, Theil's model still performs much better than the no-change extrapolation of the budget shares. This is a striking result as Theil's sample of 30 countries includes only 9 of the OECD countries.

In Appendix A4 we measure the goodness-of-fit of the models with respect to each commodity using the Strobel (1982) decomposition of information inaccuracy. The results indicate that no individual commodity dominates the lack of fit of the models.

8. DO THE DEMAND PARAMETERS DEPEND ON INCOME AND PRICES?

The conclusion of the previous section is that the demand parameters do not seem to vary across countries. This conclusion was reached by a comparison of the pooled model and the individual country model in which the coefficients are completely free to vary across countries.

Rather than allowing the parameters to vary arbitrarily across countries, it is interesting to inquire whether they vary across countries in response to systematic differences in economic characteristics. The economic characteristics we shall consider are per capita GDP and relative prices.

Column 2 of Table 8.1 presents the per capita GDP in 1980 expressed in international dollars (from Summers and Heston, Data Table, 1984) for the 15 OECD countries. Columns 3-12 present the relative prices of the 10 commodity groups in the 15 countries, where the overall price level in each country (P^c) is equal to 100. (See Appendix A5 for the data source and further details.) Rather than using per capita GDP in 1975 presented in Table 2.1 we use the values for 1980 since the relative price data refer to that year.

To analyse the effect of income and prices on the income coefficients, we regress these coefficients from the individual country models (presented in Table 3.1) on the logarithm of the per capita GDP (y^c), and the logarithm of the relative price of commodity i , $\tilde{p}_i^c = \log (p_i^c/P^c)$.

$$(8.1) \quad \beta_i^c = \alpha_i + \gamma_i y^c + \lambda_i \tilde{p}_i^c + \epsilon_i^c, \quad c=1, \dots, 15,$$

where α_i , γ_i and λ_i are parameters to be estimated; and ϵ_i^c is an

TABLE 8.1

PER CAPITA GDP AND RELATIVE PRICES OF 10 COMMODITIES
IN 1980 FOR 15 COUNTRIES

Country	Per capita GDP in international dollars	Relative prices									
		Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. U.S.	8089	89	97	88	104	87	134	89	93	117	105
2. Canada	7521	89	99	87	95	101	129	93	94	121	98
3. Sweden	7142	99	102	111	95	110	90	119	106	110	95
4. Denmark	6746	106	114	132	101	97	100	122	104	105	96
5. Australia	6308	99	102	111	95	110	90	119	106	110	95
6. France	6678	98	80	125	102	119	89	117	108	105	93
7. Belgium	6293	95	84	121	106	103	88	112	109	86	92
8. Norway	6825	110	122	134	91	131	72	151	114	160	94
9. Netherlands	5856	88	72	104	97	99	89	119	88	137	94
10. Iceland	5836	99	102	111	95	110	90	119	106	110	95
11. Finland	5939	112	157	105	93	111	66	136	132	174	97
12. Austria	6052	103	106	116	85	117	73	123	104	119	89
13. U.K.	4990	98	124	99	94	118	72	139	99	53	99
14. Spain	4264	97	88	111	101	111	93	117	103	59	95
15. Italy	4661	106	82	117	72	130	85	121	126	94	92

independent disturbance term with mean zero. We use weighted least squares to estimate (8.1), where the weights are proportional to the asymptotic standard errors of the income coefficients.

The first 10 rows of column 2 of Table 8.2 present the F-statistics for the null hypothesis that $\gamma_i = \lambda_i = 0$ for each

TABLE 8.2

F-STATISTICS AND RESTRICTED WEIGHTED LEAST SQUARES ESTIMATES
OF DEMAND PARAMETERS

(Standard errors are in parentheses)

β_i, ϕ (1)	F-statistics (2)	Restricted estimates (3)
1. Food	F(2,12) = 2.64	-.104 (.012)
2. Beverages	F(2,12) = 4.30	-.012 (.004)
3. Clothing	F(2,12) = .84	.042 (.009)
4. Housing	F(2,12) = .51	-.111 (.013)
5. Durables	F(2,12) = .35	.059 (.006)
6. Medical Care	F(2,12) = 1.71	-.013 (.004)
7. Transport	F(2,12) = 3.34	.127 (.022)
8. Recreation	F(2,12) = .67	.016 (.007)
9. Education	F(2,12) = .49	-.002 (.001)
10. Miscellaneous	F(2,12) = .22	-.000 (.010)
11. Income flexibility	F(1,13) = 1.37	-.404 (.047)

commodity. As the critical value of F(2,12) at the 5 percent level of significance is 3.89, all except one of the F-values are insignificant and at the 1 percent level all the F-values are insignificant. That is, the income coefficients are not related to income and relative prices. The first 10 rows of column 3 of Table 8.2 present the restricted weighted least squares estimates of α_i in (8.1). These estimates are similar to the weighted averages of the parameter estimates presented in row 1 of Table 4.1.

Next, we analyse the effect of income on the income flexibility by estimating

$$(8.2) \quad \phi^c = \eta_0 + \eta_1 y^c + \xi^c, \quad c=1, \dots, 15.$$

where η_0 and η_1 are parameters to be estimated; and ξ^c is an independent disturbance term with mean zero. We use the income flexibilities from the individual country model to estimate (8.2) by weighted least squares. The last row of Table 8.2 presents the F-value for the null hypothesis that $\eta_1 = 0$. The F-value is insignificant so that we are unable to reject the null hypothesis at the 5 percent level of significance. We conclude that ϕ is independent of real income. The restricted estimates of η_0 in (8.2) is given as the last entry of column 3. Again this is similar to the weighted mean presented in row 1 of Table 4.1.

The finding that ϕ is unrelated to real income is, of course, at variance with Frisch' (1959) famous conjecture. He argues that ϕ should increase in absolute value as the consumer (or country) becomes more affluent. Our rejection of Frisch, however, agrees with the previous results of Clements and Theil (1979), Theil (1980), Theil (1987) and Theil and Brooks (1970/71). It is not surprising that there is no strong evidence in support of this conjecture given that ϕ is the reciprocal of the income elasticity of the marginal utility of income: The marginal utility of income relates to a first-order derivative of the utility function, so that its income elasticity relates to a second derivative. Stating that ϕ changes systematically with real income thus amounts to a statement about the third-order derivative of the utility function. It is usually very difficult to measure such higher-order effects.

To summarize, the income coefficients and income flexibility seem to be unrelated to income and prices. The constancy of these coefficients suggests that they are fundamental parameters characterizing tastes which are invariant in a wide variety of economic circumstances.

9. CONCLUDING COMMENTS

In this paper we have presented three demand systems to explain time-series consumption data for 15 OECD countries. Model I allows consumers in different countries to be idiosyncratic; Model II is obtained by taking weighted averages of the parameter estimates of Model I; and in Model III consumers are taken to be the same across countries by specifying that the demand parameters are identical in different countries.

We found that about 95 percent of the parameter estimates of Model I lie within the two standard error bands of the estimates from Models II and III. This finding points in the direction of similarities of consumers across countries. Comparing the three models using a goodness-of-fit criterion, the information inaccuracy, we analysed the relative performance of the models. We found that Model III (identical consumers) performed quite well, again implying that OECD consumers are not too dissimilar.

We also used Theil's (1987) cross-country demand system to analyse the extent to which it explains the OECD time-series data. The result is that Models I-III explain the data better than does Theil's. However, Theil's model still does very well given that the 30 countries used to estimate it (from Kravis et al., 1982) include only 9 of our OECD countries. We also found that the demand parameters are unrelated to systematic differences in income and prices across countries; and that our data did not support Frisch's (1959) famous conjecture about the income dependence of the income elasticity of marginal utility of income.

