DATA ANALYTIC TECHNIQUES
FOR CONSUMPTION ECONOMICS

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DATA ANALYTIC TECHNIQUES FOR CONSUMPTION ECONOMICS

1. Introduction

This paper introduces a number of new data analytic techniques which can be used in consumption economics. These techniques are useful to apply before estimating demand equations to obtain a general 'feel' for the data. They provide summary measures of the data and informal estimates of key demand parameters. The applications of these techniques are illustrated with U.K. alcohol data, 1955-1975.

We commence by presenting the data and several Divisia index numbers. We then compare the U.K. data with alcohol consumption in Australia and the U.S. Finally, we apply the new techniques to the U.K. data.

2. The Basic Data

Quantity and Price Data

McGuinness (1980, App.2) gives U.K. consumption of beer, wine and spirits, 1955-1975, in terms of gallons of pure alcohol per adult, where an adult is defined to be a person aged fifteen or over. For each of the three beverages he also gives current-price expenditure per adult.

We convert McGuinness' data into litres per capita and then compute the implicit price indexes as the ratios of current-price expenditures to the corresponding quantities. The details of these conversions are given in the Appendix.

In Table 1 we give the resulting quantity and price data. In 1975, U.K. per capita consumption of beer is 118.1 litres, wine is 6.3 litres and spirits 3.7 litres. Figure 1 plots alcohol consumption, while Figure 2 plots the prices.

Logarithmic Changes

In Table 2 we give the quantity and price log-changes. The log-change in consumption and the price from year $t-1$ to t is $\log q_{it} - \log q_{i,t-1}$ and $\log p_{it} - \log p_{i,t-1}$, where q_{it} and p_{it} represent per capita consumption and the price of i (= beer, wine and spirits) in year t . As can be seen from Table 2, the average annual growth in per capita consumption is approximately 2.0, 6.9 and 4.6 percent for beer, wine and spirits. On average, the prices increased by 5.7, 4.2 and 4.3 percent for beer, wine and spirits, respectively. We note that wine has the highest annual growth in consumption and lowest price increase. Figures 3 and 4 plot the quantity and price log-changes.

Expenditures and Budget Shares

We convert McGuinness' (1980, App.2) alcohol expenditure data to a per capita basis and Table 3 contains the results; see the Appendix for details of the conversion. Table 4 gives the expenditure on each

TABLE 1
PER CAPITA ALCOHOL CONSUMPTION AND PRICE INDEXES:
UNITED KINGDOM, 1955-1975

Year	Beer		Wine		Spirits	
	Quantity	Price	Quantity	Price	Quantity	Price
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1955	79.64	53.31	1.5899	65.74	1.4785	59.46
1956	80.44	54.17	1.6652	64.10	1.5701	59.95
1957	80.83	56.42	1.7594	65.28	1.5875	60.25
1958	78.36	57.16	1.8105	63.94	1.6126	61.36
1959	81.53	53.35	1.9785	62.43	1.7324	60.91
1960	84.83	52.02	2.2872	58.24	1.8394	61.53
1961	88.78	54.39	2.4308	60.90	1.9260	66.91
1962	88.33	59.61	2.4888	59.29	1.9248	70.74
1963	87.95	62.08	2.7466	68.42	2.0441	67.31
1964	91.40	65.47	3.1200	71.17	2.1947	71.43
1965	91.55	72.39	2.9619	77.05	2.0345	78.82
1966	92.35	76.98	3.1351	81.30	2.0572	83.79
1967	93.49	81.17	3.4350	82.48	2.0620	86.37
1968	94.46	83.89	3.8281	84.97	2.1503	91.21
1969	97.70	90.92	3.6849	95.48	2.0215	96.82
1970	99.96	100.00	3.7519	100.00	2.2961	100.00
1971	103.63	108.26	4.3954	103.88	2.4308	103.57
1972	106.05	114.91	5.0374	105.64	2.7827	104.17
1973	111.26	118.80	6.3594	107.21	3.4946	107.01
1974	114.20	132.62	6.6992	121.08	3.8554	109.51
1975	118.13	165.95	6.2942	151.19	3.6775	139.56
Mean	93.57	81.61	3.4028	83.32	2.2273	82.89

Quantities are in litres per capita and the price indexes have base 1970=100.

ALCOHOL CONSUMPTION: UNITED KINGDOM, 1955-1975

Figure 1

ALCOHOL PRICE INDEXES: UNITED KINGDOM, 1955-1975

Figure 2

TABLE 2
ALCOHOL QUANTITY AND PRICE LOG-CHANGES:
UNITED KINGDOM, 1955-1975

Year	Beer		Wine		Spirits	
	Quantity	Price	Quantity	Price	Quantity	Price
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1956	.9977	1.5857	4.6253	-2.5253	6.0126	.8194
1957	.4853	4.0708	5.5024	1.8256	1.0970	.5010
1958	-3.1047	1.3176	2.8648	-2.0831	1.5735	1.8116
1959	3.9702	-6.9073	8.8728	-2.3771	7.1665	-.7312
1960	3.9681	-2.5261	14.4979	-6.9632	5.9905	1.0218
1961	4.5460	4.4614	6.0911	4.4761	4.6021	8.3738
1962	-.5052	9.1582	2.3580	-2.6864	-.0619	5.5659
1963	-.4334	4.0606	9.8575	14.3238	6.0107	-4.9642
1964	3.8481	5.3183	12.7475	3.9386	7.1095	5.9362
1965	.1638	10.0529	-5.2012	7.9479	-7.5791	9.8505
1966	.8647	6.1386	5.6834	5.3613	1.1084	6.1140
1967	1.2324	5.3116	9.1352	1.4402	.2339	3.0333
1968	1.0340	3.2961	10.8355	2.9788	4.1941	5.4420
1969	3.3741	8.0461	-3.8145	11.6635	-6.1799	5.9777
1970	2.2820	9.5145	1.8038	4.6243	12.7405	3.2278
1971	3.6055	7.9376	15.8298	3.8056	5.6988	3.5082
1972	2.3125	5.9599	13.6317	1.6784	13.5200	.5791
1973	4.7921	3.3327	23.3050	1.4752	22.7809	2.6837
1974	2.6125	11.0019	5.2057	12.1671	9.8245	2.3140
1975	3.3835	22.4207	-6.2362	22.2122	-4.7225	24.2493
Mean	1.9715	5.6776	6.8798	4.1642	4.5560	4.2657

All entries are to be divided by 100. The quantities are expressed in per capita terms. The year 1956, for example, refers to the transition from 1955 to 1956.

