DO OECD CONSUMERS OBEY

DEMAND THEORY?

by

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Abstract

Theil (1987) recently developed distribution-free Monte Carlo testing procedures for demand homogeneity and symmetry which do not use the asymptotic theory. In this paper we apply this methodology to test homogeneity and symmetry for 18 OECD countries and introduce a similar procedure to test preference independence. We find that homogeneity and symmetry are reasonably acceptable hypotheses for most of the countries. We also present some tentative support for the preference independence hypothesis.

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1. INTRODUCTION

Demand theory predicts that consumers do not suffer from money illusion (known as demand homogeneity) and that the substitution effects are symmetric. In a review article on systems of consumer demand functions, Barten (1977) summarizes the results from various empirical applications which test the validity of these restrictions. These results show that homogeneity is generally not acceptable, while symmetry is generally a bit more acceptable than homogeneity. Barten concludes that one reason for these negative results is that since the test procedures are usually based on the asymptotic distribution of the test statistic without correction for small sample bias, they are biased towards rejection of the null hypothesis.

Using simulation experiments, Laitinen (1978) and Meisner (1979) succeeded in showing that these asymptotic tests are indeed the main reason for the rejection of homogeneity and symmetry, particularly for large systems. Laitinen also derived the finite sample distribution for the test statistic for homogeneity. Theil (1987b) recently developed alternative testing procedures for homogeneity and symmetry which are distribution-free and hence do not require any asymptotic theory. These tests are based on Barnard's (1963) Monte Carlo simulation procedure.

The main aim of this paper is to apply Theil's methodology to test homogeneity and symmetry for 18 OECD countries. We also introduce a similar procedure to test preference independence. Under preference independence goods exhibit no interaction in the consumer's utility function. Cross-country comparisons of consumer expenditure patterns

have been previously analysed by Deaton (1972), Gamaletsos (1973), Goldberger and Gamaletsos (1970), Lluch and Powell (1975), Lluch et al. (1977) and Lluch and William (1975).

The main results of this paper are as follows. We find that homogeneity and symmetry are reasonably acceptable hypotheses for all 18 countries. We also find that the evidence gives some tentative support to preference independence for most countries.

The plan of this paper is as follows. In Section 2, drawing on Theil (1987b), we present the demand model, the test statistics and procedure to test homogeneity and symmetry, and discuss its application to Dutch data. Section 3 summarizes the database for 18 OECD countries. In Section 4 we discuss the problems associated with approximating the unknown error covariance matrix by its usual unbiased estimator; we then propose an alternative approach by specifying this matrix. We discuss the results for testing homogeneity and symmetry for the OECD countries in Sections 5 and 6. In Section 7 we propose a test statistic to test preference independence and present the results of its application to the OECD countries. In Section 8 we present the implied income and own-price elasticities. We give our concluding comments in Section 9.

2. THEIL'S RESULTS FOR THE NETHERLANDS

Drawing on Taylor et al. (1986) and Theil et al. (1985), Theil (1987b) develops distribution-free test statistics for homogeneity and symmetry and applies them to Dutch data. In this section, we briefly discuss Theil's methodology and its application.

The Demand Model

Let p_{it} be the price and q_{it} the per capita quantity consumed of good i during period t . Let there be n goods. Total expenditure is then $M_t = \sum_{i=1}^n p_{it}q_{it}$ and the proportion of total expenditure devoted to commodity i , called the budget share of i , is $w_{it} = p_{it}q_{it}/M_t$ for $t=1, \dots, T$, where T is the sample size.

Consider the absolute price version of the Rotterdam model,

$$(2.1) \quad \bar{w}_{it} Dq_{it} = \theta_i DQ_t + \sum_{j=1}^n \pi_{ij} Dp_{jt} + \epsilon_{it}, \quad \begin{array}{l} i=1, \dots, n, \\ t=1, \dots, T, \end{array}$$

where, in period t , $\bar{w}_{it} = \frac{1}{2}(w_{it} + w_{i,t-1})$ is the arithmetic average of the budget share of commodity i in periods $t-1$ and t ; $Dp_{it} = \log p_{it} - \log p_{i,t-1}$ and $Dq_{it} = \log q_{it} - \log q_{i,t-1}$ are the price and quantity log-changes of commodity i ; $DQ_t = \sum_{i=1}^n \bar{w}_{it} Dq_{it}$ is the Divisia volume index; ϵ_{it} is the disturbance term of the i^{th} equation; θ_i is the i^{th} marginal share; and π_{ij} is the $(i,j)^{\text{th}}$ Slutsky coefficient.

Let $y_{it} = \bar{w}_{it} Dq_{it}$, $\gamma_i = [\theta_i \pi_{i1} \dots \pi_{in}]'$ and $x_t = [DQ_t Dp_{1t} \dots Dp_{nt}]'$. Therefore (2.1) can be written as

$$(2.2) \quad y_i = X\gamma_i + \epsilon_i, \quad i=1, \dots, n,$$

where $y_i = [y_{it}]$ is a T -vector; X is a $T \times (n+1)$ matrix whose t^{th} row is x_t' ; and $\epsilon_i = [\epsilon_{it}]$ is a T -vector. It can be easily shown that the best linear unbiased estimators of the γ_i 's in the system of equations (2.2) are the least squares (LS) estimators obtained by applying LS to each equation separately.

If we sum both sides of (2.1) over $i=1, \dots, n$, we get $\sum_{i=1}^n \epsilon_{it} = 0$ for $t=1, \dots, T$, where we have used the properties that the marginal shares have unit sum and that $\sum_{i=1}^n \pi_{ij} = 0$. Therefore, the ϵ_{it} 's are linearly dependent. Thus one of the equations in (2.2) is redundant and can be deleted. We delete the n^{th} equation, and write (2.2) for $i=1, \dots, n-1$ as

$$(2.3) \quad y = (I \otimes X)\gamma + \epsilon,$$

where I is the identity matrix of order $(n-1)$; and $y = [y_i]$, $\gamma = [\gamma_i]$ and $\epsilon = [\epsilon_i]$ are vectors consisting of $(n-1)$ subvectors.

Demand homogeneity and Slutsky symmetry take the form

$$(2.4) \quad \sum_{j=1}^n \pi_{ij} = 0, \quad i=1, \dots, n,$$

and

$$(2.5) \quad \pi_{ij} = \pi_{ji}, \quad i, j=1, \dots, n.$$

The Monte Carlo Simulation Procedure

Switching to the standard notation, consider the system of linear equations $y = X\beta + \epsilon$. Suppose we are interested in testing the null hypothesis $R\beta = b$ using a test statistic τ . Obviously τ is a function of the parameter vector β . In general, the Monte Carlo test procedure applied in Theil (1987b) can be summarized as follows.

- Step 1: Estimate the unrestricted model and obtain the data-based value τ_1 of τ .
- Step 2: Estimate the model under the null hypothesis $R\beta = b$ and obtain the unbiased estimate S of the covariance matrix $\Sigma (= E[\epsilon\epsilon'])$ of the disturbances.
- Step 3: Generate quasi-normal error terms with zero mean and covariance matrix S and use these errors and the restricted estimates to generate data sets under the null hypothesis. Using the generated data estimate the unrestricted model.
- Step 4: Repeat Step 3 a certain number of times, N say, and in each case calculate the simulated value of the test statistic τ ; the number N is usually chosen to be sufficiently large to reduce the 'blurring effect', which leads to loss of power (see Theil, 1987b).
- Step 5: Let $\tau_2, \dots, \tau_M$ be the values of the test statistic τ obtained from the simulated data sets ($M=N+1$). We reject the null hypothesis for the observed sample at the α percent significance level if τ_1 is among the M' largest values of the τ_i 's such that $(M'/M) \times 100 = \alpha$.

For example, if we use the $\alpha = 5$ percent significance level and $N = 99$ simulations, for a two-tailed test, we reject the null hypothesis when the rank of the data-based value τ_1 is 1, 2, 3, 98, 99 or 100; and for a one-tailed test, we reject the null hypothesis if the rank of τ_1 is 96, 97, 98, 99 or 100.