APPENDICES

A1. THE CONSTANT TERMS FOR THE INDIVIDUAL COUNTRY MODEL

Table A1.1 gives the ML estimates of the constants for the individual country model.

A2. THE WEIGHTED MEANS OF THE INDIVIDUAL COUNTRY ESTIMATES

In this appendix we derive the weighted means of the income coefficients and income flexibilities which are presented in Section 4.

The Income Coefficients

Let $\beta^c = [\beta_1^c \dots \beta_{10}^c]'$ be the vector of the 10 income coefficients for country c . The constraint $\sum_{i=1}^{10} \beta_i^c = 0$ implies that $\Omega_c = \text{var } \beta^c$ is singular. Let $\beta^{*c} = [\beta_1^c \dots \beta_9^c]'$ and $\Omega_c^* = \text{var } \beta^{*c}$. The 9×9 covariance matrix Ω_c^* is non-singular.

Consider the weighted sum of squares

$$(A2.1) \quad L = \sum_{c=1}^{15} \epsilon_c^{*'} \Omega_c^{*-1} \epsilon_c^* = \sum_{c=1}^{15} (\beta^{*c} - \tilde{\beta}^*)' \Omega_c^{*-1} (\beta^{*c} - \tilde{\beta}^*),$$

where $\epsilon_c^* = \beta^{*c} - \tilde{\beta}^*$; and $\tilde{\beta}^*$ is an unknown vector of 9 elements. Using the first-order condition for a minimum of L with respect to $\tilde{\beta}^*$ yields

$$(A2.2) \quad \tilde{\beta}^* = \sum_{c=1}^{15} A^{*c} \beta^{*c},$$

TABLE A1.1

CONSTANT TERMS FOR 10 COMMODITIES IN 15 COUNTRIES:

INDIVIDUAL COUNTRY MODEL

(Asymptotic standard errors are in parentheses)

Country (1)	α_1^c (2)	α_2^c (3)	α_3^c (4)	α_4^c (5)	α_5^c (6)	α_6^c (7)	α_7^c (8)	α_8^c (9)	α_9^c (10)	α_{10}^c (11)
1. U.S.	-.121 (.060)	.033 (.039)	-.131 (.069)	.384 (.078)	-.203 (.052)	.351 (.053)	-.418 (.074)	.040 (.058)	.028 (.023)	.037 (.063)
2. Canada	-.213 (.124)	.046 (.085)	.014 (.120)	.581 (.144)	-.177 (.125)	-.300 (.073)	-.103 (.133)	-.026 (.109)	.047 (.038)	.136 (.119)
3. Sweden	-.099 (.071)	-.073 (.046)	-.156 (.081)	.507 (.067)	-.142 (.051)	.067 (.024)	-.108 (.065)	.110 (.058)	.004 (.006)	-.110 (.041)
4. Denmark	-.065 (.080)	-.010 (.064)	-.228 (.059)	.649 (.072)	-.286 (.055)	.017 (.025)	-.209 (.071)	.084 (.057)	.070 (.016)	-.022 (.054)
5. Australia	.122 (.112)	-.074 (.074)	-.227 (.087)	.377 (.087)	-.234 (.111)	.017 (.053)	-.015 (.111)	-.031 (.066)	-.041 (.022)	.106 (.071)
6. France	-.011 (.133)	-.027 (.062)	-.280 (.094)	.510 (.080)	-.299 (.093)	.475 (.131)	-.311 (.097)	.060 (.092)	.001 (.011)	-.118 (.087)
7. Belgium	-.008 (.161)	-.036 (.092)	-.073 (.105)	.164 (.121)	-.087 (.146)	.238 (.088)	.179 (.110)	.045 (.074)	.004 (.017)	-.426 (.155)
8. Norway	.160 (.103)	-.073 (.059)	-.204 (.075)	.502 (.083)	-.026 (.070)	.018 (.044)	-.550 (.085)	.148 (.087)	.000 (.018)	.024 (.068)
9. Netherlands	.049 (.197)	.085 (.082)	-.896 (.148)	.228 (.091)	-.129 (.174)	.316 (.057)	-.045 (.089)	.046 (.062)	.095 (.048)	.250 (.106)
10. Iceland	.033 (.187)	.106 (.133)	-.262 (.159)	.536 (.185)	-.272 (.152)	.289 (.108)	-.472 (.160)	.015 (.112)	.020 (.034)	.006 (.110)
11. Finland	-.065 (.183)	-.008 (.125)	-.441 (.135)	.419 (.169)	-.077 (.119)	.095 (.077)	-.075 (.148)	-.090 (.109)	-.058 (.045)	.300 (.124)
12. Austria	.247 (.156)	.042 (.095)	-.222 (.125)	.458 (.105)	-.270 (.115)	.158 (.059)	-.569 (.147)	.092 (.083)	.010 (.019)	.053 (.127)
13. U.K.	-.090 (.073)	-.046 (.039)	-.066 (.069)	.280 (.058)	-.216 (.053)	-.002 (.015)	.055 (.059)	.080 (.050)	.015 (.022)	-.010 (.063)
14. Spain	-.349 (.287)	-.016 (.125)	-.235 (.159)	.406 (.229)	-.216 (.137)	.220 (.105)	-.155 (.156)	.009 (.085)	.054 (.050)	.282 (.128)
15. Italy	-.225 (.121)	.023 (.072)	-.301 (.073)	.161 (.089)	-.083 (.062)	.114 (.050)	.109 (.076)	.114 (.066)	-.002 (.017)	.090 (.081)

All entries in columns 2-11 are to be divided by 100.

where

$$\Lambda^{*c} = \left[\begin{array}{cc} 15 & \\ \Sigma & \Omega_d^{*-1} \end{array} \right]_{d=1}^{-1} \Omega_c^{*-1}$$

is a weight matrix which satisfies $\sum_{c=1}^{15} \Lambda^{*c} = I_9$; and I_9 is the identity matrix of order 9.

The covariance matrix of $\tilde{\beta}^*$ is

$$\begin{aligned} \text{cov } \tilde{\beta}^* &= \left[\begin{array}{cc} 15 & \\ \Sigma & \Omega_d^{*-1} \end{array} \right]_{d=1}^{-1} \sum_{c=1}^{15} \Omega_c^{*-1} \text{cov } \beta^{*c} \Omega_c^{*-1} \left[\begin{array}{cc} 15 & \\ \Sigma & \Omega_d^{*-1} \end{array} \right]_{d=1}^{-1} \\ &= \left[\begin{array}{cc} 15 & \\ \Sigma & \Omega_d^{*-1} \end{array} \right]_{d=1}^{-1} \sum_{c=1}^{15} \Omega_c^{*-1} \Omega_c^* \Omega_c^{*-1} \left[\begin{array}{cc} 15 & \\ \Sigma & \Omega_d^{*-1} \end{array} \right]_{d=1}^{-1} \\ &= \left[\begin{array}{cc} 15 & \\ \Sigma & \Omega_d^{*-1} \end{array} \right]_{d=1}^{-1} . \end{aligned}$$

where we have assumed that the parameters are independent across countries. The standard errors of the $\tilde{\beta}_i$'s, $i=1, \dots, 9$, are the square roots of the diagonal elements of $\text{cov } \tilde{\beta}^*$.