LOG-CHANGES IN ALCOHOL CONSUMPTION:
UNITED KINGDOM, 1955-1975

Figure 3

LOG-CHANGES IN ALCOHOL PRICES:
UNITED KINGDOM, 1955-1975

Figure 4

TABLE 3
EXPENDITURE ON ALCOHOL:
UNITED KINGDOM, 1955-1975

Year (1)	Beer (2)	Wine (3)	Spirits (4)	Total (5)
1955	10.36	1.66	4.22	16.24
1956	10.64	1.69	4.51	16.84
1957	11.13	1.82	4.59	17.54
1958	10.93	1.83	4.75	17.51
1959	10.62	1.96	5.06	17.64
1960	10.77	2.11	5.43	18.31
1961	11.79	2.35	6.18	20.31
1962	12.85	2.34	6.53	21.72
1963	13.33	2.98	6.60	22.90
1964	14.61	3.52	7.52	25.64
1965	16.18	3.62	7.69	27.49
1966	17.35	4.04	8.27	29.66
1967	18.53	4.49	8.54	31.56
1968	19.35	5.15	9.41	33.91
1969	21.69	5.57	9.39	36.65
1970	24.40	5.94	11.01	41.36
1971	27.39	7.23	12.07	46.70
1972	29.75	8.43	13.90	52.08
1973	32.27	10.80	17.93	61.00
1974	36.97	12.85	20.25	70.07
1975	47.86	15.08	24.61	87.55
Mean	19.46	5.02	9.45	33.94

Expenditures are expressed in current pounds per capita.

TABLE 4
 CONDITIONAL BUDGET SHARES FOR ALCOHOL:
 UNITED KINGDOM, 1955-1975

Year (1)	Beer (2)	Wine (3)	Spirits (4)
1955	63.83	10.20	25.97
1956	63.15	10.04	26.81
1957	63.47	10.38	26.16
1958	62.44	10.47	27.09
1959	60.21	11.10	28.70
1960	58.83	11.52	29.65
1961	58.03	11.55	30.43
1962	59.17	10.76	30.06
1963	58.19	13.00	28.81
1964	56.96	13.72	29.32
1965	58.86	13.16	27.98
1966	58.51	13.62	27.88
1967	58.71	14.22	27.07
1968	57.06	15.20	27.74
1969	59.18	15.21	25.61
1970	59.00	14.37	26.63
1971	58.65	15.49	25.86
1972	57.12	16.19	26.69
1973	52.90	17.71	29.40
1974	52.76	18.34	28.90
1975	54.66	17.22	28.12
Mean	58.65	13.50	27.85

All entries are to be divided by 100.

beverage expressed as a fraction of total alcohol expenditure, $w_{it} = p_{it}q_{it}/M_{gt}$, where $M_{gt} = \sum_{i=1}^3 p_{it}q_{it}$ is total alcohol expenditure; these fractions are the conditional budget shares. Table 4 shows that the share for beer has decreased by about 9 percentage points over this period and is 54.7 percent in 1975. Most of that fall has been offset by the increase in the wine share, which is 10.2 percent in 1955 and 17.2 in 1975. The share for spirits has been much more constant than those of the other two beverages over this period. In Figure 5 we plot the conditional budget shares.

Other Data

Table 5 gives the consumer price index (from International Monetary Fund, International Financial Statistics, 1981); total (current-price) per capita consumption expenditure M_t (Economic Trends, Annual Supplement, 1977/79); and the expenditure on each of the three beverages as a proportion of total consumption expenditure, $w_{it} = p_{it}q_{it}/M_t$, which is known as the unconditional budget share of i . Column 7 of Table 5 contains the unconditional budget share for total alcohol, defined as $W_{gt} = \sum_{i=1}^3 w_{it}$. As can be seen, the budget share for total alcohol first declines and then increases slowly. These unconditional budget shares are plotted in Figure 6.

Table 6 presents the unconditional budget shares, the budget share for total alcohol and the conditional budget shares, all in arithmetic average form. Here

$$(2.1) \quad \bar{w}_{it} = \frac{1}{2} (w_{it} + w_{i,t-1}), \quad i=1(\text{beer}), 2(\text{wine}), 3(\text{spirits}).$$

CONDITIONAL BUDGET SHARES OF BEER, WINE AND SPIRITS:
UNITED KINGDOM, 1955-1975

Figure 5

UNCONDITIONAL BUDGET SHARES OF
BEER, WINE, SPIRITS AND TOTAL ALCOHOL:
UNITED KINGDOM, 1955-1975

Figure 6

TABLE 5

CPI, TOTAL CONSUMPTION EXPENDITURE AND UNCONDITIONAL BUDGET SHARES

FOR ALCOHOL: UNITED KINGDOM, 1955-1975

Year	CPI (1970=100)	Total Consumption		Unconditional Budget Shares		
		Expenditure (Pounds/Capita)	Beer	Wine	Spirits	Total Alcohol
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1955	60.33	255	4.0647	.6494	1.6535	6.3676
1956	62.92	267	3.9836	.6334	1.6909	6.3078
1957	64.94	281	3.9616	.6476	1.6325	6.2417
1958	66.79	295	3.7067	.6217	1.6086	5.9370
1959	67.16	309	3.4364	.6333	1.6378	5.7075
1960	67.90	322	3.3455	.6553	1.6858	5.6867
1961	69.74	337	3.4979	.6960	1.8340	6.0278
1962	72.51	354	3.6309	.6604	1.8447	6.1359
1963	73.99	374	3.5637	.7960	1.7644	6.1241
1964	76.38	397	3.6795	.8861	1.8938	6.4594
1965	79.89	421	3.8430	.8588	1.8269	6.5287
1966	83.03	444	3.9083	.9095	1.8620	6.6797
1967	85.24	464	3.9927	.9673	1.8409	6.8009
1968	89.30	497	3.8926	1.0369	1.8925	6.8220
1969	94.10	526	4.1229	1.0597	1.7846	6.9672
1970	100.00	571	4.2735	1.0410	1.9285	7.2431
1971	109.41	635	4.3130	1.1392	1.9014	7.3536
1972	117.34	715	4.1607	1.1791	1.9444	7.2842
1973	128.04	807	3.9984	1.3385	2.2223	7.5592
1974	148.52	927	3.9885	1.3863	2.1843	7.5591
1975	184.50	1134	4.2203	1.3295	2.1706	7.7204
Mean	90.57	492	3.8850	.9107	1.8478	6.6435

All entries in the last four columns are to be divided by 100.

TABLE 6

ARITHMETIC AVERAGES OF UNCONDITIONAL AND CONDITIONAL BUDGET SHARES:

UNITED KINGDOM, 1955-1975

Year	Unconditional				Conditional		
	Beer $\bar{w}_1$	Wine $\bar{w}_2$	Spirits $\bar{w}_3$	Total Alcohol $\bar{w}_g$	Beer $\bar{w}'_1$	Wine $\bar{w}'_2$	Spirits $\bar{w}'_3$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1956	4.0241	.6414	1.6722	6.3377	63.495	10.120	26.385
1957	3.9726	.6405	1.6617	6.2748	63.311	10.207	26.482
1958	3.8342	.6346	1.6205	6.0893	62.965	10.422	26.613
1959	3.5715	.6275	1.6232	5.8222	61.343	10.778	27.879
1960	3.3909	.6443	1.6618	5.6971	59.521	11.310	29.169
1961	3.4217	.6756	1.7599	5.8573	58.418	11.535	30.046
1962	3.5644	.6782	1.8393	6.0819	58.607	11.150	30.243
1963	3.5973	.7282	1.8045	6.1300	58.683	11.879	29.438
1964	3.6216	.8411	1.8291	6.2918	57.561	13.368	29.071
1965	3.7612	.8725	1.8603	6.4941	57.918	13.435	28.647
1966	3.8756	.8842	1.8444	6.6042	58.684	13.388	27.928
1967	3.9505	.9384	1.8514	6.7403	58.610	13.922	27.468
1968	3.9426	1.0021	1.8667	6.8114	57.882	14.712	27.405
1969	4.0077	1.0483	1.8385	6.8946	58.129	15.205	26.666
1970	4.1982	1.0504	1.8566	7.1051	59.087	14.783	26.130
1971	4.2932	1.0901	1.9150	7.2983	58.825	14.936	26.239
1972	4.2369	1.1591	1.9229	7.3189	57.889	15.838	26.273
1973	4.0796	1.2588	2.0833	7.4217	54.968	16.961	28.071
1974	3.9935	1.3624	2.2033	7.5591	52.830	18.023	29.147
1975	4.1044	1.3579	2.1775	7.6398	53.724	17.774	28.502
Mean	3.8721	.9068	1.8446	6.6235	58.623	13.487	27.890