Application to Homogeneity

Let $\hat{\pi}_{ij}$ be the LS estimate of π_{ij} . Then the test statistic for the homogeneity of the i^{th} equation is

$$(2.6) \quad \hat{\tau}_i = \sum_{j=1}^n \hat{\pi}_{ij}$$

and the test statistic for the homogeneity of all n equations jointly is

$$(2.7) \quad \hat{\tau}_h = \sum_{i=1}^n |\hat{\tau}_i|.$$

Column 2 of Table 2.1 reproduces the values of these data-based test-statistics and their standard errors from Theil (1987b) for Dutch data consisting of $n = 14$ commodities.

TABLE 2.1

MONTE CARLO TEST OF HOMOGENEITY: DUTCH DATA

(Standard errors are in parentheses)

Commodity (1)	$\hat{\tau}_i, \hat{\tau}_h$ (2)	Rank of the data-based test statistic (3)
1. Bread	-.006 (.005)	13
2. Groceries	-.002 (.010)	44
3. Dairy products	.017 (.011)	90
4. Vegetables, fruit	-.004 (.012)	38
5. Meat	.024 (.021)	90
6. Fish	.002 (.006)	64
7. Beverages	.019 (.008)	97
8. Tobacco	.007 (.007)	90
9. Pastry, ice cream	-.013 (.005)	1
10. Textiles	-.100 (.024)	1
11. Footwear	.005 (.004)	93
12. Other durables	.045 (.012)	100
13. Water, light, heat	.010 (.013)	82
14. Other non-durables	-.002 (.029)	44
All 14 goods	.256 (-)	98

To assess the significance of the values of these test statistics, the Monte Carlo simulation procedure described above is used. To simulate data sets under the null hypothesis, we estimate the homogeneity-constrained version of (2.1),

$$(2.8) \quad \bar{w}_{it} Dq_{it} = \theta_i DQ_t + \sum_{j=1}^{n-1} \pi_{ij} (Dp_{jt} - Dp_{nt}) + \epsilon_{it}, \quad \begin{matrix} i=1, \dots, n, \\ t=1, \dots, T, \end{matrix}$$

and obtain the data-based unbiased estimate S_h of the contemporaneous covariance matrix of the ϵ_{it} 's. Then we generate quasi-normal error terms with zero mean and covariance matrix equal to S_h . This procedure is repeated 99 times and in each case we estimate the unconstrained model (2.1) and compute the simulated values of the test statistics (2.6) and (2.7). Then we obtain the rank of the data-based value of the test statistic among the simulated values.

Column 3 of Table 2.1 reproduces Theil's (1987b) results. As can be seen from the last row of the table, the value of the data-based test statistic for the homogeneity of the 14 equations jointly is $\hat{\tau}_h = .256$. Its rank is 98 and thus homogeneity is rejected at the 5 percent significance level but not at the 1 percent significance level (using a one-tail test as $\hat{\tau}_h > 0$). A two-tailed test for the individual equations at the 5 percent significance level shows that equations 9, 10 and 12 are the troublemakers which leads to the overall rejection of homogeneity.

Application to Symmetry

Under the homogeneity restriction (2.4), the demand model (2.1) can be written as (2.8). Let $\hat{\pi}_{ij}^h$ be the homogeneity-constrained LS estimate of π_{ij} . The test statistic for symmetry given homogeneity is

$$(2.9) \quad \hat{\tau}_s = \sum_{i=1}^n \sum_{j=1}^n |\hat{\pi}_{ij}^h - \hat{\pi}_{ji}^h|.$$

From Theil (1987b), for Dutch data, the data-based value of $\hat{\tau}_s = 1.392$. To assess the significance of this data-based value, we use the Monte Carlo simulation procedure. To simulate data sets under the null hypothesis, we estimate the symmetry-constrained version of (2.8). Let $\gamma_i^h = [\theta_i \ \pi_{i1} \ \dots \ \pi_{i,n-1}]'$ and $x_t^h = [DQ_t \ Dp_{1t} \ Dp_{nt} \ \dots \ Dp_{n-1,t} \ Dp_{nt}]'$. Then (2.8) can be written as

$$(2.10) \quad y_i = X^h \gamma_i^h + \epsilon_i, \quad i=1, \dots, n,$$

where X^h is a $T \times n$ matrix whose t^{th} row is x_t^h ; and y_i and ϵ_i are as before. As for the unconstrained case, it can be easily shown that the best linear unbiased estimators of the γ_i^h 's in (2.10) are the least squares estimators obtained by applying LS to each equation separately.

As before, we delete the n^{th} equation and write (2.10) in matrix form as

$$(2.11) \quad y = (I \otimes X^h) \gamma^h + \epsilon,$$

where I is the identity matrix of order $(n-1)$; $\gamma^h = [\gamma_i^h]$ is a vector

consisting of $(n-1)$ subvectors; and y and ϵ are as before. Let $\hat{\tau}_i^h$ be the LS estimator of τ_i^h for $i=1, \dots, n$. Therefore, $\hat{\tau}_i^h = (X^{h'}X^h)^{-1}X^{h'}y_i$. Let $\hat{\tau}^h = [\hat{\tau}_i^h]$ be the vector consisting of $n-1$ subvectors. That is, $\hat{\tau}^h$ is the LS estimator of τ^h .

In matrix form, the symmetry constraint (2.5) can be written as

$$(2.12) \quad R\tau^h = 0,$$

where R is a $q \times n(n-1)$ matrix with $q = \frac{1}{2}(n-1)(n-2)$ and each row of R consists of zeros except for a 1 and a -1 corresponding to π_{ij} and π_{ji} for some $i \neq j$. The best linear unbiased estimator of τ^h of the symmetry constrained model is given by

$$(2.13) \quad \hat{\tau}^s = \hat{\tau}^h - C(\Sigma)R'[RC(\Sigma)R']^{-1}RC(\Sigma)\hat{\tau}^h,$$

where $C(\Sigma) = \Sigma \otimes (X'X)^{-1}$. Since Σ is typically unknown, we replace Σ by the homogeneity constrained data-based unbiased estimate S_h of the contemporaneous covariance matrix of the ϵ_{it} 's in (2.8).

Using the symmetry-constrained estimator (2.13), we obtain the data-based unbiased estimate S_s of Σ . Then we generate quasi-normal error terms with zero mean and covariance matrix equal to S_s . This procedure is repeated 99 times and in each case we estimate the homogeneity-constrained model (2.8) and compute the simulated value of the test statistic (2.9). Then we obtain the rank of the data-based value of the test statistic $\hat{\tau}_s$ among the simulated values. For Dutch data, Theil (1987b) found the rank of the data-based $\hat{\tau}_s$ to be 16 in 99 simulations. Thus Slutsky symmetry may not be rejected at the 5 percent significance level for these data.

3. THE OECD DATABASE

The basic data consisting of annual consumption expenditures (in current and constant prices) and mid-year populations for 18 OECD countries are from Stening (1985). There are 25 OECD countries of which we omit 7 due to the unavailability of data. For most countries, goods and services are classified into 10 commodity groups which we give in Table 3.1.