Recall that β^{*c} and $\tilde{\beta}^*$ involve only 9 income coefficients. We use the constraint $\sum_{i=1}^{10} \tilde{\beta}_i = 0$ to obtain

$$(A2.3) \quad \tilde{\beta}_{10} = - \sum_{i=1}^9 \tilde{\beta}_i \quad \text{and} \quad \text{SE}[\tilde{\beta}_{10}] = \left[\sum_{i=1}^9 \sum_{j=1}^9 \text{cov}[\tilde{\beta}_i, \tilde{\beta}_j] \right]^{1/2}.$$

Let

$$\Lambda^c = \begin{bmatrix} \Lambda^{*c} & 0 \\ -\iota' \Lambda^{*c} & 0 \end{bmatrix}.$$

Then combining (A2.2) and (A2.3) it can be easily shown that

$$\tilde{\beta} = \sum_{c=1}^{15} A^c \beta^c.$$

Following Barten (1969), it can also be easily shown that

$$(A2.4) \quad \epsilon_c^* \Omega_c^{*-1} \epsilon_c^* = \epsilon_c' \left(\Omega_c + \frac{1}{10} \iota \iota' \right)^{-1} \epsilon_c,$$

where $\epsilon_c = \beta^c - \tilde{\beta}$ and $\iota = [1 \dots 1]'$ is a vector of 10 elements. The expression on the left-hand side of (A2.4) involves only the first 9 commodities, whereas the right-hand side involves all 10. Any commodity can be taken as the 10th and (A2.4) would still be valid, since the derivation of (A2.4) does not depend on deleting a specific commodity. Therefore, since the right-hand side of (A2.4) includes all 10 commodities and is invariant to the index of the commodity dropped in the left-hand side, equation (A2.1) does not depend on the index of the commodity deleted to form ϵ_c^* . It also follows that the values of $\tilde{\beta}^*$ and $\tilde{\beta}_{10}$ do not depend on which commodity is dropped.

Next, consider the role of the constraint $\sum_{i=1}^{10} \tilde{\beta}_i = 0$, which we write in the form $\iota' \tilde{\beta} = 0$. We form the new Lagrangean function

$$(A2.5) \quad L = \sum_{c=1}^{15} \epsilon_c^* \Omega_c^{*-1} \epsilon_c^* + \lambda \iota' \tilde{\beta},$$

where λ is a Lagrangean multiplier. The first-order condition for a minimum of L with respect to the last element of $\tilde{\beta}$, $\tilde{\beta}_{10}$, is

$$\frac{\partial L}{\partial \tilde{\beta}_{10}} = \lambda = 0.$$

Therefore, since $\tilde{\beta}_{10}$ is a free parameter in (A2.5), the unconstrained minimum of $\sum_{c=1}^{15} \epsilon_c^* \Omega_c^{*-1} \epsilon_c^*$ is equal to the constrained minimum. Furthermore, result (A2.4) establishes that the constraint also plays no role with respect to the value of the 10-element vector $\tilde{\beta}$.

The Income Flexibility

To obtain the weighted mean $\tilde{\phi}$ of the income flexibilities ϕ^c , $c=1, \dots, 15$, we minimize the weighted sum of squares

$$(A2.6) \quad L = \sum_{c=1}^{15} \left[\frac{1}{\text{var } \phi^c} \right] (\phi^c - \tilde{\phi})^2.$$

The first-order condition for a minimum of (A2.6) is

$$\frac{\partial L}{\partial \tilde{\phi}} = -2 \sum_{c=1}^{15} \left[\frac{1}{\text{var } \phi^c} \right] (\phi^c - \tilde{\phi}) = 0$$

which yields

$$\tilde{\phi} = \left[\sum_{d=1}^{15} \left[\frac{1}{\text{var } \phi^d} \right] \right]^{-1} \sum_{c=1}^{15} \left[\frac{1}{\text{var } \phi^c} \right] \phi^c.$$

It can be easily verified that the standard error of $\tilde{\phi}$ is

$$SE[\tilde{\phi}] = \left[\sum_{d=1}^{15} \left[\frac{1}{\text{var } \phi^d} \right] \right]^{-1/2}.$$

A3. ESTIMATION OF THE POOLED DEMAND SYSTEM

The Model

For convenience, we reproduce the pooled model (5.1),

$$(A3.1) \quad y_{it}^c = \alpha_i^c + \beta_i DQ_t^c + \phi z_{it}^c + \epsilon_{it}^c, \quad \begin{array}{l} i=1, \dots, 10; \\ t=1, \dots, T_c; \\ c=1, \dots, 15. \end{array}$$

As one equation is redundant for each country, the total number of free parameters to be estimated in (A3.1) is $15 \times 9 = 135$ intercept terms, 9 income coefficients and 1 income flexibility. That is, we have $135 + 9 + 1 = 145$ free parameters to be estimated. As this is very large, we eliminate the constants by using deviations from means. This reduces the number of parameters to be estimated to 10.

Deviations from the Mean

Let $\bar{y}_{it}^c$, $\bar{DQ}_t^c$, $\bar{z}_i^c$ and $\bar{\epsilon}_i^c$ be the sample means of y_{it}^c , DQ_t^c , z_{it}^c and ϵ_{it}^c , respectively. Since the disturbances are assumed to have zero means, without loss generality we can take $\bar{\epsilon}_i^c = 0$. Taking the sum of both sides of (A3.1) over $t=1, \dots, T_c$ and dividing through by T_c we get

$$(A3.2) \quad \bar{y}_i^c = \alpha_i^c + \beta_i \bar{DQ}^c + \phi \bar{z}_i^c, \quad \begin{array}{l} i=1, \dots, 10; \\ c=1, \dots, 15. \end{array}$$

From equations (A3.1) and (A3.2) we have

$$(A3.3) \quad y_{it}^{*c} = \beta_i DQ_t^{*c} + \phi z_{it}^{*c} + \epsilon_{it}^c, \quad \begin{array}{l} i=1, \dots, 10; \\ t=1, \dots, T_c; \\ c=1, \dots, 15, \end{array}$$

where $y_{it}^{*c} = y_{it}^c - \bar{y}_i^c$; $DQ_t^{*c} = DQ_t^c - \bar{DQ}^c$; and $z_{it}^{*c} = z_{it}^c - \bar{z}_i^c$.

Since for each country c , $\sum_{i=1}^{10} \epsilon_{it}^c = 0$, for $t=1, \dots, T_c$, the ϵ_{it}^c 's are linearly dependent. Thus one of the equations is redundant and can be deleted. Therefore we confine ourselves to (A3.3) for $i=1, \dots, 9$.

The Model in Vector Form

Let $y_t^{*c} = [y_{it}^{*c}]$, $z_t^{*c} = [z_{it}^{*c}]$, $\epsilon_t^c = [\epsilon_{it}^c]$ and $\beta = [\beta_1 \dots \beta_9]'$ be vectors of 9 elements. Then for $i=1, \dots, 9$, (A3.3) can be written in vector form as

$$(A3.4) \quad y_t^{*c} = DQ_t^{*c} \beta + \phi z_t^{*c} + \epsilon_t^c, \quad \begin{array}{l} t=1, \dots, T_c; \\ c=1, \dots, 15. \end{array}$$

We assume that the disturbances ϵ_{it}^c 's are normally distributed with zero means and $\text{cov}[\epsilon_{it}^c, \epsilon_{jt}^c] = \sigma_{ij}^c$, and are independent over time and countries. That is,

$$E[\epsilon_{it}^c \epsilon_{js}^d] = \sigma_{ij}^c \delta_{ts} \delta_{cd}, \quad \begin{array}{l} i, j=1, \dots, 10; \\ t, s=1, \dots, T_c; \\ c, d=1, \dots, 15. \end{array}$$

where δ_{ts} is the Kronecker delta. Let $\Sigma_c = [\sigma_{ij}^c]$. Then

$$(A3.5) \quad E[\epsilon_t^c \epsilon_s^{c'}] = \delta_{ts} \Sigma_c.$$

Let $\theta = [\beta' \ \phi']'$ be a vector of 10 parameters. Then (A3.4) takes the form

$$\begin{aligned} y_t^{*c} &= [DQ_t^{*c} I_9 \quad z_t^{*c}] \theta + \epsilon_t^c \\ &= X_t^{*c} \theta + \epsilon_t^c, \end{aligned} \quad \begin{array}{l} t=1, \dots, T_c; \\ c=1, \dots, 15. \end{array}$$

where $X_t^{*c} = [DQ_t^{*c} I_9 \quad z_t^{*c}]$ is a matrix of order 9×10 and I_9 is the identity matrix of order 9. For $t=1, \dots, T_c$, this can be written as

$$(A3.6) \quad y^{*c} = X^{*c} \theta + \epsilon^c, \quad c=1, \dots, 15,$$

where $y^{*c} = [y_t^{*c}]$ and $\epsilon^c = [\epsilon_t^c]$ are vectors of $9T_c$ elements; and $X^{*c} = [X_t^{*c}]$ is a matrix of order $9T_c \times 10$. It follows from (A3.5) that

$$(A3.7) \quad E[\epsilon^c \epsilon^{c'}] = I_{T_c} \otimes \Sigma_c, \quad c=1, \dots, 15.$$