The year 1956, for example, refers to the transition from 1955 to 1956. All entries are to be divided by 100.

is the arithmetic average of the unconditional budget share of commodity i ;

$$(2.2) \quad \bar{w}_{gt} = \sum_{i=1}^3 \bar{w}_{it}$$

is the arithmetic average of the budget share of total alcohol; and

$$(2.3) \quad \bar{w}'_{it} = \frac{\bar{w}_{it}}{\bar{w}_{gt}}, \quad i=1,2,3,$$

is the arithmetic average of the conditional budget share of commodity i .

3. Divisia Indexes

In this section we give within group (or conditional) versions of Divisia indexes and then apply them to the U.K. alcohol data. See Theil (1975/76) for details of these indexes.

The within-alcohol versions of the Divisia volume and price indexes are

$$(3.1) \quad DQ_{gt} = \sum_{i=1}^3 \bar{w}'_{it} Dq_{it}, \quad DP_{gt} = \sum_{i=1}^3 \bar{w}'_{it} Dp_{it}$$

and the Divisia variances and covariance are

$$(3.2) \quad K_{gt} = \sum_{i=1}^3 \bar{w}'_{it} (Dq_{it} - DQ_{gt})^2, \quad \Pi_{gt} = \sum_{i=1}^3 \bar{w}'_{it} (Dp_{it} - DP_{gt})^2,$$

$$(3.3) \quad \Gamma_{gt} = \sum_{i=1}^3 \bar{w}'_{it} (Dp_{it} - DP_{gt})(Dq_{it} - DQ_{gt}),$$

where $\bar{w}'_{it}$ is the conditional budget share defined in (2.3) and Dp_{it} and Dq_{it} are the price and quantity log-changes.

The Divisia volume and price indexes defined by (3.1) are given in columns 2 and 3 of Table 7. On average, per capita alcohol consumption increased by about 3.3 percent per annum while alcohol prices rose by 5.1 percent. We give the Divisia variances and covariance defined by (3.2) and (3.3) in columns 4, 5 and 6 of the same table. In 14 out of 20 cases, the quantity variance K_g exceeds the price variance Π_g , which agrees with the results of Clements (1982,1983), Meisner (1979) and Suhm (1979). We also present the Divisia price-quantity correlation,

$$(3.4) \quad \rho_{gt} = \frac{\Gamma_{gt}}{\sqrt{K_{gt}\Pi_{gt}}},$$

in column 7 of Table 7. The correlation is negative in 16 cases out of 20 with an average of $-.42$. This reflects the tendency of the drinker to move away from those beverages having above-average price increases.

The within-alcohol version of the Divisia variance of the logarithmic changes in the budget shares, in terms of finite changes, is

$$(3.5) \quad \psi_{gt} = \sum_{i=1}^3 \bar{w}'_{it} \left[Dw'_{it} - \sum_{j=1}^3 \bar{w}'_{jt} Dw'_{jt} \right]^2,$$

TABLE 7
DIVISIA INDEXES OF ALCOHOL CONSUMPTION:
UNITED KINGDOM, 1955-1975

Year	Volume DQ_g x100	Price DP_g x100	Quantity Variance K_g x10000	Price Variance Π_g x10000	Price- Quantity Covariance Γ_g x10000	Price- Quantity Correlation ρ_g	Budget Share Variance ψ_g x10000	Γ_g/Π_g
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1956	2.69	.97	5.11	1.48	-1.48	-.537	3.64	-.997
1957	1.16	2.90	2.21	2.51	-.94	-.397	2.85	-.373
1958	-1.24	1.10	6.05	1.22	-1.08	-.399	5.10	-.889
1959	5.39	-4.70	3.42	7.96	4.76	.912	20.91	.598
1960	5.75	-1.99	10.56	5.61	-4.14	-.538	7.90	-.738
1961	4.74	5.64	.24	3.21	-.16	-.184	3.13	-.050
1962	-.05	6.75	.77	13.75	-3.17	-.976	8.18	-.231
1963	2.69	2.62	15.07	34.42	-.09	-.004	49.32	-.003
1964	5.99	5.31	9.11	.37	-1.05	-.573	7.38	-2.860
1965	-2.78	9.71	12.40	.49	.96	.391	14.82	1.965
1966	1.58	6.03	2.62	.07	-.42	-1.000	1.84	-6.167
1967	2.06	4.15	8.29	2.16	-2.67	-.632	5.10	-1.240
1968	3.34	3.84	11.54	.98	.15	.045	12.83	.154
1969	-.27	8.05	18.94	3.13	1.31	.170	24.69	.419
1970	4.94	7.15	21.53	8.27	-10.54	-.790	8.72	-1.275
1971	5.98	6.16	17.83	4.53	-5.75	-.640	10.86	-1.269
1972	7.05	3.87	30.85	6.13	-13.61	-.989	9.77	-2.219
1973	12.98	2.84	81.90	.46	-5.04	-.824	72.28	-11.042
1974	5.18	8.68	9.77	16.85	-11.75	-.916	3.12	-.697
1975	-.64	22.91	19.01	.73	-1.92	-.517	15.90	-2.646
Mean	3.33	5.10	14.36	5.72	-2.83	-.420	14.42	-1.478

The year 1956, for example, refers to the transition from 1955 to 1956.

where $Dw'_{it} = \log w'_{it} - \log w'_{i,t-1}$. Column 8 of Table 7 gives this variance. The Divisia variances and covariance satisfy

$$\psi_{gt} = K_{gt} + \Pi_{gt} + 2\Gamma_{gt}.$$

In the last column of Table 7, we give the ratio of Γ_g to Π_g . We will come back to this ratio in Section 6.

4. A Cross-Country Comparison

In Tables 8 and 9 we compare key aspects of the British data with those for Australia and America. Looking at column 2 of Table 8, at means the Australians devote 5.9 percent of their income to alcohol while the figures for the British and the Americans are 6.6 and 3.9 percent, respectively. For the Australians beer accounts for 72 percent of total alcohol expenditure at sample means (see, column 3), while the corresponding figures for the British and the Americans are 59 and 47 percent. The mean shares for spirits are 16, 28 and 46 percent for Australia, the U.K. and the U.S., respectively. Relative to the British, the Australians accordingly are bigger beer drinkers and smaller consumers of spirits. On the other hand, the Americans are bigger spirits drinkers and smaller consumers of beer than the British. The share for wine is quite similar in Australia and the U.K, whereas it is smaller in the U.S.