Table 3.2 summarizes the general characteristics of the database. Column 2 presents the sample period for each country and column 3 gives the sample size (less 1 lag). In column 4 we present per capita gross domestic product (GDP) expressed in 1975 international dollars (from Data Table, Summers and Heston, 1984). Column 5 gives per capita GDP in index form with U.S.= 100. The 18 countries are listed in the order of declining per capita GDP. As can be seen, the U.S. has the highest GDP

TABLE 3.1

DETAILS OF THE COMMODITY GROUPS

Commodity	Details
1. Food	Food
2. Beverages	Beverages and tobacco
3. Clothing	Clothing and footwear
4. Housing	Gross rent, fuel and power
5. Durables	Furniture, furnishings and household equipment
6. Medical care	Medical care
7. Transport	Transport and communication
8. Recreation	Recreation, entertainment and cultural services
9. Education	Education and Research
10. Miscellaneous	Miscellaneous goods and services

TABLE 3.2
CHARACTERISTICS OF THE DATABASE

Country	Sample period	Sample size	Per capita GDP in 1975		Number of commodity groups	Comments
			International dollars	(4) with U.S.=100		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1. United States	1960-1981	21	7132	100	10	
2. Canada	1960-1981	21	6788	95	10	
3. Sweden	1964-1981	17	6749	95	10	
4. Switzerland	1960-1981	21	6082	85	9	Education is included in recreation
5. Denmark	1966-1981	15	5969	84	10	
6. Australia	1960-1981	21	5919	83	10	
7. France	1964-1981	17	5864	82	10	
8. Germany	1960-1981	21	5758	81	8	Beverages are included in food, and education is included in recreation
9. Belgium	1960-1981	21	5554	78	10	
10. Norway	1964-1981	17	5419	76	10	
11. Netherlands	1952-1977	25	5321	75	10	
12. Iceland	1960-1973	13	5201	73	10	
13. Finland	1960-1977	17	5192	73	10	
14. Austria	1964-1981	17	4994	70	10	
15. Japan	1970-1981	11	4905	69	8	Beverages are included in food, and education is included in recreation
16. United Kingdom	1964-1981	17	4601	65	10	
17. Spain	1964-1977	13	4032	57	10	
18. Italy	1964-1981	17	3870	54	10	

Note: Sample size is after lagging.

per capita while Italy has the lowest, it being 54 percent of the U.S. value. Column 6 gives the number of commodity groups considered in each country. All except three of the countries have 10 commodity groups. Germany and Japan have 8, as beverages are included in food and education is included in recreation. Switzerland has 9 groups as education is included in recreation.

Columns 2-11 of Table 3.3 present the mean log-changes in per capita consumption of each commodity,

$$\bar{Dq}_i = \frac{1}{T} \sum_{t=1}^T Dq_{it}, \quad i=1, \dots, n.$$

for each of the 18 countries. Column 12 of that table presents the mean of the Divisia volume index for each country,

$$DQ = \frac{1}{T} \sum_{t=1}^T DQ_t = \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^n \bar{w}_{it} Dq_{it},$$

where $\bar{w}_{it}$ is the arithmetic average of the budget shares in periods t and $t-1$. The last row of the table presents the mean quantity log-changes and Divisia volume indexes averaged over all countries. As can be seen, the average growth in OECD per capita consumption of food is approximately 1.4 percent per annum, 2.5 percent for beverages, 2.4 percent for clothing and so on. Per capita consumption as a whole, as measured by the Divisia volume index, on average, increased by 3.0 percent per annum.

TABLE 3.3

PER CAPITA QUANTITIES CONSUMED OF 10 COMMODITIES

AND DIVISIA VOLUME INDEX FOR 18 COUNTRIES

(Mean Log-Changes $\times$ 100)

Country	Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous	Divisia volume index
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. U.S.	.60	1.41	2.33	2.97	1.64	4.49	2.80	4.10	3.33	1.77	2.43
2. Canada	.91	2.57	3.12	3.17	3.02	.22	4.17	6.27	4.20	2.79	2.87
3. Sweden	.50	.80	2.66	2.35	1.18	3.02	2.06	4.53	3.17	-.95	1.71
4. Switzerland	1.72	1.57	.75	2.08	.62	2.85	3.63	2.98		2.79	2.13
5. Denmark	.17	1.43	.59	3.23	-1.01	1.50	1.49	3.34	8.53	.75	1.42
6. Australia	1.35	1.03	.71	3.54	4.45	1.23	3.49	4.33	-2.40	1.95	2.37
7. France	1.62	1.37	1.71	4.89	3.31	6.68	4.65	5.37	2.43	3.20	3.57
8. Germany	2.02		2.36	3.24	3.51	1.45	5.69	3.90		3.77	3.15
9. Belgium	1.53	2.58	2.76	2.80	4.20	5.13	4.52	4.04	1.98	2.98	3.11
10. Norway	1.35	2.08	1.40	3.43	3.21	2.14	4.06	5.56	1.26	2.17	2.68
11. Netherlands	2.22	4.39	3.19	3.93	6.91	5.53	7.52	4.07	5.15	5.20	4.34
12. Iceland	1.64	5.11	5.65	2.54	8.12	7.20	8.58	5.92	4.54	8.56	5.07
13. Finland	1.77	4.89	1.62	3.60	5.41	6.43	6.48	6.18	-1.60	5.58	3.86
14. Austria	1.90	2.34	4.26	4.29	3.61	2.62	5.58	4.35	1.94	1.98	3.27
15. Japan	1.50		1.47	4.93	1.58	6.07	4.10	3.26		3.40	3.13
16. U.K.	.15	1.48	2.52	1.87	1.58	.90	2.81	3.65	2.10	1.70	1.76
17. Spain	2.82	4.01	3.00	3.79	3.44	9.24	10.66	5.31	4.24	5.72	4.40
18. Italy	2.20	3.15	2.81	2.80	3.90	6.82	5.71	3.98	1.61	3.65	3.33
Mean	1.44	2.51	2.39	3.30	3.26	4.08	4.89	4.51	2.70	3.17	3.04

Table 3.4 contains the mean price log-changes of each commodity,

$$\bar{Dp}_i = \frac{1}{T} \sum_{t=1}^T Dp_{it}, \quad i=1, \dots, n,$$

for all countries. The last column of that table presents the mean of the Divisia price index for each country,

$$DP = \frac{1}{T} \sum_{t=1}^T DP_t = \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^n \bar{w}_{it} Dp_{it}.$$

The last row of the table presents the mean price changes averaged over all countries. On average, the annual growth rates in all prices lie between 6 and 9 percent and their weighted average is 7 percent.

Table 3.5 presents the budget shares at sample means for each commodity,

$$\bar{w}_i = \frac{1}{T} \sum_{t=1}^T w_{it}, \quad i=1, \dots, n,$$

for the 18 countries. The last row of the table presents the budget share of each commodity group averaged over the 18 countries. As can be seen, the average OECD consumer spends 23 percent of his total expenditure on food. The food, housing and transport in total occupy about 50 percent of the total in most countries.

TABLE 3.4
PRICES OF 10 COMMODITIES AND DIVISIA PRICE INDEX FOR 18 COUNTRIES
(Mean Log-Changes $\times$ 100)

Country	Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous	Divisia price index
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. U.S.	5.00	4.85	3.29	4.67	4.25	5.81	4.87	3.56	5.91	5.37	4.78
2. Canada	6.15	4.71	3.87	5.18	4.49	4.96	4.72	3.89	8.43	6.73	5.27
3. Sweden	7.17	7.27	4.82	8.50	7.30	6.14	7.66	6.57	9.13	9.10	7.43
4. Switzerland	4.00	4.06	3.39	5.43	3.96	6.49	4.55	4.10		5.47	4.55
5. Denmark	8.69	6.68	6.57	11.43	8.55	8.91	8.79	7.62	9.54	9.30	8.90
6. Australia	6.05	6.58	6.12	7.43	4.33	8.42	6.31	6.97	8.38	7.53	6.58
7. France	7.30	6.47	6.74	8.80	6.76	6.99	8.13	6.13	9.02	8.28	7.48
8. Germany	3.23		3.67	5.57	3.04	5.58	4.13	3.74		4.61	3.15
9. Belgium	4.56	4.61	3.89	5.76	4.36	6.02	5.28	5.00	4.54	6.88	5.13
10. Norway	6.96	6.99	6.74	7.20	7.02	8.25	7.32	5.36	7.85	7.91	7.05
11. Netherlands	3.71	2.91	3.48	5.08	2.29	7.73	3.38	5.51	5.91	4.80	4.07
12. Iceland	13.74	11.25	9.69	11.66	10.13	14.10	11.25	12.66	12.78	11.48	11.79
13. Finland	7.75	6.87	6.09	5.90	6.25	6.22	7.16	6.32	9.44	8.17	7.02
14. Austria	4.57	2.93	3.67	7.49	3.59	8.94	6.12	4.21	4.20	6.53	5.28
15. Japan	7.61		7.99	7.58	6.71	7.36	8.70	7.35		8.88	7.85
16. U.K.	9.00	8.49	6.88	11.28	8.68	9.36	10.25	8.85	10.73	9.99	9.56
17. Spain	9.79	7.57	11.24	8.58	11.07	11.69	7.89	12.54	12.82	10.87	10.06
18. Italy	9.67	7.07	10.37	10.89	10.37	8.72	10.83	9.25	10.03	11.65	10.13
Mean	6.94	6.21	6.03	7.69	6.28	7.87	7.08	6.64	8.58	7.98	7.01