For $c=1, \dots, 15$, (A3.6) can be written as

$$(A3.8) \quad y^* = X^* \theta + \epsilon,$$

where $y^* = [y^{*c}]$ and $\epsilon = [\epsilon^c]$ are vectors of $9T$ elements, where $T = \sum_{c=1}^{15} T_c$; and $X^* = [X^{*c}]$ is a matrix of order $9T \times 10$. The vector of disturbances ϵ has expectation zero and covariance matrix

$$(A3.9) \quad E[\epsilon \epsilon'] = \left[\delta_{cd} (I_{T_c} \otimes \Sigma_c) \right] = \Sigma \text{ (say),}$$

where we have used (A3.7) and assumed that the disturbances are independent across countries.

The Log-likelihood Function

Under normality, the log-likelihood function is

$$(A3.10) \quad L^*(\Sigma, \theta; y^*) = C^* + \frac{1}{2} \log |\Sigma^{-1}| - \frac{1}{2} \left[(y^* - X^* \theta)' \Sigma^{-1} (y^* - X^* \theta) \right],$$

where C^* is a constant. Consider the term in the square brackets on the right-hand side of (A3.10). From (A3.8) and (A3.9),

$$\begin{aligned}
 (y^* - X^* \theta)' \Sigma^{-1} (y^* - X^* \theta) &= \epsilon' \Sigma^{-1} \epsilon \\
 &= \sum_{c=1}^{15} \epsilon^c' \left[(I_{T_c} \otimes \Sigma_c)^{-1} \right] \epsilon^c = \sum_{c=1}^{15} \sum_{t=1}^{T_c} \epsilon_t^{*c} \Sigma_c^{-1} \epsilon_t^{*c} \\
 &= \sum_{c=1}^{15} \sum_{t=1}^{T_c} (y_t^{*c} - X_t^{*c} \theta)' \Sigma_c^{-1} (y_t^{*c} - X_t^{*c} \theta).
 \end{aligned}$$

We also have

$$\begin{aligned}
 \log |\Sigma^{-1}| &= \log \left[\prod_{c=1}^{15} |I_{T_c} \otimes \Sigma_c^{-1}| \right] \\
 &= \sum_{c=1}^{15} \log |I_{T_c} \otimes \Sigma_c^{-1}| = \sum_{c=1}^{15} \log \left[|\Sigma_c^{-1}| \right]^{T_c} \\
 &= \sum_{c=1}^{15} T_c \log |\Sigma_c^{-1}|.
 \end{aligned}$$

Substituting these in (A3.10), the log-likelihood function takes the form

$$\begin{aligned}
 L^* &= C^* + \frac{1}{2} \sum_{c=1}^{15} T_c \log |\Sigma_c^{-1}| - \frac{1}{2} \sum_{c=1}^{15} \sum_{t=1}^{T_c} (y_t^{*c} - X_t^{*c} \theta)' \Sigma_c^{-1} (y_t^{*c} - X_t^{*c} \theta) \\
 &= \sum_{c=1}^{15} \left[C^{*c} + \frac{T_c}{2} \log |\Sigma_c^{-1}| - \frac{1}{2} \sum_{t=1}^{T_c} (y_t^{*c} - X_t^{*c} \theta)' \Sigma_c^{-1} (y_t^{*c} - X_t^{*c} \theta) \right] \\
 (A3.11) \quad &= \sum_{c=1}^{15} L^{*c},
 \end{aligned}$$

where

$$(A3.12) \quad L^{*c} = C^{*c} + \frac{T_c}{2} \log |\Sigma_c^{-1}| - \frac{1}{2} \sum_{t=1}^{T_c} (y_t^{*c} - X_t^{*c} \theta)' \Sigma_c^{-1} (y_t^{*c} - X_t^{*c} \theta)$$

is the log-likelihood function of country c ; and C^{*c} is a constant term satisfying $C^* = \sum_{c=1}^{15} C^{*c}$.

Specifying the Covariance Matrix

In general the covariance matrix Σ_c is unknown. Selvanathan (1987) specifies a simple and plausible covariance structure,

$$\Sigma_c = \lambda_c^2 \Omega_c,$$

where λ_c^2 is a constant to be estimated;

$$\Omega_c = \bar{W}^c - \frac{\bar{C}^c \bar{C}^c}{\bar{W}^c \bar{W}^c};$$

$\bar{W}^c = \text{diag}[\bar{w}_1^c \dots \bar{w}_9^c]$; $\bar{C}^c = [\bar{w}_1^c \dots \bar{w}_9^c]'$; and $\bar{w}_i^c = (1/T_c) \sum_{t=1}^{T_c} w_{it}^c$, $i=1, \dots, 9$. Therefore

$$(A3.13) \quad \Sigma = \left[(I_{T_c} \otimes \Sigma_c) \delta_{cd} \right] = \left[\lambda_c^2 (I_{T_c} \otimes \Omega_c) \delta_{cd} \right].$$

Therefore, when the covariance matrix is specified as (A3.13), the log-likelihood function for country c takes the form

$$(A3.14) \quad L^{*c} = C_1^{*c} - \frac{9T_c}{2} \log \lambda_c^2 - \frac{1}{2\lambda_c^2} \sum_{t=1}^{T_c} (y_t^{*c} - X_t^{*c} \theta)' \Omega_c^{-1} (y_t^{*c} - X_t^{*c} \theta),$$

where C_1^{*c} is a constant.

The First-order Conditions

The first-order conditions for a maximum of (A3.11) with respect to λ_c^2 , $c=1, \dots, 15$, and θ' are

$$\begin{aligned} \frac{\partial L^*}{\partial \lambda_c^2} &= \frac{\partial L^{*c}}{\partial \lambda_c^2} \\ (A3.15) \quad &= -\frac{9T_c}{2\lambda_c^2} + \frac{1}{2\lambda_c^4} \sum_{t=1}^{T_c} (y_t^{*c} - X_t^{*c}\theta)' \Omega_c^{-1} (y_t^{*c} - X_t^{*c}\theta) = 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial L^*}{\partial \theta'} &= \sum_{c=1}^{15} \frac{\partial L^{*c}}{\partial \theta'} \\ (A3.16) \quad &= \sum_{c=1}^{15} \left[\frac{1}{\lambda_c^2} \sum_{t=1}^{T_c} (y_t^{*c} - X_t^{*c}\theta)' \Omega_c^{-1} \frac{\partial (X_t^{*c}\theta)}{\partial \theta'} \right] = 0, \end{aligned}$$