It can be seen from column 2 of Table 9 that total per capita alcohol consumption in Australia increased at a slower rate than it did in the U.K. and at about the same rate in the U.S. Alcohol prices

TABLE 8
 ARITHMETIC AVERAGES OF GROUP AND CONDITIONAL
 BUDGET SHARES FOR ALCOHOL:
 AUSTRALIA, THE UNITED KINGDOM AND THE UNITED STATES

Year	Conditional Budget Shares			
	Total Alcohol $\bar{w}_g$	Beer $\bar{w}_1$	Wine $\bar{w}_2$	Spirits $\bar{w}_3$
(1)	(2)	(3)	(4)	(5)
<u>Australia</u>				
1956/57	6.12	75.73	8.59	15.67
1976/77	6.07	65.75	18.62	15.64
Sample Mean	5.94	71.51	12.71	15.78
<u>United Kingdom</u>				
1955/56	6.34	63.50	10.12	26.39
1974/75	7.64	53.72	17.77	28.50
Sample Mean	6.62	58.62	13.49	27.89
<u>United States</u>				
1949/50	4.77	50.77	5.95	43.29
1981/82	3.07	52.01	12.31	35.68
Sample Mean	3.87	47.13	7.34	45.54

The Australian data are from Clements and Johnson (1983, Table 2) and the American are from Clements and Selvanathan (1987). All entries are to be divided by 100.

TABLE 9
DIVISIA VOLUME AND PRICE INDEXES.
RELATIVE QUANTITY AND PRICE LOG-CHANGES OF ALCOHOL:
AUSTRALIA, THE UNITED KINGDOM, AND THE UNITED STATES
(SAMPLE MEANS)

Country	Divisia	Divisia	Relative Quantity			Relative Price		
	Volume	Price	Log-changes			Log-changes		
	Index	Index	Beer	Wine	Spirits	Beer	Wine	Spirits
	DQ_g	DP_g	$Dq_1 - DQ_g$	$Dq_2 - DQ_g$	$Dq_3 - DQ_g$	$Dp_1 - DP_g$	$Dp_2 - DP_g$	$Dp_3 - DP_g$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Australia	1.81	5.49	-.62	2.66	.38	.00	1.05	-.73
U.K.	3.33	5.10	-1.36	3.55	1.23	.58	-.93	-.83
U.S.A.	1.46	3.07	-.39	1.38	.09	.42	.96	-.65

The quantities are expressed in per capita terms. The Australian data are from Clements and Johnson (1983, Table 1 and Appendix, Table 7). The American data are from Clements and Selvanathan (1987). All entries are to be divided by 100.

(as measured by the Divisia index) increased by about 5 percent per annum in Australia and the U.K. and by about 3 percent in the U.S. We also give the within-alcohol relative quantity changes, $(Dq_i - DQ_g)$, and price changes, $(Dp_i - DP_g)$, for the three countries in columns 4-9 of the same table. Wine and spirits, in all three countries, have a positive relative consumption growth rate while that of beer is negative. The relative growth rate of wine in all three countries is much higher than that of the other two beverages. The relative price change for beer in the U.K. and the U.S. and for wine in Australia and the U.S. are in the same order of magnitude. In Australia and the U.S., the relative price of wine increased by about 1 percent per annum, while

it fell by .9 percent in the U.K. The relative price change for spirits is of the same order of magnitude in the three countries.

Comparing the relative quantity changes for each beverage with the corresponding relative price changes, we see that except for wine in Australia and the U.S., all have opposite signs. This implies that a higher relative price inhibits consumption.

5. Scatter Plots

In this section, we use differential demand equations to provide a framework for a preliminary analysis of the U.K. alcohol data. The Rotterdam parameterization of the composite demand equation under block independence is (Selvanathan, 1987)

$$(5.1) \quad \bar{W}_{gt} DQ_{gt} = \theta_g DQ_t + \phi \theta_g (DP'_{gt} - DP'_t),$$

where DQ_{gt} is defined by equation (3.1); θ_g is the marginal share of the group; DQ_t is the Divisia index of the change in real income; ϕ is the income flexibility; $DP'_{gt} = \sum_{i=1}^3 \theta'_i Dp_{it}$ is the Frisch price index of alcohol; $\theta'_i = \theta_i / \theta_g$ is the conditional marginal share of i ; θ_i is the unconditional marginal share of i ; and $DP'_t = \sum_{j=1}^n \theta_j Dp_{jt}$ is the overall Frisch price index.

If we replace the marginal shares, θ_i and θ'_i , with the corresponding budget shares, $\bar{w}_{it}$ and $\bar{w}'_{it}$, then the group marginal share θ_g becomes the group budget share $\bar{W}_g$ and the composite demand equation (5.1) simplifies to

$$(5.2) \quad DQ_{gt} - DQ_t = \phi (DP_{gt} - DP_t),$$

where DP_{gt} is defined by equation (3.1) and

$$(5.3) \quad DP_t = \sum_{j=1}^n \bar{w}_{jt} DP_{jt}$$

is the overall Divisia price index.

Equation (5.2) shows that growth in consumption of alcohol relative to that of real income, $(DQ_{gt} - DQ_t)$, depends only on the change in the relative price of the group $(DP_{gt} - DP_t)$. Using (5.2), if we plot $(DQ_{gt} - DQ_t)$ against $(DP_{gt} - DP_t)$ for $t=1, \dots, T$, the points should be scattered around a straight line with slope ϕ . In order to construct this scatter plot, we give in Table 10 the values of DQ_t , which is approximated by $(DM_t - DP_t^*)$, where DM_t is the log-change in income (total expenditure) and DP_t^* is the CPI log-change; DM_t and DP_t^* are also given in that table. We approximate the overall Divisia price index DP_t in (5.3) with DP_t^* .

Figure 7 plots $(DQ_g - DQ)$ against $(DP_g - DP^*)$, where the data are from Tables 7 and 10. This figure also gives the least-squares regression line. As can be seen, the observations fall around a straight line with negative slope, so that alcohol consumption grows less rapidly when its relative price rises. Under (5.2), the slope of this line is interpreted as an estimate of the income flexibility ϕ ,

$$(5.4) \quad \text{Estimate of } \phi \text{ under (5.2)} = -.610 (.173),$$

where the figure in parenthesis is the standard error. This estimate is significantly negative. The intercept ($\times 100$) of regression line is 1.155 (.558) which is also highly significant. This value of the

TABLE 10
TOTAL CONSUMPTION EXPENDITURE, CPI AND
REAL EXPENDITURE LOG-CHANGES:
UNITED KINGDOM, 1955-1975

Year	Total Consumption Expenditure DM	CPI DP*	Real Total Expenditure DQ=DM-DP*
(1)	(2)	(3)	(4)
1956	4.599	4.203	.395
1957	5.111	3.160	1.951
1958	4.862	2.809	2.053
1959	4.637	.552	4.084
1960	4.121	1.096	3.025
1961	4.553	2.674	1.879
1962	4.921	3.895	1.026
1963	5.496	2.021	3.475
1964	5.968	3.179	2.789
1965	5.870	4.493	1.377
1966	5.319	3.855	1.464
1967	4.406	2.627	1.779
1968	6.871	4.653	2.217
1969	5.671	5.236	.435
1970	8.209	6.081	2.128
1971	10.624	8.993	1.630
1972	11.866	6.997	4.868
1973	12.104	8.727	3.377
1974	13.863	14.838	-.975
1975	20.155	21.693	-1.538
Mean	7.461	5.589	1.872

All entries are to be divided by 100. Expenditures are expressed in per capita terms. The year 1956, for example, refers to the transition from 1955 to 1956.