TABLE 3.5
BUDGET SHARES OF 10 COMMODITIES FOR 18 COUNTRIES
(Means $\times$ 100)

Country	Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1. U.S.	14.38	4.24	8.03	19.29	7.28	9.46	15.93	6.25	1.81	13.33
2. Canada	16.42	6.21	8.41	18.85	8.73	3.84	15.02	5.96	2.33	14.22
3. Sweden	20.72	7.96	8.41	22.79	7.66	2.27	13.65	9.18	.18	7.18
4. Switzerland	22.73	9.42	6.77	18.06	7.80	6.52	10.99	9.36		8.35
5. Denmark	19.18	9.09	6.97	20.92	8.98	1.92	15.04	7.88	.83	9.19
6. Australia	19.48	8.96	9.13	15.77	8.19	6.04	15.13	5.71	.69	10.89
7. France	21.20	4.69	8.22	14.41	10.23	10.50	11.99	6.05	.29	12.42
8. Germany	30.38		10.36	15.79	12.04	2.68	13.06	7.44		8.55
9. Belgium	23.11	6.95	7.53	15.72	14.14	6.74	10.67	3.98	.23	10.93
10. Norway	23.45	7.56	9.97	14.54	8.86	4.20	13.31	7.68	.54	9.89
11. Netherlands	26.20	6.38	14.78	10.81	12.48	6.67	6.67	3.41	3.10	9.50
12. Iceland	23.50	7.86	10.66	19.06	10.00	6.07	12.13	5.24	.44	5.05
13. Finland	28.24	8.62	8.26	14.36	7.51	2.54	13.60	5.92	1.44	9.49
14. Austria	22.06	7.29	11.37	11.99	8.89	3.49	13.36	5.67	.35	15.54
15. Japan	27.45		7.76	16.78	6.76	8.92	8.97	9.05		14.32
16. U.K.	18.92	6.26	8.40	18.20	7.78	.85	13.13	7.88	1.98	16.50
17. Spain	38.53	3.51	10.58	13.99	8.55	4.62	9.50	4.28	2.02	8.87
18. Italy	30.85	5.69	9.41	12.94	6.79	3.73	10.48	7.05	.43	12.62
Mean	23.71	6.92	9.16	16.35	9.04	5.05	12.37	6.56	1.11	10.94

4. SPECIFYING THE COVARIANCE MATRIX

In general, the covariance matrix Σ of the disturbance terms of the demand equations is unknown. It is common practice to approximate Σ by its unbiased estimator S , the matrix of mean squares and cross products of the LS residuals. Laitinen (1978) showed that for large demand systems without a sufficiently large sample, S becomes singular or ill-conditioned. In this case the usual (asymptotic) test statistics for homogeneity and symmetry, which involve the inverse of S , become infinity. Laitinen also proved that a necessary and sufficient condition for the non-singularity of S is that $T - 2n \geq 0$ for model (2.1), where T is the sample size and n is the number of equations in the system. It can be easily seen from columns 3 and 6 of Table 3.2 that for 11 countries $T - 2n$ is less than 0; that is, S will be singular. Therefore, to test homogeneity and symmetry, the usual tests involving the inverse of S cannot be used for these countries.

Also among all countries the largest sample size is 25 (for Netherlands), but even this cannot be considered a large sample for a model with 10 equations. Therefore, even when S^{-1} exists, it would be unwise to rely on the large-sample tests of homogeneity or symmetry. Instead we shall use Theil's methodology described in Section 2 for those seven countries with non-singular covariance matrix S . For the rest of the countries, since S is not positive definite, it cannot be used to generate quasi-normal error terms for our simulation experiments. Therefore, as an alternative to S we specify a simple and plausible covariance structure,

$$(4.1) \quad \Sigma^* = \lambda^2 \Omega,$$

where $\Omega = \bar{W} - \bar{w}\bar{w}'$; $\bar{W} = \text{diag}[\bar{w}_1, \dots, \bar{w}_{n-1}]$; $\bar{w} = [\bar{w}_1 \dots \bar{w}_{n-1}]'$; $\bar{w}_i = (1/T) \sum_{t=1}^T \bar{w}_{it}$; and the constant parameter λ^2 is to be estimated. It can be easily shown that Ω is non-singular and that an unbiased estimator of λ^2 for model (2.1) is

$$(4.2) \quad \hat{\lambda}^2 = \frac{1}{(T-n-1)(n-1)} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \omega^{ij} \hat{\epsilon}_i \hat{\epsilon}_j,$$

where $\hat{\epsilon}_i$ is the LS residual vector of the i^{th} equation and ω^{ij} is the $(i,j)^{\text{th}}$ element of Ω^{-1} .

Let $\Sigma^* = [\sigma_{ij}^*]$ and $\Omega = [\omega_{ij}]$. Then $\omega_{ii} = \bar{w}_i(1 - \bar{w}_i)$ and $\sigma_{ii}^* = \lambda^2 \omega_{ii}$. It can be easily seen that, for $0 \leq \bar{w}_i \leq .5$, σ_{ii}^* increases with $\bar{w}_i$. Therefore, since $\bar{w}_i$ is always less than .5 for all the OECD countries, σ_{ii}^* increases with $\bar{w}_i$. Specification (4.1) allows for larger error variances for goods which occupy larger shares of the budget which is plausible. This specification has been previously considered by Deaton (1975) and Selvanathan (1985).

5. TESTING HOMOGENEITY

In this section we use Theil's methodology described in Section 2 to test homogeneity for the OECD countries. Our demand model slightly differs from that used by Theil in the sense that, instead of specifying constant marginal shares, we use Working's (1943) model, where the marginal share and the budget share for a commodity differs by a

constant (see Clements, 1987). That is, the marginal share of commodity i is

$$(5.1) \quad \theta_{it} = \bar{w}_{it} + \beta_i, \quad i=1, \dots, n,$$

where β_i is a constant satisfying $\sum_{i=1}^n \beta_i = 0$. Substituting (5.1) in (2.1) and rearranging yields the demand model,

$$(5.2) \quad \bar{w}_{it}(Dq_{it} - DQ_t) = \beta_i DQ_t + \sum_{j=1}^n \pi_{ij} Dp_{jt} + \epsilon_{it}, \quad \begin{matrix} i=1, \dots, n, \\ t=1, \dots, T. \end{matrix}$$

For the 7 countries with non-singular S , we obtain the ranks in 99 simulations of the data-based test statistics for testing the homogeneity of each equation and the homogeneity of all n equations jointly. Next, we include a constant term in each equation to account for the trend-like changes in tastes etc. and repeat the above procedure for all countries. Table 5.1 summarizes these results, where the upper half gives the results with no constant terms while the lower half presents the results with constants. Columns 2-11 present the ranks of the data-based values of the test statistics for each equation. Columns 12 and 13 give the percentage significance of homogeneity over all equations for each country at the 5 and 1 percent levels. The rank of the data-based test statistic for the homogeneity of the n equations jointly is presented in column 14. The second last and last rows of the upper and lower parts of the table present the percent rejected at the 5 and 1 percent levels.