where

$$\begin{aligned} \frac{\partial (X_t^{*c}\theta)}{\partial \theta'} &= \begin{bmatrix} \frac{\partial (X_t^{*c}\theta)}{\partial \beta'} & \frac{\partial (X_t^{*c}\theta)}{\partial \phi} \end{bmatrix} \\ &= \begin{bmatrix} DQ_t^{*c} I_9 + \phi \frac{\partial z_t^{*c}}{\partial \beta'} & z_t^{*c} \end{bmatrix}; \\ \frac{\partial z_t^{*c}}{\partial \beta'} &= \begin{bmatrix} \frac{\partial z_{it}^{*c}}{\partial \beta_j} \end{bmatrix}; \end{aligned}$$

and

$$\begin{aligned}
\frac{\partial z_{it}^{*c}}{\partial \beta_j} &= \frac{\partial (z_{it}^c - \bar{z}_i^c)}{\partial \beta_j} \\
&= \frac{\partial z_{it}^c}{\partial \beta_j} - \frac{1}{T_c} \sum_{s=1}^{T_c} \frac{\partial z_{is}^c}{\partial \beta_j} \\
&= -(\beta_i + \bar{w}_{it}^c) Dp_{jt}^{*c} + \delta_{ij} \left[Dp_{it}^{*c} - \sum_{k=1}^9 (\beta_k + \bar{w}_{kt}^c) Dp_{kt}^{*c} \right] \\
&\quad + \frac{1}{T_c} \sum_{s=1}^{T_c} \left[(\beta_i + \bar{w}_{is}^c) Dp_{js}^{*c} - \delta_{ij} \left[Dp_{is}^{*c} - \sum_{k=1}^9 (\beta_k + \bar{w}_{ks}^c) Dp_{ks}^{*c} \right] \right].
\end{aligned}$$

The Information Matrix

From (A3.16), for $c=1, \dots, 15$,

$$(A3.17) \quad \frac{\partial^2 L^*}{\partial \lambda_c^2 \partial \theta'} = \frac{1}{\lambda_c^4} \sum_{t=1}^{T_c} (y_t^{*c} - X_t^{*c} \theta)' \Omega_c^{-1} \frac{\partial (X_t^{*c} \theta)}{\partial \theta'}.$$

Since $E[y_t^{*c} - X_t^{*c} \theta] = E[\epsilon_t^c] = 0$, from (A3.17), the information matrix of the ML procedure is block-diagonal with respect to θ and λ_c^2 , $c=1, \dots, 15$.

From (A3.16) we also have

$$\begin{aligned}
\frac{\partial^2 L^*}{\partial \theta \partial \theta'} &= \sum_{c=1}^{15} \left[-\frac{1}{\lambda_c^2} \sum_{t=1}^{T_c} \frac{\partial (X_t^{*c} \theta)'}{\partial \theta} \Omega_c^{-1} \frac{\partial (X_t^{*c} \theta)}{\partial \theta'} \right. \\
&\quad \left. + \frac{1}{\lambda_c^2} \sum_{t=1}^{T_c} (y_t^{*c} - X_t^{*c} \theta)' \Omega_c^{-1} \frac{\partial^2 (X_t^{*c} \theta)}{\partial \theta \partial \theta'} \right].
\end{aligned}$$

The second term on the right-hand side has zero expectation. Therefore, the asymptotic covariance matrix of the ML estimator of θ is

$$V^* = - \left[E \left[\frac{\partial^2 L^*}{\partial \theta \partial \theta'} \right] \right]^{-1} = \left[\sum_{c=1}^{15} \frac{1}{\lambda_c^2} \sum_{t=1}^{T_c} \frac{\partial(X_t^{*c} \theta)'}{\partial \theta} \Omega_c^{-1} \frac{\partial(X_t^{*c} \theta)}{\partial \theta'} \right]^{-1}.$$

An Iterative Procedure

The ML estimator of θ is obtained by means of a Newton's iterative scheme based on successive estimates of V^* and λ_c^2 , $c=1, \dots, 15$. The asymptotic standard errors are the square roots of the diagonal elements of V^* with ML estimates substituted for the unknown parameters in V^* .

The Constant Terms

Using the ML estimator of θ , $\hat{\theta}$, we obtain the ML estimators of α_i^c , $i=1, \dots, 10$, $c=1, \dots, 15$ from (A3.2),

$$(A3.18) \quad \hat{\alpha}_i^c = \bar{y}_i^c - \hat{\beta}_i D\bar{Q}^c - \hat{\phi}_{z_i}^c.$$

For $i=1, \dots, 9$, this can be written as

$$\begin{aligned} \hat{\alpha}_i^c &= \bar{y}_i^c - [0 \dots D\bar{Q}^c \dots 0 \quad \bar{z}_i^c] \hat{\theta} \\ &= \bar{y}_i^c - a_i^c \hat{\theta} \\ &= \alpha_i^c(\hat{\theta}), \end{aligned}$$

where $a_i^c = [0 \dots D\bar{Q}^c \dots 0 \quad \bar{z}_i^c]'$ is a vector of 10 elements with $D\bar{Q}^c$ as the i^{th} element, $\bar{z}_i^c$ as the 10th and all other elements zero. Therefore $\hat{\alpha}_i^c$ is a totally differentiable nonlinear function of $\hat{\theta}$. Thus

$\sqrt{T}_c[\alpha_i^c(\hat{\theta}) - \alpha_i^c(\theta)]$ converges in distribution to a normal random variate with mean zero and asymptotic variance $[\partial\alpha_i^c/\partial\theta']\hat{\Sigma}_\theta[\partial\alpha_i^c/\partial\theta]$, where $\hat{\Sigma}_\theta = \text{var } \hat{\theta}$. Therefore the asymptotic standard errors of $\hat{\alpha}_i^c$'s are given by

$$\text{ASE}[\hat{\alpha}_i^c] = \left[\frac{\partial\hat{\alpha}_i^c}{\partial\hat{\theta}'} \hat{\Sigma}_\theta \frac{\partial\hat{\alpha}_i^c}{\partial\hat{\theta}} \right]^{1/2},$$

where

$$\frac{\partial\hat{\alpha}_i^c}{\partial\hat{\theta}'} = a_i^c + \hat{\theta}' \frac{\partial a_i^c}{\partial\hat{\theta}'}.$$

Tables A3.1 presents the estimates of the constants for the pooled model.

Testing Pooling

When the model is estimated for each country separately, we estimate 15×9 free income coefficients (β_i^c) and 15×1 income flexibilities (ϕ^c). Under pooling, β_i takes the same value in each country, so that we have only 9 free income coefficients. Consequently, pooling involves $15 \times 9 - 9 = 126$ restrictions on the income coefficients. Also, the income flexibility is taken to be the same for all countries when we pool, which involves a further 14 restrictions. Relative to the 15 individual country models, pooling therefore involves a total of $126 + 14 = 140$ restrictions.

TABLE A3.1
CONSTANT TERMS FOR 10 COMMODITIES IN 15 COUNTRIES:

POOLED MODEL

(Asymptotic standard errors are in parentheses)