RELATIVE CONSUMPTION OF ALCOHOL AGAINST RELATIVE PRICE:
UNITED KINGDOM, 1955-1975

Figure 7

ALCOHOL CONSUMPTION SCATTERGRAM:
UNITED KINGDOM, 1955-1975

Figure 8

intercept can be interpreted as the residual exponential trend rate of growth in per capita alcohol consumption. Thus U.K. alcohol consumption increases autonomously at 1.2 percent per annum.

We return to (5.1) with DP_g and DP_g^* replacing DP'_g and DP'_t , as before. We write this as

$$(5.5) \quad \bar{W}_{gt} DQ_{gt} = \theta_g [DQ_t + \phi(DP_{gt} - DP_t^*)].$$

The term in square brackets on the right represents the change in real income adjusted for the change in the relative price of alcohol. Thus if we specify the value of ϕ and plot $\bar{W}_g DQ_g$ against $[DQ + \phi(DP_g - DP_g^*)]$, the observations should be scattered around a line with slope θ_g . We take ϕ as $-.7$ which is in consistent with previous econometric findings (Theil, 1980), as well as (5.4). This scatter plot is given in Figure 8. The slope of the least-squares line fitted to the points in Figure 8 is .062 and has a standard error of .013. Under (5.5), this is interpreted as an estimate of θ_g :

$$(5.6) \quad \text{Estimate of } \theta_g \text{ under (5.5)} = .062 (.013),$$

which is highly significant. This means that a one-dollar increase in income results in a 6.2 cent increase in spending on alcohol. Dividing this estimate by the mean of $\bar{W}_{gt}$ of .066 (see Table 6), yields an implied income elasticity of $.062/.066 = .94$. Thus alcohol as a whole has an income elasticity of about unity.

The i^{th} equation of the relative price version of the conditional

Rotterdam model (again under block independence, see Selvanathan, 1987) is

$$(5.7) \quad \bar{w}_{it} Dq_{it} = \theta'_i \bar{w}_{gt} DQ_{gt} + \sum_{j=1}^3 v_{ij} (Dp_{jt} - DP'_{gt}),$$

where DP'_{gt} is defined below equation (5.1); and v_{ij} is the $(i,j)^{th}$ price coefficient satisfying $\sum_{j=1}^3 v_{ij} = \phi \theta_i = \phi \theta_g \theta'_i$, $i=1,2,3$. Under preference independence within alcohol, we have

$$v_{ij} = 0, \quad i \neq j=1,2,3,$$

which implies that

$$v_{ii} = \phi \theta_g \theta'_i, \quad i=1,2,3.$$

Thus (5.7) becomes

$$(5.8) \quad \bar{w}_{it} Dq_{it} = \theta'_i \bar{w}_{gt} DQ_{gt} + \phi \theta_g \theta'_i (Dp_{it} - DP'_{gt}).$$

To eliminate some of the unknown parameters from (5.8), we replace θ'_i with $\bar{w}'_{it}$ and θ_g with $\bar{w}_{gt}$, so that the equation becomes

$$(5.9) \quad Dq_{it} - DQ_{gt} = \phi (Dp_{it} - DP_{gt}).$$

Equation (5.9) is a lower-case version of equation (5.2). Using (5.9) for a given i , we can plot the growth in consumption of i relative to the group average $(Dq_{it} - DQ_{gt})$ against the change in its relative price

$(Dp_{it} - DP_{gt})$ for $t=1, \dots, T$. The points should be scattered around a straight line with slope ϕ . As ϕ is the same for all $i=1,2,3$, we can also plot on the same graph $(Dq_{it} - DQ_{gt})$ against $(Dp_{it} - DP_{gt})$ for $i=1,2,3$ and $t=1, \dots, T$. This scatter plot is shown in Figure 9. Although there is quite a lot of dispersion, most of the observations do fall around a negatively-sloped line. Thus, when the price of a beverage increases more than average ($Dp_i > DP_g$), its consumption grows at a rate less than average ($Dq_i < DQ_g$). The slope of the least-squares regression line in Figure 9 is

$$(5.10) \quad \text{Estimate of } \phi \text{ under (5.9)} = -.332 (.181),$$

which is not highly significant. Four of the points in Figure 9 are outliers; these are indicated by an arrow. The slope of the least-squares line with those four outliers omitted is $-.887$ with standard error $.263$. This estimate of ϕ is highly significant and close to our previous findings.

From (5.8), we also have

$$(5.11) \quad \bar{w}_{it} Dq_{it} = \theta'_i [\bar{w}_{gt} DQ_{gt} + \phi \theta_g (Dp_{it} - DP_{gt})],$$

where DP'_{gt} has been replaced by DP_{gt} , as before. Now, if we plot $\bar{w}_i Dq_i$ against $[\bar{w}_g DQ_g + \phi \theta_g (Dp_i - DP_g)]$, the observations should be scattered around a straight line with slope θ'_i . Figures 10-12 give these plots with $\phi = -.7$ and $\theta_g = .062$ [from (5.6)] for beer, wine and spirits, respectively. The slopes of the least-squares regression lines are

$$(5.12) \quad \text{Estimate of } \theta'_1 \text{ under (5.11)} = .212 (.052),$$

$$(5.13) \quad \text{Estimate of } \theta'_2 \text{ under (5.11)} = .206 (.042)$$

RELATIVE CONSUMPTION OF BEER, WINE AND SPIRITS
AGAINST RELATIVE PRICES:
UNITED KINGDOM, 1955-1975

Figure 9

CONSUMPTION OF BEER AGAINST
ADJUSTED TOTAL ALCOHOL CONSUMPTION:
UNITED KINGDOM, 1955-1975

Figure 10

CONSUMPTION OF WINE AGAINST
ADJUSTED TOTAL ALCOHOL CONSUMPTION:
UNITED KINGDOM, 1955-1975

Figure 11

CONSUMPTION OF SPIRITS AGAINST
ADJUSTED TOTAL ALCOHOL CONSUMPTION:
UNITED KINGDOM, 1955-1975

Figure 12

and

$$(5.14) \quad \text{Estimate of } \theta'_3 \text{ under (5.11)} = .429 (.048).$$

These estimated conditional marginal shares are all positive and highly significant. It is to be noted that these estimates do not sum to one because we estimate (5.11) for $i=1,2,3$ separately. Thus we normalize these marginal shares so that they add up to one. This yields

$$(5.15) \quad \text{Normalized estimate of } \theta'_1 \text{ under (5.11)} = .250,$$

$$(5.16) \quad \text{Normalized estimate of } \theta'_2 \text{ under (5.11)} = .243$$

and

$$(5.17) \quad \text{Normalized estimate of } \theta'_3 \text{ under (5.11)} = .507.$$

These figures indicate that when total expenditure on alcohol increases by one dollar, expenditure on beer, wine and spirits increases by 25, 24 and 51 cents, respectively.