TABLE 5.1
RANK OF THE DATA-BASED TEST STATISTIC FOR HOMOGENEITY
IN 99 SIMULATIONS FOR 7 COUNTRIES

(Based on S)

Country (1)	Food (2)	Beverages (3)	Clothing (4)	Housing (5)	Durables (6)	Medical care (7)	Transport (8)	Recreation (9)	Education (10)	Miscellaneous (11)	Percent significant at 5% level (12)	Percent significant at 1% level (13)	All goods (14)
No constant													
1. U.S.	9	67	56	97	4	100	29	6	82	71	10	10	91
2. Canada	66	2	68	76	21	74	15	47	39	39	10	0	45
3. Switzerland	46	19	1	97	1	65	5	100		100	44	44	99
4. Australia	77	2	51	42	75	66	27	29	2	97	20	0	35
5. Germany	12		2	100	13	90	18	95		97	25	10	100
6. Belgium	15	33	68	6	99	97	6	94	65	25	10	0	93
7. Netherlands	42	37	81	91	10	64	71	29	78	58	0	0	44
Percent significant at 5% level	0	33	28	14	28	14	0	14	20	14			28
Percent significant at 1% level	0	0	14	14	14	14	0	14	0	14			14
With constant													
1. U.S.	64	97	57	23	2	61	84	8	93	31	10	0	70
2. Canada	73	2	67	78	23	87	10	51	36	33	10	0	59
3. Switzerland	11	45	12	74	16	24	24	100		97	10	10	78
4. Australia	27	11	90	6	96	57	42	14	3	95	10	0	90
5. Germany	17		23	75	55	82	43	87		65	0	0	25
6. Belgium	12	28	75	5	98	93	6	90	67	34	10	0	92
7. Netherlands	42	37	92	95	8	46	48	32	69	37	0	0	52
Percent significant at 5% level	0	17	0	0	28	0	0	14	0	0			0
Percent significant at 1% level	0	0	0	0	0	0	0	14	0	0			0

Looking at the second last and last rows of the upper half of the table, we see that beverages, clothing and durables are the troublesome equations at the 5 percent level, while beverages are not significant at the 1 percent level. This finding is in agreement with Theil's results for the Netherlands reproduced in Table 2.1, where textiles and durables are significant and the rank of the beverages is on the high side. As can be seen from the lower half of the table, homogeneity of the n equations jointly is acceptable at the 5 percent significance level for all 7 countries.

To test homogeneity for the 11 countries with singular S , we replace S by $\Sigma^* = \lambda^2 \Omega$ and estimate λ^2 . For comparison, we also use Σ^* for the other 7 countries. We then follow the same procedure as before to test homogeneity for all 18 countries with and without constant terms. Table 5.2 presents the results. As can be seen from the lower part of column 14 of the table, homogeneity of the entire model (with constant terms) is acceptable for all countries except Belgium at the 5 percent level and is acceptable for all countries at the 1 percent level.

Tables 5.1 and 5.2 have 7 countries in common. A comparison of the results for all goods presented in column 14 of the tables shows that for these countries things do not change much. For example, homogeneity is rejected at the 5 percent level for Germany and Switzerland when there are no constant terms, while it is accepted for both countries when constant terms are included irrespective of the choice of Σ . Consequently, there is no evidence that the use of Σ^* distorts the results.

TABLE 5.2
RANK OF THE DATA-BASED TEST STATISTIC FOR HOMOGENEITY
IN 99 SIMULATIONS FOR 18 COUNTRIES

(Based on Σ^*)

Country (1)	Food (2)	Beverages (3)	Clothing (4)	Housing (5)	Durables (6)	Medical care (7)	Transport (8)	Recreation (9)	Education (10)	Miscellaneous (11)	Percent significant at 5% level (12)	Percent significant at 1% level (13)	All goods (14)
No constant													
1. U.S.	5	65	56	95	4	100	28	12	59	61	10	10	90
2. Canada	62	19	68	73	37	100	9	50	47	42	10	10	23
3. Sweden	88	57	43	81	31	69	3	90	53	6	10	0	87
4. Switzerland	45	27	1	100	2	59	8	100		97	44	30	100
5. Denmark	78	49	23	99	5	70	7	31	90	77	10	0	92
6. Australia	68	15	56	45	94	69	30	26	6	69	0	0	20
7. France	99	29	5	92	9	95	1	70	67	42	20	10	100
8. Germany	20		2	100	16	78	5	78		83	25	10	100
9. Belgium	10	44	62	3	100	96	23	80	53	13	20	10	100
10. Norway	77	39	23	95	68	79	2	80	41	35	10	0	83
11. Netherlands	45	38	80	82	1	64	68	24	64	66	10	10	25
12. Iceland	41	54	24	90	28	100	2	85	70	52	20	10	92
13. Finland	41	72	43	80	42	39	46	50	25	38	0	0	2
14. Austria	39	2	81	74	40	66	69	34	50	58	10	0	18
15. Japan	8		1	97	14	91	87	9		97	13	10	99
16. U.K.	16	59	39	67	20	32	44	71	50	88	0	0	25
17. Spain	60	55	68	35	54	61	35	69	37	26	0	0	1
18. Italy	12	60	1	77	20	95	99	84	55	77	20	10	99
Percent significant at 5% level	6	6	22	22	17	17	28	6	0	0			33
Percent significant at 1% level	0	0	17	11	11	17	6	6	0	0			22
With constant													
1. U.S.	76	89	53	40	1	77	98	3	70	25	30	10	80
2. Canada	65	18	68	68	37	100	8	49	47	39	10	10	29
3. Sweden	91	74	95	24	35	49	2	58	48	37	10	0	75
4. Switzerland	16	45	17	92	29	40	16	96		94	0	0	82
5. Denmark	77	55	14	29	51	23	84	78	13	31	0	0	25
6. Australia	37	35	81	6	100	63	34	9	7	72	10	10	87
7. France	83	60	31	67	95	76	2	58	28	34	10	0	61
8. Germany	23		26	83	52	77	29	72		55	0	0	14
9. Belgium	10	37	70	3	100	93	25	79	55	29	20	10	99
10. Norway	83	1	77	76	61	12	70	63	51	27	10	10	64
11. Netherlands	39	38	92	91	2	55	55	26	59	45	10	0	37
12. Iceland	68	17	48	69	83	57	12	98	44	5	10	0	52
13. Finland	61	48	41	65	66	34	62	70	10	23	0	0	3
14. Austria	40	2	81	72	40	58	78	33	47	58	10	0	18
15. Japan	85		9	50	2	84	81	16		78	13	0	89
16. U.K.	19	81	74	25	85	41	20	64	50	83	0	0	43
17. Spain	63	56	73	35	51	58	37	76	32	25	0	0	2
18. Italy	20	52	41	32	73	95	87	55	32	55	0	0	29
Percent significant at 5% level	0	13	0	6	28	6	17	11	0	0			6
Percent significant at 1% level	0	6	0	0	17	6	0	0	0	0			0

Now we compare the upper half of Table 5.2 with its lower half. As can be seen, all the commodities except beverages, durables and recreation perform better when a constant term is added to the equations at the 5 percent level. Also, looking at the last column of the table we see that the homogeneity of the n equations jointly becomes more acceptable when constant terms are included in the model. These findings point to the importance of including constant terms when testing homogeneity.

6. TESTING SYMMETRY

In this section we use Theil's methodology described in Section 2 to test Slutsky symmetry given homogeneity for the OECD countries. As before, we use model (5.2). Given homogeneity, (5.2) takes the form

$$(6.1) \quad \bar{w}_{it}(Dq_{it} - DQ_t) = \beta_1 DQ_t + \sum_{j=1}^{n-1} \pi_{ij}(Dp_{jt} - Dp_{nt}) + e_{it}, \quad \begin{matrix} i=1, \dots, n, \\ t=1, \dots, T. \end{matrix}$$

For the 7 countries with non-singular S , we obtain the ranks of the data-based test statistics in 99 simulations. We repeat this procedure with a constant term added to each equation. Columns 2 and 3 of Table 6.1 give the results. The last two rows give the percentage significance at the 5 and 1 percent levels. As can be seen, symmetry is acceptable at the 5 percent level for all 7 countries whether or not there are constant terms in the equations.