Country (1)	α_1^c (2)	α_2^c (3)	α_3^c (4)	α_4^c (5)	α_5^c (6)	α_6^c (7)	α_7^c (8)	α_8^c (9)	α_9^c (10)	α_{10}^c (11)
1. U.S.	-.025 (.019)	-.013 (.011)	-.181 (.019)	.381 (.018)	-.208 (.014)	.264 (.006)	-.213 (.015)	.022 (.015)	.026 (.002)	-.053 (.019)
2. Canada	-.013 (.019)	.006 (.013)	-.161 (.020)	.388 (.020)	-.173 (.017)	-.100 (.008)	-.194 (.017)	.111 (.017)	.043 (.002)	.091 (.024)
3. Sweden	-.098 (.013)	-.045 (.007)	-.131 (.017)	.411 (.008)	-.129 (.009)	.046 (.006)	-.114 (.009)	.188 (.010)	.005 (.001)	-.133 (.022)
4. Denmark	-.100 (.011)	-.047 (.010)	-.217 (.015)	.644 (.006)	-.288 (.008)	.019 (.004)	-.128 (.009)	.088 (.010)	.063 (.001)	-.033 (.024)
5. Australia	.007 (.020)	-.085 (.010)	-.260 (.015)	.477 (.014)	-.079 (.018)	.006 (.005)	-.131 (.014)	.080 (.011)	-.020 (.002)	.004 (.021)
6. France	-.066 (.030)	-.077 (.018)	-.328 (.024)	.604 (.022)	-.260 (.021)	.353 (.011)	-.229 (.022)	.004 (.022)	.004 (.003)	-.006 (.026)
7. Belgium	-.085 (.026)	-.011 (.015)	-.207 (.022)	.324 (.020)	-.072 (.018)	.204 (.008)	-.210 (.018)	-.016 (.016)	.003 (.004)	.071 (.025)
8. Norway	-.061 (.022)	-.012 (.012)	-.253 (.017)	.422 (.019)	-.098 (.015)	.025 (.006)	-.106 (.016)	.104 (.018)	-.001 (.002)	-.020 (.020)
9. Netherlands	-.110 (.036)	.030 (.021)	-.335 (.027)	.451 (.028)	-.042 (.028)	.218 (.009)	-.345 (.029)	-.044 (.019)	.058 (.004)	.119 (.031)
10. Iceland	-.166 (.035)	.057 (.023)	-.235 (.037)	.148 (.037)	-.038 (.032)	.246 (.012)	-.194 (.034)	.011 (.024)	.009 (.005)	.162 (.035)
11. Finland	-.162 (.028)	.124 (.017)	-.387 (.026)	.383 (.033)	-.137 (.023)	.106 (.012)	-.128 (.024)	.049 (.022)	-.044 (.003)	.197 (.029)
12. Austria	-.022 (.028)	-.088 (.018)	-.110 (.024)	.509 (.016)	-.225 (.021)	.054 (.005)	.010 (.020)	-.023 (.019)	.000 (.004)	-.104 (.032)
13. U.K.	-.147 (.016)	-.021 (.010)	-.144 (.019)	.283 (.007)	-.150 (.011)	.013 (.005)	.019 (.011)	.096 (.010)	.019 (.001)	.032 (.022)
14. Spain	-.152 (.037)	.014 (.025)	-.253 (.025)	.402 (.038)	-.261 (.023)	.264 (.011)	-.196 (.033)	.022 (.018)	.025 (.004)	.134 (.037)
15. Italy	-.069 (.028)	-.036 (.021)	-.175 (.020)	.316 (.023)	-.136 (.018)	.150 (.012)	-.104 (.020)	-.052 (.019)	-.001 (.004)	.108 (.028)

All entries in columns 2-11 are to be divided by 100.

We shall test these restrictions by means of a likelihood ratio test. Under the null hypothesis of pooling, the test statistic $-2(L_r - L_u)$ has an asymptotic $\chi^2(140)$ distribution, where L_r is the log-likelihood value with the restrictions and L_u is the unrestricted value. Under the assumption that the observations are independent across countries, the unrestricted log-likelihood value is the sum of the log-likelihood values for the 15 countries individually (L_u^c).

$$L_u = \sum_{c=1}^{15} L_u^c = 3037.1.$$

Under pooling the restricted log-likelihood value $L_r = 2844.4$. Thus the value of the test statistic $-2(L_r - L_u)$ is $-2(2844.4 - 3037.1) = 385.4$. As this value is too high for $\chi^2(140)$, we are unable to accept the pooled model.

A4. THE GOODNESS-OF-FIT OF THE MODELS

The Information Inaccuracy

For country c , let w_{it}^c be the observed value of the budget share of commodity i in year t and $\hat{w}_{it}^c$ be the predicted share implied by the demand model. Then as a measure of poorness of fit of the model with respect to the observed shares, we use the information inaccuracy of the predictions defined as

$$(A4.1) \quad I_t^c = \sum_{i=1}^{10} w_{it}^c \log \left[\frac{w_{it}^c}{\hat{w}_{it}^c} \right].$$

Clearly $I_t^c \geq 0$ for all $t=1, \dots, T_c$ and $c=1, \dots, 15$. When the predictions are perfect (i.e., $\hat{w}_{it}^c = w_{it}^c$ for $i=1, \dots, 10$), $I_t^c = 0$. We shall discuss methods of obtaining the budget share forecasts for each model later in this appendix.

The average information inaccuracy for each country is given by

$$\bar{I}_c = \frac{1}{T_c} \sum_{t=1}^{T_c} I_t^c.$$

Columns 2-5 of Table A4.1 presents the average information inaccuracies for the 15 countries for Models I, III, IV and the naive model. In contrast to the information inaccuracies in Table 7.1, these inaccuracies are not adjusted for degrees of freedom; later in this appendix we present such an adjustment. Comparing columns 4-7 of Table 7.1 with columns 2-5 of Table A4.1 we see that both tables display the same pattern. However, the degrees-of-freedom-adjusted inaccuracies of the pooled model are closer to the individual country model than their unadjusted counterparts. The inaccuracies associated with Theil's model also move closer to those of the pooled and the individual country model after the adjustment.

The Budget Share Predictions from Model I: The Individual Country Model

The individual country model is

$$(A4.2) \quad y_{it}^c = \alpha_i^c + \beta_i^c DQ_t^c + \phi^c (\beta_i^c + \bar{w}_{it}^c) \left[Dp_{it}^{*c} - \sum_{j=1}^9 (\beta_j^c + \bar{w}_{jt}^c) Dp_{jt}^{*c} \right] + \epsilon_{it}^c.$$

TABLE A4.1
UNADJUSTED AVERAGE INFORMATION INACCURACIES IN 15 COUNTRIES
FOR FOUR DEMAND MODELS

Country (1)	Model I: Individual country model (2)	Model III: Pooled model (3)	Model IV: Theil's estimates (4)	Naive model (5)
1. U.S.	1.22	1.39	2.19*	3.18
2. Canada	4.49	5.18	5.65	5.45
3. Sweden	1.54	1.71	3.10	4.05
4. Denmark	2.00	2.20	3.91*	7.48
5. Australia	2.61	2.84	3.10	5.15
6. France	.84	.89	1.12*	3.52
7. Belgium	3.01	3.53	3.80*	6.16
8. Norway	1.88	2.59	3.82	4.98
9. Netherlands	4.70	6.02	6.22*	8.23
10. Iceland	9.24	11.72	20.39	27.73
11. Finland	5.93	6.32	7.66	10.65
12. Austria	2.40	2.93	4.02*	7.09
13. U.K.	1.37	1.51	2.17*	4.05
14. Spain	2.27	2.46	3.03*	7.40
15. Italy	1.55	2.12	2.28*	4.21
Mean	3.00	3.56	4.83	7.29

All entries in columns 2-5 are to be divided by 10^4 . A * denotes that the country is included in Theil's (1987) sample.

where $y_{it}^c = \bar{w}_{it}^c (Dq_{it}^c - DQ_t^c)$ and $Dp_{it}^{*c} = Dp_{it}^c - Dp_{10,t}^c$. Let $\hat{\alpha}_i^c$, $\hat{\beta}_i^c$ and $\hat{\phi}^c$ be the ML estimates of α_i^c , β_i^c and ϕ^c , respectively. Then the ML-residual of the i^{th} equation for country c is

$$(A4.3) \quad \hat{\epsilon}_{it}^c = y_{it}^c - \hat{\alpha}_i^c - \hat{\beta}_i^c DQ_t^c - \hat{\phi}^c (\hat{\beta}_i^c + \bar{w}_{it}^c) \left[Dp_{it}^{*c} - \sum_{j=1}^9 (\hat{\beta}_j^c + \bar{w}_{jt}^c) Dp_{jt}^{*c} \right].$$

Thus the predicted value of y_{it}^c is

$$(A4.4) \quad \hat{y}_{it}^c = y_{it}^c - \hat{\epsilon}_{it}^c.$$

Our objective is to transform this prediction into a budget share prediction, $\hat{w}_{it}^c$.