6. Further Estimates of the Income Flexibility

Now we introduce another procedure to estimate the income flexibility ϕ . Consider again equation (5.8), the conditional demand equation for beverage i under preference independence. We divide both sides of this equation by $\bar{W}_{gt}$ and add a disturbance (ϵ'_{it}) to give

$$(6.1) \quad \bar{w}'_{it} Dq_{it} = \theta'_i DQ_{gt} + \phi \frac{\theta_g}{\bar{w}_{gt}} \theta'_i (Dp_{it} - DP'_{gt}) + \epsilon'_{it}.$$

Again we eliminate the unknown parameters θ'_i and θ_g by replacing them with $\bar{w}'_{it}$ and $\bar{w}_{gt}$, respectively. Therefore (6.1) becomes

$$(6.2) \quad \bar{w}'_{it} (Dq_{it} - DQ_{gt}) = \phi \bar{w}'_{it} (Dp_{it} - DP_{gt}) + \epsilon'_{it}.$$

We assume that the disturbances ϵ'_{it} have zero means and

$$(6.3) \quad \text{cov}[\epsilon'_{it}, \epsilon'_{jt}] = \sigma_t^2 \bar{w}'_{it} \delta_{ij},$$

where δ_{ij} is the Kronecker delta. Specification (6.3) states that the error variance is larger for those goods having a larger conditional budget share, which is plausible. As $\sum_{i=1}^3 \epsilon'_{it} = 0$, the assumption that $\text{cov}[\epsilon'_{it}, \epsilon'_{jt}] = 0$ for $i \neq j$ cannot be exactly true. However, as our analysis is designed only to obtain a preliminary feel for the data, we shall use this assumption as an approximation. Accordingly, the transformed disturbances $\epsilon'_{it} / \sqrt{\bar{w}'_{it}}$ are uncorrelated with zero mean and the same variance σ_t^2 . This means that if we divide both sides of (6.2) by $\sqrt{\bar{w}'_{it}}$ and then regress $\sqrt{\bar{w}'_{it}} (Dq_{it} - DQ_{gt})$ on $\sqrt{\bar{w}'_{it}} (Dp_{it} - DP_{gt})$ for $i=1,2,3$, we obtain for year t the least-squares estimator of ϕ as

$$(6.4) \quad \hat{\phi}_t = \frac{\sum_{i=1}^3 \bar{w}'_{it} (Dp_{it} - DP_{gt}) (Dq_{it} - DQ_{gt})}{\sum_{i=1}^3 \bar{w}'_{it} (Dp_{it} - DP_{gt})^2}.$$

Using (3.2) and (3.3) we can write (6.4) as

$$(6.5) \quad \hat{\phi}_t = \frac{\Gamma_{gt}}{\Pi_{gt}}.$$

It follows that the sampling variance of this estimator is

$$(6.6) \quad \text{var } \hat{\phi}_t = \frac{\sigma_t^2}{\Pi_{gt}}$$

and that σ_t^2 is estimated unbiasedly by

$$(6.7) \quad \hat{\sigma}_t^2 = \frac{1}{n_g - 1} \left[K_{gt} - \hat{\phi}_t \Gamma_{gt} \right],$$

where $n_g = 3$ and K_{gt} is defined in (3.2).

Equation (6.5) shows that, for each t , we can estimate ϕ as the ratio of Γ_{gt} to Π_{gt} . We have tabulated these ratios in the last column of Table 7. As expected, these values fluctuate quite a bit. As can be seen, there are two outliers, in 1966 and 1973. As shown in the last entry of column 9 of the table, with the outliers included, we obtain an average value of these $\hat{\phi}_t$ of -1.5, which is much larger (in absolute value) than the previous estimates. When we re-calculate this mean omitting these outliers, we get -.62, which is much closer to the previous findings.

Equation (6.5) provides a one-period estimator of ϕ . Thus if we have T periods we get T estimates of ϕ . If these are independent, an optimal estimator of ϕ for all T periods is given by a weighted average of $\hat{\phi}_1, \dots, \hat{\phi}_T$ with weights being inversely proportional to their

variances. This yields

$$(6.8) \quad \hat{\phi} = \frac{\sum_{t=1}^T \frac{\Pi_{gt}}{\sum_{\tau=1}^T \Pi_{g\tau}} \hat{\phi}_t}{\sum_{t=1}^T \frac{\Pi_{gt}}{\sum_{\tau=1}^T \Pi_{g\tau}}} = \frac{\sum_{t=1}^T \frac{\Pi_{gt}}{\sum_{\tau=1}^T \Pi_{g\tau}} \frac{\Gamma_{gt}}{\Pi_{gt}}}{\sum_{t=1}^T \frac{\Pi_{gt}}{\sum_{\tau=1}^T \Pi_{g\tau}}} = \frac{\sum_{t=1}^T \Gamma_{gt}}{\sum_{t=1}^T \Pi_{gt}} = \frac{\bar{\Gamma}_g}{\bar{\Pi}_g},$$

where $\bar{\Gamma}_g = (1/T) \sum_{t=1}^T \Gamma_{gt}$; $\bar{\Pi}_g = (1/T) \sum_{t=1}^T \Pi_{gt}$; and where we have also assumed that $\sigma_t^2 = \sigma^2$. In Section 7 we show that the sampling variance of $\hat{\phi}$ is

$$(6.9) \quad \text{var } \hat{\phi} = \frac{\sigma^2}{\sum_{t=1}^T \Pi_{gt}} = \frac{\sigma^2}{T \bar{\Pi}_g}$$

and that an unbiased estimator of σ^2 is given by

$$(6.10) \quad \hat{\sigma}^2 = \frac{1}{T n_g - 1} \sum_{t=1}^T (K_{gt} - \hat{\phi} \Gamma_{gt})^2 = \frac{T}{T n_g - 1} (\bar{K}_g - \hat{\phi} \bar{\Gamma}_g)^2,$$

where $\bar{K}_g = (1/T) \sum_{t=1}^T K_{gt}$. (For a similar approach, see Clements and Theil, 1978.)

Using our data to evaluate (6.8) and (6.9) yields

$$(6.11) \quad \hat{\phi} = -.500 (.159),$$

which is significant and consistent with the previous findings.

7. Derivations

In this section we derive equations (6.9) and (6.10).

Under the assumption that the estimates $\hat{\phi}_t$ are independent, it follows from (6.6) and (6.8) that

$$(7.1) \quad \text{var } \hat{\phi} = \sum_{t=1}^T \frac{\Pi_{gt}^2}{\left[\sum_{\tau=1}^T \Pi_{g\tau} \right]^2} \text{var } \hat{\phi}_t = \frac{\sigma^2}{\left[\sum_{\tau=1}^T \Pi_{g\tau} \right]},$$

where we have used the assumption $\sigma_t^2 = \sigma^2$. Equation (7.1) is equivalent to (6.9).

We now show that $\hat{\sigma}^2$ defined in (6.10) is an unbiased estimator of σ^2 by establishing that

$$E \left[\sum_{t=1}^T \left[K_{gt} - \hat{\phi} \Gamma_{gt} \right] \right] = (Tn_g - 1)\sigma^2.$$

We proceed in two steps. In the first step we express K_{gt} and Γ_{gt} in terms of the disturbances ϵ'_{it} . In the second step we find the expectation $E \left[\sum_{t=1}^T \left[K_{gt} - \hat{\phi} \Gamma_{gt} \right] \right]$.