Next, we replace S by Σ^* and follow the same procedure as before. Columns 4 and 5 of Table 6.1 present the results. At the 5 percent

TABLE 6.1
RANK OF THE DATA-BASED TEST STATISTIC FOR SYMMETRY
IN 99 SIMULATIONS FOR 18 COUNTRIES

Country (1)	Based on S		Based on Σ^*	
	No constant (2)	With constant (3)	No constant (4)	With constant (5)
1. U.S.	29	16	90	71
2. Canada	61	21	72	32
3. Sweden	..	..	51	14
4. Switzerland	54	59	95	96
5. Denmark	..	..	31	41
6. Australia	3	1	18	2
7. France	..	..	42	80
8. Germany	47	72	74	90
9. Belgium	17	5	56	30
10. Norway	..	..	85	11
11. Netherlands	46	20	91	56
12. Iceland	..	..	19	38
13. Finland	..	..	49	42
14. Austria	..	..	6	21
15. Japan	..	..	11	56
16. U.K.	..	..	96	30
17. Spain	..	..	28	87
18. Italy	..	..	47	49
Percent significant at 5% level	0	0	6	6
Percent significant at 1% level	0	0	0	0

significance level, symmetry is acceptable for all countries except the U.K. when there are no constant terms in the model; and is acceptable for all countries except Switzerland when constant terms are included. Symmetry is acceptable for all countries at the 1 percent significance level.

We now compare the results with the alternative covariance matrices for the 7 countries that have non-singular covariance matrix. In Table 6.1, we compare column 2 with 4 and column 3 with 5. This shows that the ranks tend to increase when we use Σ^* . On the whole, adding constant terms to the model does not make any major difference to the symmetry results.

7. TESTING PREFERENCE INDEPENDENCE

In this section we propose a test of the preference independence hypothesis, that is, whether or not the commodity groups interact with each other in the consumer's utility function.

Under preference independence the Slutsky coefficients take the form (see Clements, 1987)

$$(7.1) \quad \pi_{ij} = \phi \theta_i (\delta_{ij} - \theta_j), \quad i, j = 1, \dots, n,$$

where ϕ is the income flexibility (the reciprocal of the income elasticity of the marginal utility of income); θ_i is the marginal share of commodity i ; and δ_{ij} is the Kronecker delta. Since the marginal shares sum to one, (7.1) satisfies demand homogeneity and Slutsky

symmetry. Under Working's model, (7.1) becomes

$$(7.2) \quad \pi_{ij} = \phi(\bar{w}_{it} + \beta_i)(\delta_{ij} - \bar{w}_{jt} - \beta_j).$$

Therefore the preference independence version of (5.2) is

$$(7.3) \quad \bar{w}_{it}(Dq_{it} - DQ_t) = \beta_i DQ_t + \phi(\bar{w}_{it} + \beta_i) \left[Dp_{it} - \sum_{j=1}^n (\bar{w}_{jt} + \beta_j) Dp_{jt} \right] + \epsilon_{it}.$$

Since this model is nonlinear in the parameters, we use a maximum likelihood (ML) procedure for estimation. See Appendix A1 for details.

Let $\hat{\pi}_{ij}^s$ be the homogeneity- and symmetry-constrained estimate of the $(i,j)^{th}$ Slutsky coefficient. Let $\hat{\pi}_{ij}^p$ be the estimate under preference independence defined as

$$(7.4) \quad \hat{\pi}_{ij}^p = \hat{\phi}(\bar{\bar{w}}_i + \hat{\beta}_i)(\delta_{ij} - \bar{\bar{w}}_j - \hat{\beta}_j),$$

where $\hat{\phi}$ and the $\hat{\beta}_i$'s are the ML estimates of the model under preference independence; and where we have replaced $\bar{w}_{it}$ by its sample mean $\bar{\bar{w}}_i = (1/T) \sum_{t=1}^T \bar{w}_{it}$. We define the test statistic for preference independence as

$$(7.5) \quad \hat{\tau}_p = \sum_{i=1}^n \sum_{j < i} |\hat{\pi}_{ij}^s - \hat{\pi}_{ij}^p|.$$

To assess the significance of the data-based value of the test statistic we use the Monte Carlo simulation procedure described in Section 2. To generate data sets under the null hypothesis, we first calculate the consistent estimator S_p of the contemporaneous covariance

matrix of the ϵ_{it} 's in (7.3). Then we generate quasi-normal error terms with zero mean and covariance matrix S_p . This procedure is repeated a certain number of times and in each trial we generate data sets and estimate the homogeneity- and symmetry-constrained model to calculate the simulated values of the test statistic (7.5).

Columns 2 and 3 of Table 7.1 give the ranks in 99 simulations of the data-based values of the test statistics for the 7 countries with nonsingular covariance matrix, with and without constants. As can be seen, preference independence is rejected at the 5 percent level of significance for 4 out of 7 countries. Columns 4 and 5 of the table present the corresponding ranks of the data-based test statistics when we specify the covariance structure for all 18 countries. As can be seen, for all countries, preference independence may not be rejected at the 5 percent level when constant terms are included.

In contrast to homogeneity and symmetry, when testing preference independence, there is notable difference between the results based on S and Σ^* ; for Table 7.1 compare column 2 with 4 and column 3 with 5. It should be noted that the data-based and simulated values of the test statistics for both homogeneity and symmetry do not depend on the covariance matrix. The covariance matrix is used only to generate the error terms under the null hypotheses; and to obtain the parameter estimates under the null hypothesis for testing symmetry. However, this is not the case with testing preference independence. As described in Appendix A1, the ML estimates of the parameters are obtained using Newton's iterative procedure based on successive estimates of the covariance matrix of the parameters of the model and the disturbance covariance matrix, S or Σ^* . Therefore, the data-based and the simulated

TABLE 7.1
RANK OF THE DATA-BASED TEST STATISTIC FOR PREFERENCE INDEPENDENCE
IN 99 SIMULATIONS FOR 18 COUNTRIES

Country (1)	Based on S		Based on Σ^*	
	No constant (2)	With constant (3)	No constant (4)	With constant (5)
1. U.S.	100	100	58	70
2. Canada	98	96	23	15
3. Sweden	..	..	22	89
4. Switzerland	91	86	95	82
5. Denmark	..	..	47	58
6. Australia	86	70	93	92
7. France	..	..	91	86
8. Germany	95	50	84	30
9. Belgium	99	100	43	49
10. Norway	..	..	97	93
11. Netherlands	98	99	39	25
12. Iceland	..	..	7	24
13. Finland	..	..	91	59
14. Austria	..	..	81	79
15. Japan	..	..	50	79
16. U.K.	..	..	43	5
17. Spain	..	..	56	36
18. Italy	..	..	84	77
Percent significant at 5% level	57	57	6	0
Percent significant at 1% level	14	29	0	0

values of the test statistics are not the same for different specifications of the covariance matrix. In Appendix A2, we present further results on testing preference independence to identify the extent to which the specifications of the covariance matrix affects the test results at different stages of the procedure. Because of the effects of the specification of the covariance matrix on the test results, it would seem best to conclude that the results of this section give some tentative support to the hypothesis of preference independence.

8. THE IMPLIED DEMAND ELASTICITIES

In this section we present the income and own-price elasticities implied by the demand model under preference independence. We also present the income flexibility for each country. Let $\hat{\phi}$ and $\hat{\beta}_i$, $i=1,\dots,n$, be the ML estimates of the parameters of model (7.3). The i^{th} income elasticity implied by (7.3) is

$$(8.1) \quad \eta_{it} = 1 + \frac{\hat{\beta}_i}{\bar{w}_{it}}.$$

Since the budget share of a commodity is fairly stable over time, we replace $\bar{w}_{it}$ by its sample mean $\bar{\bar{w}}_i = (1/T) \sum_{t=1}^n \bar{w}_{it}$ in (8.1) and write

$$(8.2) \quad \eta_i = 1 + \frac{\hat{\beta}_i}{\bar{\bar{w}}_i}.$$

The standard error of η_i is then given by

$$\text{s.e.}[\eta_i] = \frac{1}{\bar{\bar{w}}_i} \text{s.e.}[\hat{\beta}_i].$$

Similarly, the i^{th} Slutsky (or compensated) own-price elasticity implied by (7.3) at means is

$$\eta_{ii} = \frac{\hat{\phi}(\hat{\beta}_i + \bar{\bar{w}}_i)(1 - \hat{\beta}_i - \bar{\bar{w}}_i)}{\bar{\bar{w}}_i}.$$