Let $\Delta w_{it}^c = w_{it}^c - w_{i,t-1}^c$. Then the predicted value of w_{it}^c can be written as

$$(A4.5) \quad \hat{w}_{it}^c = w_{i,t-1}^c + \Delta \hat{w}_{it}^c,$$

where $\Delta \hat{w}_{it}^c$ is the predicted value of Δw_{it}^c . To obtain an expression for $\Delta \hat{w}_{it}^c$, first we take the differential of $w_i^c = p_i^c q_i^c / M^c$ to give

$$dw_i^c = w_i^c d(\log q_i^c) + w_i^c d(\log p_i^c) - w_i^c d(\log M^c).$$

In finite-change form, ignoring the remainder term of the third degree in prices, quantity and income log-changes, this can be written as

$$\begin{aligned} \Delta w_{it}^c &= \bar{w}_{it}^c Dq_{it}^c + \bar{w}_{it}^c Dp_{it}^c - \bar{w}_{it}^c DM_t^c \\ &= y_{it}^c + \bar{w}_{it}^c (Dp_{it}^c + DQ_t^c - DM_t^c) \\ (A4.6) \quad &= y_{it}^c + \bar{w}_{it}^c (Dp_{it}^c - DP_t^c). \end{aligned}$$

where $DP_t^C = DM_t^C - DQ_t^C$ is the Divisia price index. Since the last term on the right-hand side of (A4.6) is known, the forecast of Δw_{it}^C is

$$\hat{\Delta w}_{it}^C = \hat{y}_{it}^C + \bar{w}_{it}^C (DP_{it}^C - DP_t^C).$$

Therefore using (A4.6) again and then (A4.4) we can write

$$\begin{aligned} \hat{\Delta w}_{it}^C &= \hat{y}_{it}^C + (\Delta w_{it}^C - y_{it}^C) \\ &= \Delta w_{it}^C - \hat{\epsilon}_{it}^C. \end{aligned}$$

Thus from (A4.5),

$$\hat{w}_{it}^C = w_{it}^C - \hat{\epsilon}_{it}^C.$$

Therefore, by substituting for $\hat{\epsilon}_{it}^C$ from (A4.3) we can obtain the predicted value of w_{it}^C . However, since $\bar{w}_{it}^C$ occurs on the right-hand side of (A4.3), assuming that $\bar{w}_{it}^C$ is known to obtain $\hat{\epsilon}_{it}^C$ is not really legitimate. This is because $\bar{w}_{it}^C$ is the average of w_{it}^C and $w_{i,t-1}^C$, and w_{it}^C is unknown since it is the object of prediction. Theil (1967) notes that this problem can be corrected using the following iterative procedure. We first replace $\bar{w}_{it}^C$ by $w_{i,t-1}^C$ which leads to a forecast $w_{it}^{C'}$ of w_{it}^C . Then we replace $\bar{w}_{it}^C$ in (A4.3) by the average of $w_{it}^{C'}$ and $w_{i,t-1}^C$, which leads to a new forecast of w_{it}^C , and so on until convergence is reached. This is the procedure we use.

The Budget Share Predictions from Model III: The Pooled Model

The pooled model (5.1) is exactly the same as the individual country model (A4.2) except that the income coefficients β_i and the

income flexibility ϕ now do not have a country superscript. Therefore, to obtain the budget share forecasts of the pooled model we follow the same procedure described for the individual country model.

The Budget Share Predictions from Model IV: Theil's Cross-Country Demand System

Table 5.1 gives the estimates of Theil's (1987) cross-country demand system. To apply these estimates to time-series data we add constant terms for each good and each country. We estimate these as the mean of the time-series residuals,

$$\hat{\alpha}_i^{cT} = \frac{1}{T_c} \sum_{t=1}^{T_c} \hat{\xi}_{it}^c,$$

where

$$\hat{\xi}_{it}^c = y_{it}^c - \hat{\beta}_i^{cT} DQ_t^c - \hat{\phi}^{cT} z_{it}^c$$

is the i^{th} residual in year t for country c ; and the superscript T denotes a Theil estimate. With these estimates we use the same procedure described for the pooled model to obtain the budget share predictions for this model.

The Budget Share Predictions from the Naive Model

The naive prediction model of no-change extrapolation is given by $\Delta w_{it}^c = 0$. That is,

$$\hat{w}_{it}^c = w_{i,t-1}^c.$$

Using this as the forecasts of the budget shares in (A4.1) we obtain the average information inaccuracies for the 15 countries.

Correction for Degrees of Freedom

In the individual country model, each observation yields $n-1 = 9$ degrees of freedom. Therefore, the total number of degrees of freedom supplied by T_c observations is $(n-1)T_c = 9T_c$, where T_c is the sample size for country c . The number of unconstrained coefficients is $2(n-1) + 1 = (2 \times 9) + 1 = 19$. Therefore, due to statistical adjustment of the coefficients of the model, the correction of the average information inaccuracy for country c is (Theil, 1980)

$$\frac{(n-1)T_c}{(n-1)T_c - [2(n-1) + 1]} = \frac{9T_c}{9T_c - 19}.$$

For the pooled model, the total number of degrees of freedom is $(n-1) \sum_{c=1}^{15} T_c = 9 \times 269 = 2421$ and the number of unconstrained parameters is $15(n-1) + (n-1) + 1 = 135 + 9 + 1 = 145$. Thus the correction factor for the average information inaccuracy for the pooled model is

$$\frac{\sum_{c=1}^{15} \frac{(n-1)T_c}{(n-1)T_c - [2(n-1) + 1]}}{15} = \frac{2421}{2421-145} = 1.06.$$

The correction for Theil's model is the same as that for the pooled model, while no adjustment is needed for the naive model. Table A4.2 presents the degrees of freedom correction factors for the individual country model and the pooled model. Columns 4-7 of Table 7.1 of the text are derived from columns 2-5 of Table A4.1 by multiplying by the corresponding correction factors. That is, column 4 of Table 7.1 is column 2 of Table A4.1 multiplied by the correction factors for each country presented in Table A4.2; columns 5 and 6 are columns 3 and 4 of Table A4.1 multiplied by the correction factor for the pooled model, 1.06; and column 7 is column 5 of Table A4.1.

TABLE A4.2
DEGREES OF FREEDOM CORRECTION FACTORS
FOR THE INDIVIDUAL COUNTRY MODEL AND THE POOLED MODEL

Country (1)	Sample size (2)	Correction factor (3)
1. U.S.	21	1.11
2. Canada	21	1.11
3. Sweden	17	1.14
4. Denmark	15	1.16
5. Australia	21	1.11
6. France	17	1.14
7. Belgium	21	1.11
8. Norway	17	1.14
9. Netherlands	25	1.09
10. Iceland	13	1.19
11. Finland	17	1.14
12. Austria	17	1.14
13. U.K.	17	1.14
14. Spain	13	1.19
15. Italy	17	1.14
Pooled model	269	1.06

The Relative Strobel Measures and Their Entropy

The information inaccuracy defined above refers to all 10 commodities jointly. It is also possible to consider the goodness-of-fit of the model with respect to each commodity. Strobel (1982) proposes the following decomposition of the information inaccuracy:

$$(A4.7) \quad I_{it}^c = \hat{w}_{it}^c - w_{it}^c + w_{it}^c \log \left[\frac{w_{it}^c}{\hat{w}_{it}^c} \right].$$

Obviously, $I_{it}^c \geq 0$ and vanishes only when the prediction is perfect. Summing both sides of (A4.7) over $i=1, \dots, 10$, and using (A4.1) and the fact that $\sum_{i=1}^{10} w_{it}^c = \sum_{i=1}^{10} \hat{w}_{it}^c = 1$, we get

$$\sum_{i=1}^{10} I_{it}^c = \sum_{i=1}^{10} w_{it}^c - \sum_{i=1}^{10} \hat{w}_{it}^c + \sum_{i=1}^{10} w_{it}^c \log \left[\frac{w_{it}^c}{\hat{w}_{it}^c} \right] = I_t^c.$$

We also have that $0 \leq I_{it}^c \leq I_t^c$ for every i . Therefore the Strobel measure provides a simple decomposition of the information inaccuracy of the model over the individual commodity groups. We analyse the fit of the model with respect to commodity i by examining the ratio I_{it}^c/I_t^c which is the proportion of the inaccuracy of the model due to the misfit of the demand equation for commodity i . This ratio is called the relative Strobel measure.