Step 1

From equation (6.2) we have that

$$\sqrt{\bar{w}'_{it}}(Dq_{it} - DQ_{gt}) = \phi \sqrt{\bar{w}'_{it}}(Dp_{it} - DP_{gt}) + \frac{\epsilon'_{it}}{\sqrt{\bar{w}'_{it}}}.$$

Substituting the right-hand side of the above equation for

$\sqrt{\bar{w}'_{it}}(Dq_{it} - DQ_{gt})$ in (3.2) yields

$$\begin{aligned}
 K_{gt} &= \sum_{i=1}^3 \left[\phi \sqrt{\bar{w}'_{it}}(Dp_{it} - DP_{gt}) + \frac{\epsilon'_{it}}{\sqrt{\bar{w}'_{it}}} \right]^2 \\
 (7.2) \quad &= \phi^2 \Pi_{gt} + 2\phi \sum_{i=1}^3 (Dp_{it} - DP_{gt})\epsilon'_{it} + \sum_{i=1}^3 \frac{\epsilon'^2_{it}}{\bar{w}'_{it}}.
 \end{aligned}$$

Substituting in equation (3.3) the right-hand side of (6.2) for $\bar{w}'_{it}(Dq_{it} - DQ_{gt})$ yields

$$\begin{aligned}
 \Gamma_{gt} &= \sum_{i=1}^3 (Dp_{it} - DP_{gt})[\phi \bar{w}'_{it}(Dp_{it} - DP_{gt}) + \epsilon'_{it}] \\
 (7.3) \quad &= \phi \Pi_{gt} + \sum_{i=1}^3 (Dp_{it} - DP_{gt})\epsilon'_{it}.
 \end{aligned}$$

Using equations (7.2) and (7.3), we obtain

$$\begin{aligned}
 (7.4) \quad K_{gt} - \hat{\phi} \Gamma_{gt} &= (\phi - \hat{\phi})\phi \Pi_{gt} + 2\phi \sum_{i=1}^3 (Dp_{it} - DP_{gt})\epsilon'_{it} \\
 &\quad + \sum_{i=1}^3 \frac{\epsilon'^2_{it}}{\bar{w}'_{it}} - \hat{\phi} \sum_{i=1}^3 (Dp_{it} - DP_{gt})\epsilon'_{it}.
 \end{aligned}$$

Step 2

As $\hat{\phi}_t$ is an unbiased estimator of ϕ , it follows from (6.8) that $E[\hat{\phi}] = \phi$. Taking the expectation of both sides of (7.4) yields

$$(7.5) \quad E[K_{gt} - \hat{\phi} \Gamma_{gt}] = \sum_{i=1}^3 \sigma_t^2 - E \left[\hat{\phi} \sum_{i=1}^3 (Dp_{it} - DP_{gt})\epsilon'_{it} \right],$$

where we have used $E[\hat{\phi}] = \phi$ and $E[\epsilon'_{it}] = 0$.

We now simplify the second term on the right of (7.5). Dividing both sides of (7.3) by Π_{gt} and using (6.5), we obtain

$$(7.6) \quad \hat{\phi}_s = \phi + \frac{1}{\Pi_{gs}} \sum_{i=1}^3 (Dp_{is} - DP_{gs}) \epsilon'_{is}, \quad s=1, \dots, T.$$

Multiplying both sides of (7.6) by $\lambda_{gs} = \Pi_{gs} / \sum_{\tau=1}^T \Pi_{g\tau}$ and then taking the sum over $s=1, \dots, T$, we obtain

$$\hat{\phi} = \phi + \sum_{s=1}^T \frac{\lambda_{gs}}{\Pi_{gs}} \left[\sum_{i=1}^3 (Dp_{is} - DP_{gs}) \epsilon'_{is} \right],$$

where we have used (6.8) and $\sum_{s=1}^T \lambda_{gs} = 1$. Multiplying both sides of the above equation by $\sum_{i=1}^3 (Dp_{it} - DP_{gt}) \epsilon'_{it}$ and taking the expectation gives

$$\begin{aligned} E \left[\hat{\phi} \sum_{i=1}^3 (Dp_{it} - DP_{gt}) \epsilon'_{it} \right] &= E \left[\sum_{s=1}^T \frac{\lambda_{gs}}{\Pi_{gs}} \left[\sum_{i=1}^3 (Dp_{is} - DP_{gs}) \epsilon'_{is} \right] \times \right. \\ &\quad \left. \left[\sum_{i=1}^3 (Dp_{it} - DP_{gt}) \epsilon'_{it} \right] \right] \\ &= E \left[\frac{\lambda_{gt}}{\Pi_{gt}} \left[\sum_{i=1}^3 (Dp_{it} - DP_{gt}) \epsilon'_{it} \right]^2 \right], \end{aligned}$$

where the second step uses $E[\epsilon'_{it}, \epsilon'_{is}] = 0$ for $t \neq s$. As $E[\epsilon'^2_{it}] = \bar{w}'_{it} \sigma_t^2$,

$$\begin{aligned} E \left[\hat{\phi} \sum_{i=1}^3 (Dp_{it} - DP_{gt}) \epsilon'_{it} \right] &= \frac{\lambda_{gt}}{\Pi_{gt}} \sum_{i=1}^3 \bar{w}'_{it} (Dp_{it} - DP_{gt})^2 \sigma_t^2 \\ (7.7) \quad &= \lambda_{gt} \sigma_t^2. \end{aligned}$$

Substituting (7.7) in (7.5), we obtain

$$\begin{aligned} E \left[K_{gt} - \hat{\phi} \Gamma_{gt} \right] &= \sum_{i=1}^3 \sigma_t^2 - \lambda_{gt} \sigma_t^2 \\ &= (n_g - \lambda_{gt}) \sigma^2, \end{aligned}$$

where $n_g = 3$ and we have assumed that $\sigma_t^2 = \sigma^2$. Therefore, we have

$$\begin{aligned} E \left[\sum_{t=1}^T \left[K_{gt} - \hat{\phi} \Gamma_{gt} \right] \right] &= \sum_{t=1}^T (n_g - \lambda_{gt}) \sigma^2 \\ &= (T n_g - 1) \sigma^2. \end{aligned}$$

This shows that $\hat{\sigma}^2$ defined in equation (6.10) is an unbiased estimator of σ^2 .

8. Comments on Assumptions

We conclude this paper with a brief recapitulation of the assumptions under which we obtained the preliminary estimates of the income flexibility and the marginal shares. The estimate (5.4) is derived under block-independence with marginal shares replaced by the corresponding budget shares. As the income elasticity is the ratio of the marginal share to budget share, this amounts to assuming unitary income elasticities. The estimate of θ_g given in (5.6) is also derived under a similar assumption.

The assumptions underlying the estimate of ϕ given in (5.10) are

(i) unitary income elasticities and (ii) that the three alcoholic beverages are preference independent in the consumer's utility function. The estimates obtained in Section 6 are also based on these two assumptions.