We show in Appendix A3 that the asymptotic standard error of η_{ii} is

$$\begin{aligned} \text{a.s.e.}[\eta_{ii}] &= \left[\left(\frac{\partial \eta_{ii}}{\partial \hat{\phi}} \right)^2 \text{var } \hat{\phi} + \left(\frac{\partial \eta_{ii}}{\partial \hat{\beta}_i} \right)^2 \text{var } \hat{\beta}_i + 2 \left(\frac{\partial \eta_{ii}}{\partial \hat{\phi}} \right) \left(\frac{\partial \eta_{ii}}{\partial \hat{\beta}_i} \right) \text{cov}[\hat{\phi}, \hat{\beta}_i] \right]^{1/2} \\ &= \left[\left(\frac{\eta_{ii}}{\hat{\phi}} \right)^2 \text{var } \hat{\phi} + \left(\frac{\hat{\phi}(1 - 2\hat{\beta}_i - 2\bar{\bar{w}}_i)}{\bar{\bar{w}}_i} \right)^2 \text{var } \hat{\beta}_i \right. \\ &\quad \left. + 2 \left(\frac{\eta_{ii}}{\hat{\phi}} \right) \left(\frac{\hat{\phi}(1 - 2\hat{\beta}_i - 2\bar{\bar{w}}_i)}{\bar{\bar{w}}_i} \right) \text{cov}[\hat{\phi}, \hat{\beta}_i] \right]^{1/2}. \end{aligned}$$

To calculate the elasticities, we use the ML-estimates of the parameters of model (7.3) with constant terms included and specified covariance matrix Σ^* . Columns 2-11 of Table 8.1 present the income elasticities and their asymptotic standard errors for each commodity group in each country. From the table, we see that most of the income elasticities are significantly different from zero and all but five of them are positive. We shall come back to these five at the end of this section. In all countries food is a necessity. The income elasticity for food in Canada and Switzerland seems on the high side. In all

TABLE 8.1

INCOME ELASTICITIES OF 10 COMMODITIES AND INCOME FLEXIBILITY FOR 18 COUNTRIES

(Asymptotic standard errors are in parentheses)

Country	Food	Beverages	Clothing	Housing	Durables	Medical care	Transport	Recreation	Education	Miscellaneous	Income flexibility
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. U.S.	.61 (.14)	.28 (.32)	1.33 (.26)	.41 (.14)	1.74 (.23)	.37 (.22)	2.31 (.15)	1.22 (.29)	.82 (.49)	.70 (.17)	-.38 (.07)
2. Canada	.96 (.22)	.59 (.35)	.82 (.35)	.03 (.22)	1.57 (.39)	2.45 (.54)	1.56 (.24)	1.86 (.45)	1.05 (.62)	.93 (.31)	-.55 (.12)
3. Sweden	.55 (.12)	1.10 (.19)	1.45 (.27)	.16 (.14)	1.87 (.27)	.05 (.20)	2.01 (.19)	1.59 (.22)	.58 (.64)	1.03 (.29)	-.58 (.09)
4. Switzerland	.97 (.11)	1.35 (.18)	1.82 (.24)	.15 (.11)	2.10 (.24)	.30 (.22)	1.58 (.19)	1.01 (.24)		.61 (.26)	-.55 (.08)
5. Denmark	.36 (.17)	.69 (.20)	1.63 (.23)	.38 (.11)	1.59 (.18)	.49 (.39)	2.25 (.15)	1.24 (.22)	-.14 (.63)	.95 (.20)	-.46 (.08)
6. Australia	.26 (.19)	.83 (.28)	1.29 (.33)	.63 (.21)	2.22 (.38)	.70 (.41)	1.49 (.26)	2.25 (.42)	2.82 (1.65)	.54 (.25)	-.46 (.08)
7. France	.46 (.16)	.48 (.30)	1.29 (.28)	.44 (.17)	1.57 (.22)	.56 (.31)	2.21 (.21)	1.02 (.34)	.78 (1.16)	1.31 (.19)	-.53 (.08)
8. Germany	.62 (.10)		1.50 (.19)	.16 (.13)	1.44 (.17)	.83 (.36)	2.30 (.16)	1.00 (.23)		.71 (.19)	-.60 (.10)
9. Belgium	.49 (.19)	.98 (.36)	1.14 (.37)	.58 (.21)	1.49 (.28)	.58 (.37)	.88 (.29)	.94 (.52)	.06 (1.96)	2.37 (.37)	-.13 (.07)
10. Norway	.23 (.12)	1.14 (.20)	1.21 (.21)	.00 (.17)	1.29 (.22)	.79 (.33)	3.20 (.18)	1.03 (.28)	.52 (1.12)	.84 (.22)	-.48 (.09)
11. Netherlands	.50 (.15)	.62 (.20)	2.06 (.19)	.58 (.18)	1.41 (.23)	.60 (.21)	1.77 (.24)	.95 (.49)	.80 (.40)	.76 (.26)	-.84 (.11)
12. Iceland	.45 (.10)	.74 (.21)	1.34 (.17)	-.04 (.11)	1.89 (.18)	.76 (.29)	2.49 (.17)	1.44 (.33)	-.01 (1.25)	1.72 (.28)	-.69 (.09)
13. Finland	.55 (.13)	1.28 (.29)	1.67 (.32)	.15 (.23)	1.56 (.31)	.65 (.60)	1.80 (.21)	1.89 (.36)	1.16 (.71)	.71 (.29)	-.38 (.08)
14. Austria	.21 (.19)	.50 (.32)	1.79 (.27)	.18 (.25)	1.95 (.32)	-.73 (.53)	3.11 (.26)	.80 (.38)	-.40 (1.29)	.54 (.23)	-.15 (.09)
15. Japan	.62 (.14)		1.88 (.39)	.22 (.23)	2.08 (.32)	.78 (.26)	1.35 (.29)	1.15 (.29)		1.49 (.24)	-.37 (.10)
16. U.K.	.33 (.14)	1.03 (.18)	1.20 (.24)	.35 (.13)	2.14 (.24)	.61 (.62)	1.66 (.16)	1.34 (.23)	1.01 (.47)	1.17 (.14)	-.40 (.07)
17. Spain	.85 (.19)	.91 (.67)	1.29 (.34)	.19 (.32)	1.46 (.35)	.95 (.53)	2.37 (.34)	1.36 (.46)	.40 (.58)	.62 (.31)	-.29 (.07)
18. Italy	.86 (.10)	.70 (.28)	1.81 (.19)	.49 (.18)	1.53 (.22)	1.03 (.31)	1.42 (.18)	.67 (.23)	.62 (.96)	.95 (.16)	-.18 (.07)
Mean	.55	.83	1.47	.28	1.72	.65	1.99	1.26	.67	1.00	-.45

countries except Canada clothing is a luxury. Column 12 gives the estimate of income flexibility ϕ and its asymptotic standard error for each country. As can be seen, all ϕ -estimates are negative, as they should be, and the average is $-.45$. These values are in broad agreement with previous studies (Theil, 1980; Theil and Brookes, 1970/71).

Table 8.2 presents the Slutsky own-price elasticities and their asymptotic standard errors. With only a few exceptions, these elasticities are less than 1 in absolute value.

We noted previously that some (five) of the income elasticities in Table 8.1 are negative; that is, these goods are inferior. It should be noted that none of these negative elasticities are significant. These five cases also correspond to positive own-price elasticities (see Table 8.2). However, preference independence rules out inferior goods (see Clements, 1987). Therefore, we shall now re-estimate the model for those countries with the constraint that all income elasticities are positive. To do this, we specify the coefficient β_i of good i with negative income elasticity to be $\beta_i = -\min_t \bar{w}_{it}$, so that the income elasticity $\eta_{it} = 1 + \beta_i / \bar{w}_{it}$ of good i is positive.

For Denmark we specify β_9 ; for Iceland we specify β_4 and β_9 ; and for Austria we specify β_6 and β_9 . All other parameters are estimated in the same way as before by ML using model (7.3) with constant terms included and specified covariance matrix Σ^* . Table 8.3 presents the income elasticities, income flexibility and the own-price elasticities for these three countries before and after the imposition of the constraint. Comparing the two rows for each country, we see that there are only minor changes in the elasticities and the income flexibility for Denmark and Austria, while for Iceland the changes are somewhat larger.