The entropy of the relative Strobel measures is defined as

$$H_t^c = - \sum_{i=1}^{10} \left[\frac{I_{it}^c}{I_t^c} \right] \log \left[\frac{I_{it}^c}{I_t^c} \right].$$

The measure H_t^C varies between 0 and $\log 10 \approx 2.30$. It takes values close to the maximum when all 10 commodities contribute equally to the information inaccuracy and it takes the minimum value when one of the commodities has a relative measure close to 1 and the others are close to zero.

Columns 2-11 of Table A4.3 present the average relative Strobel measures for the 15 countries for each commodity for Models I, III, IV and the naive model. These measures are averaged over the T_c observations. As can be seen, in all models transport is consistently the largest contributor to the misfit of the models. We also see that education is the smallest contributor. However, there is not too much difference between the contribution of the 10 commodities to the misfit of any of the four models. Column 12 gives the average entropy of the relative Strobel measures for each country for the four models. For Model I, the entropy takes values between 1.45 (Canada) and 1.70 (Spain) with an average of 1.59. For Model III the entropy has an average value of 1.56, for Model IV it is 1.56 and for the naive model 1.58. These values are not too close to zero. This shows that, in all countries, on average no individual commodity dominates the lack of fit of any model.

A5. THE RELATIVE PRICE DATA

In this appendix we give the data source for relative prices used in Section 8 of the text. These data are derived from Tables B2 and J of the Statistical Appendix of Ward (1985). In Ward's paper, beverages are split into beverages and tobacco; recreation into

TABLE A4.3
AVERAGE RELATIVE STROBEL MEASURES FOR 10 COMMODITIES
AND THEIR ENTROPY IN 15 COUNTRIES FOR FOUR MODELS

Country	Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous	Entropy
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
<u>Model I: Individual country model</u>											
U.S.	20	3	8	4	12	12	22	5	6	7	1.56
Canada	6	4	6	5	5	34	11	7	16	4	1.46
Sweden	11	13	14	6	9	19	16	12	1	8	1.55
Denmark	10	3	16	12	8	6	19	13	5	8	1.57
Australia	5	2	8	7	23	14	13	16	7	6	1.50
France	5	5	10	14	21	15	14	5	5	5	1.53
Belgium	8	6	9	9	14	11	13	5	0	24	1.59
Norway	3	22	9	7	9	12	21	4	4	9	1.53
Netherlands	8	10	16	8	19	7	16	6	4	7	1.69
Iceland	11	9	12	8	10	7	23	12	1	9	1.64
Finland	3	9	20	2	12	2	27	12	11	3	1.61
Austria	11	12	10	11	19	7	17	4	1	7	1.67
U.K.	7	11	7	8	8	9	24	8	8	10	1.60
Spain	14	8	8	8	11	17	15	7	4	8	1.70
Italy	7	5	11	3	13	15	24	10	5	7	1.63
Mean	9	8	11	7	13	12	18	8	5	8	1.59
<u>Model III: Pooled model</u>											
U.S.	19	3	7	4	12	11	30	5	4	6	1.51
Canada	12	4	6	5	4	28	12	8	16	5	1.66
Sweden	13	15	13	6	9	7	16	13	1	7	1.59
Denmark	11	4	15	12	8	6	17	13	6	8	1.52
Australia	6	2	8	7	24	15	11	15	7	5	1.40
France	4	5	11	14	21	14	13	5	6	6	1.49
Belgium	7	6	5	8	14	10	15	4	0	30	1.49
Norway	4	18	8	5	10	10	29	3	4	8	1.41
Netherlands	6	9	18	11	17	6	20	4	4	6	1.67
Iceland	9	7	14	6	10	7	25	10	2	9	1.50
Finland	3	9	20	2	12	2	26	12	10	4	1.55
Austria	8	11	11	8	16	10	23	5	2	7	1.69
U.K.	7	11	8	8	9	9	22	8	8	9	1.68
Spain	14	7	8	8	12	15	16	7	6	8	1.70
Italy	7	4	10	4	15	11	32	8	3	5	1.56
Mean	9	8	11	7	13	11	20	8	5	8	1.56
<u>Model IV: Theil's cross-country demand system</u>											
U.S.	16	3	7	8	9	11	33	4	3	7	1.47
Canada	15	3	5	11	4	29	12	7	13	3	1.60
Sweden	12	9	8	12	9	14	17	10	3	6	1.60
Denmark	7	4	14	17	8	10	19	9	4	8	1.53
Australia	4	2	8	9	24	16	13	14	6	5	1.45
France	3	7	10	21	17	12	16	5	5	4	1.43
Belgium	6	5	6	15	15	11	9	4	0	29	1.51
Norway	2	15	9	14	8	11	28	3	3	8	1.42
Netherlands	6	8	20	8	18	10	15	4	4	7	1.66
Iceland	7	5	9	23	8	9	29	5	1	5	1.50
Finland	3	5	21	12	10	4	24	9	8	5	1.65
Austria	6	11	11	11	14	10	25	4	1	6	1.63
U.K.	4	11	7	13	8	14	22	8	6	6	1.62
Spain	14	7	7	16	10	10	15	5	4	12	1.65
Italy	9	4	13	10	12	10	22	9	3	7	1.62
Mean	8	7	10	13	12	12	20	7	4	8	1.56
<u>Naive model</u>											
U.S.	17	7	8	6	6	20	24	5	3	4	1.52
Canada	10	6	9	13	7	12	13	9	15	5	1.58
Sweden	13	11	8	19	7	7	16	11	1	6	1.58
Denmark	10	8	10	30	10	3	13	5	7	5	1.62
Australia	15	5	10	14	18	11	10	9	4	5	1.56
France	19	10	10	20	9	16	11	2	1	1	1.62
Belgium	13	6	5	15	13	11	8	3	0	25	1.49
Norway	15	6	10	11	6	8	31	4	2	8	1.54
Netherlands	16	7	17	7	17	12	10	4	4	6	1.68
Iceland	13	5	11	19	9	6	24	6	1	6	1.59
Finland	12	5	17	8	11	2	26	7	6	6	1.58
Austria	14	10	6	19	10	7	27	2	2	3	1.50
U.K.	16	7	10	16	11	4	23	6	3	4	1.55
Spain	18	7	4	9	3	22	14	6	5	10	1.71
Italy	15	10	9	7	11	9	24	4	2	9	1.59
Mean	14	7	10	14	10	10	18	6	4	7	1.58

All entries in columns 2-11 are to be divided by 100.

equipment/recreation and cultural services/recreation; and education into books/magazines and education. To aggregate these into relative prices for our 10 groups, we take conditional budget-share-weighted-averages of these three pairs.

Sweden, Australia and Iceland are excluded from Ward's (1985) study. For these three countries we use the relative prices averaged over the 12 included countries.

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