Some of these assumptions are obviously quite strong. Accordingly, the estimates in this paper are only meant to have a preliminary status.

APPENDIX

McGuinness (1980, App.2) gives U.K. consumption of beer, wine and spirits, 1955-1975, in terms of gallons of pure alcohol per adult, where an adult is defined to be a person aged fifteen or over. The conventional liquid volume units of measurement are standard gallons of beer, gallons of wine and proof gallons of spirits. McGuinness (1980, App.1) converts these liquid volumes to gallons of pure alcohol by multiplying by .048, .16 and .4, respectively. For each of the three beverages he also gives current-price expenditure per adult; expenditure on wine includes cider. Wherever possible, we cross-checked McGuinness' data and conversion factors back to original sources. The original quantity data from McGuinness and the implicit prices (calculated as the ratios of current-price expenditures to the corresponding quantities) are given in columns 2-7 of Table A1. Here and elsewhere the years are calendar years.

Walsh (1982, p.441) points out that the conversion factor for spirits should be .57 rather than .4. In columns 8-9 of Table A1 we give the quantity and price data for spirits with this correction made. The corrected price is calculated as the ratio of the given current-price expenditure on spirits to the corrected quantity.

Table A2 gives the quantity data in terms of litres per adult and the corresponding implicit prices. The quantities are derived from Table A1 as follows. For beer, we divide the quantity in Table A1 by $.22 \times .032637 = .00718014$. The .22 factor converts imperial gallons into litres. The .032637 factor is the average over the 21-year period of the ratio of the quantity of beer in terms of gallons of pure alcohol

TABLE A1
PER ADULT PURE ALCOHOL CONSUMPTION AND PRICES:
UNITED KINGDOM, 1955-1975

Year	McGuinness' Data						Corrected Data	
	Beer		Wine		Spirits		Spirits	
	Quantity	Price	Quantity	Price	Quantity	Price	Quantity	Price
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1955	.7459	18.13	.0730	29.59	.1188	46.30	.1693	32.49
1956	.7548	18.42	.0766	28.85	.1264	46.68	.1801	32.76
1957	.7591	19.18	.0810	29.38	.1279	46.91	.1823	32.92
1958	.7363	19.44	.0834	28.78	.1300	47.77	.1852	33.52
1959	.7658	18.14	.0911	28.10	.1396	47.42	.1989	33.28
1960	.7967	17.69	.1053	26.21	.1482	47.91	.2112	33.62
1961	.8344	18.49	.1120	27.41	.1553	52.09	.2213	36.56
1962	.8275	20.27	.1143	26.68	.1547	55.07	.2204	38.65
1963	.8230	21.11	.1260	30.79	.1641	52.41	.2338	36.78
1964	.8563	22.26	.1433	32.03	.1764	55.61	.2514	39.03
1965	.8581	24.61	.1361	34.68	.1636	61.37	.2331	43.07
1966	.8670	26.17	.1443	36.59	.1657	65.24	.2361	45.78
1967	.8794	27.60	.1584	37.12	.1664	67.25	.2371	47.19
1968	.8884	28.52	.1765	38.24	.1735	71.01	.2472	49.83
1969	.9200	30.91	.1701	42.97	.1633	75.38	.2327	52.90
1970	.9418	34.00	.1733	45.01	.1856	77.86	.2645	54.64
1971	.9854	36.81	.2049	46.75	.1983	80.64	.2826	56.59
1972	1.0062	39.07	.2343	47.55	.2265	81.10	.3228	56.91
1973	1.0517	40.39	.2947	48.25	.2834	83.31	.4038	58.46
1974	1.0752	45.09	.3092	54.50	.3114	85.26	.4437	59.83
1975	1.1072	56.42	.2892	68.05	.2957	108.66	.4214	76.25
Mean	.8800	27.75	.1570	37.50	.1798	64.54	.2561	45.29

Quantities are in gallons of pure alcohol per adult and the prices are in current pounds per gallon of pure alcohol. The pure alcohol conversion factor for spirits in McGuinness' data is .4, whereas it is .57 in the corrected data (see text).

(from Table A1) to the quantity in terms of gallons, from Monthly Digest of Statistics, various issues. The conversion factor .032637 is interpreted as the pure alcohol concentration of beer by volume, and is consistent with the information given in (i) The Economist Guide to Weights and Measures (London, 1962), pp. 58-9, and (ii) Monthly Digest of Statistics, January Supplements. We used this conversion factor together with the information given in (i) and (ii) above to calculate the pure alcohol content of 1 imperial gallon of beer and found the value to be .0485, which is close to the conversion factor .048 used by McGuinness.

For wine we divide the quantity data in Table A1 by $.22 \times .16 = .0352$, where .16 is the alcohol conversion factor used by McGuinness. For spirits we divide the Table A1 quantity data by $.22 \times .40 = .088$, where .40 is again the alcohol conversion factor used by McGuinness, and then multiply by $10/7$. The factor $10/7$ converts proof gallons to gallons of spirits.

The per capita consumption data presented in Table 1 are obtained from the per adult consumption data given in Table A2, using the adult and total (mid-year) populations obtained from McGuinness and various issues of the Annual Abstract of Statistics. We also use these population figures to convert the per adult expenditure on each beverage given by McGuinness (1980, App.2) to a per capita basis. These per capita expenditures are presented in Table 3.

TABLE A2
PER ADULT ALCOHOL CONSUMPTION AND PRICES:
UNITED KINGDOM, 1955-1975

Year	Beer		Wine		Spirits	
	Quantity	Price	Quantity	Price	Quantity	Price
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1955	103.88	.1301	2.0739	1.0415	1.9286	2.8519
1956	105.12	.1322	2.1761	1.0156	2.0519	2.8753
1957	105.72	.1377	2.3011	1.0343	2.0763	2.8898
1958	102.55	.1395	2.3693	1.0129	2.1104	2.9426
1959	106.66	.1302	2.5881	.9892	2.2662	2.9211
1960	110.96	.1270	2.9915	.9226	2.4058	2.9511
1961	116.21	.1328	3.1818	.9649	2.5211	3.2089
1962	115.25	.1455	3.2472	.9393	2.5114	3.3926
1963	114.62	.1515	3.5795	1.0839	2.6640	3.2283
1964	119.26	.1598	4.0710	1.1275	2.8636	3.4257
1965	119.51	.1767	3.8665	1.2207	2.6558	3.7803
1966	120.75	.1879	4.0994	1.2880	2.6899	4.0187
1967	122.48	.1982	4.5000	1.3067	2.7013	4.1425
1968	123.73	.2048	5.0142	1.3462	2.8166	4.3741
1969	128.13	.2220	4.8324	1.5127	2.6510	4.6436
1970	131.17	.2441	4.9233	1.5843	3.0130	4.7959
1971	137.24	.2643	5.8210	1.6458	3.2192	4.9671
1972	140.14	.2805	6.6562	1.6736	3.6769	4.9960
1973	146.47	.2900	8.3722	1.6985	4.6006	5.1319
1974	149.75	.3237	8.7841	1.9182	5.0552	5.2520
1975	154.20	.4051	8.2159	2.3954	4.8003	6.6933
Mean	122.56	.1992	4.4602	1.3201	2.9181	3.9754

Quantities are in litres per adult and the prices are in current pounds per litre.

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