TABLE 8.2

SLUTSKY OWN-PRICE ELASTICITIES OF 10 COMMODITIES FOR 18 COUNTRIES

(Asymptotic standard errors are in parentheses)

Country (1)	Food (2)	Beverages (3)	Clothing (4)	Housing (5)	Durables (6)	Medical care (7)	Transport (8)	Recreation (9)	Education (10)	Miscellaneous (11)
1. U.S.	-.22 (.05)	-.11 (.12)	-.46 (.12)	-.14 (.05)	-.58 (.12)	-.14 (.08)	-.56 (.09)	-.43 (.13)	-.31 (.19)	-.24 (.07)
2. Canada	-.45 (.11)	-.32 (.19)	-.42 (.19)	-.02 (.12)	-.75 (.24)	-1.23 (.35)	-.66 (.15)	-.92 (.29)	-.57 (.34)	-.45 (.17)
3. Sweden	-.28 (.07)	-.58 (.10)	-.74 (.17)	-.09 (.08)	-.93 (.18)	-.03 (.12)	-.84 (.14)	-.79 (.15)	-.33 (.32)	-.55 (.17)
4. Switzerland	-.42 (.06)	-.65 (.11)	-.88 (.15)	-.08 (.06)	-.97 (.16)	-.16 (.12)	-.72 (.12)	-.50 (.14)		-.32 (.14)
5. Denmark	-.16 (.08)	-.30 (.10)	-.67 (.15)	-.16 (.05)	-.63 (.12)	-.22 (.18)	-.69 (.12)	-.52 (.13)	.06 (.30)	-.40 (.11)
6. Australia	-.11 (.08)	-.35 (.12)	-.53 (.14)	-.26 (.09)	-.84 (.18)	-.31 (.18)	-.53 (.11)	-.91 (.18)	-1.28 (.76)	-.23 (.11)
7. France	-.22 (.08)	-.25 (.15)	-.61 (.15)	-.22 (.09)	-.70 (.12)	-.28 (.16)	-.86 (.12)	-.50 (.18)	-.41 (.60)	-.58 (.10)
8. Germany	-.31 (.07)		-.76 (.16)	-.09 (.08)	-.72 (.13)	-.49 (.22)	-.97 (.15)	-.56 (.15)		-.40 (.12)
9. Belgium	-.06 (.04)	-.12 (.08)	-.13 (.09)	-.07 (.04)	-.15 (.09)	-.07 (.06)	-.10 (.06)	-.11 (.09)	-.01 (.25)	-.22 (.10)
10. Norway	-.11 (.06)	-.50 (.10)	-.51 (.12)	-.00 (.08)	-.55 (.13)	-.37 (.16)	-.89 (.16)	-.46 (.15)	-.25 (.55)	-.37 (.11)
11. Netherlands	-.36 (.12)	-.50 (.16)	-1.21 (.15)	-.46 (.14)	-.98 (.18)	-.49 (.17)	-1.32 (.18)	-.78 (.40)	-.66 (.33)	-.59 (.21)
12. Iceland	-.28 (.07)	-.49 (.14)	-.80 (.12)	.03 (.08)	-1.07 (.15)	-.50 (.20)	-1.21 (.15)	-.92 (.23)	.01 (.87)	-1.09 (.20)
13. Finland	-.18 (.05)	-.43 (.12)	-.54 (.15)	-.06 (.08)	-.52 (.14)	-.24 (.23)	-.51 (.10)	-.64 (.17)	-.43 (.27)	-.25 (.11)
14. Austria	-.03 (.03)	-.07 (.06)	-.22 (.14)	-.03 (.04)	-.25 (.16)	.12 (.11)	-.28 (.17)	-.12 (.09)	.06 (.20)	-.08 (.06)
15. Japan	-.19 (.06)		-.59 (.21)	-.08 (.08)	-.66 (.17)	-.27 (.10)	-.44 (.14)	-.38 (.13)		-.43 (.13)
16. U.K.	-.12 (.06)	-.38 (.07)	-.43 (.11)	-.13 (.05)	-.71 (.15)	-.24 (.25)	-.51 (.09)	-.47 (.11)	-.39 (.19)	-.37 (.08)
17. Spain	-.18 (.05)	-.26 (.19)	-.32 (.11)	-.05 (.09)	-.37 (.11)	-.27 (.15)	-.53 (.12)	-.37 (.14)	-.12 (.17)	-.17 (.09)
18. Italy	-.12 (.05)	-.12 (.07)	-.28 (.11)	-.09 (.05)	-.25 (.10)	-.18 (.09)	-.22 (.09)	-.12 (.06)	-.11 (.18)	-.15 (.07)
Mean	-.21	-.34	-.56	-.11	-.65	-.30	-.66	-.53	-.32	-.38

TABLE 8.3

DEMAND ELASTICITIES FOR 3 COUNTRIES WITH SPECIFIED INCOME COEFFICIENTS

(Asymptotic standard errors are in parentheses)

Country (1)	Food (2)	Beverages (3)	Clothing (4)	Housing (5)	Durables (6)	Medical care (7)	Transport (8)	Recreation (9)	Education (10)	Miscellaneous (11)	Income flexibility (12)
Income Elasticities and Income Flexibility											
1. Denmark: U	.36 (.17)	.69 (.20)	1.63 (.23)	.38 (.11)	1.59 (.18)	.49 (.39)	2.25 (.15)	1.24 (.22)	-.14 (.63)	.95 (.20)	-.46 (.08)
R	.35 (.17)	.69 (.20)	1.63 (.23)	.37 (.12)	1.59 (.19)	.48 (.39)	2.24 (.15)	1.24 (.22)	.46 (-)	.95 (.20)	-.47 (.08)
2. Iceland: U	.45 (.10)	.74 (.21)	1.34 (.17)	-.04 (.11)	1.89 (.18)	.76 (.29)	2.49 (.17)	1.44 (.33)	-.01 (1.25)	1.72 (.28)	-.69 (.09)
R	.27 (.08)	.57 (.18)	1.23 (.14)	.23 (-)	1.92 (.17)	.84 (.28)	2.50 (.15)	1.53 (.32)	.11 (-)	1.78 (.31)	-.70 (.09)
3. Austria: U	.21 (.19)	.50 (.32)	1.79 (.27)	.18 (.25)	1.95 (.32)	-.73 (.53)	3.11 (.26)	.80 (.38)	-.40 (1.29)	.54 (.23)	-.15 (.09)
R	.18 (.19)	.43 (.31)	1.77 (.28)	.16 (.25)	1.92 (.32)	.27 (-)	3.03 (.26)	.76 (.38)	.31 (-)	.50 (.24)	-.18 (.10)
Slutsky Own-Price Elasticities											
1. Denmark: U	-.16 (.08)	-.30 (.10)	-.67 (.15)	-.16 (.05)	-.63 (.12)	-.22 (.18)	-.69 (.12)	-.52 (.13)	.06 (.30)	-.40 (.11)	
R	-.15 (.08)	-.30 (.10)	-.68 (.15)	-.16 (.05)	-.64 (.12)	-.22 (.18)	-.70 (.12)	-.52 (.13)	-.21 (-)	-.40 (.11)	
2. Iceland: U	-.28 (.07)	-.49 (.14)	-.80 (.12)	.03 (.08)	-1.07 (.15)	-.50 (.20)	-1.21 (.15)	-.92 (.23)	.01 (.87)	-1.09 (.20)	
R	-.18 (.05)	-.39 (.12)	-.75 (.11)	-.16 (-)	-1.10 (.16)	-.56 (.21)	-1.23 (.17)	-.99 (.24)	-.08 (-)	-1.14 (.25)	
3. Austria: U	-.03 (.03)	-.07 (.06)	-.22 (.14)	-.03 (.04)	-.25 (.16)	.12 (.11)	-.28 (.17)	-.12 (.09)	.06 (.20)	-.08 (.06)	
R	-.03 (.04)	-.07 (.06)	-.25 (.15)	-.03 (.05)	-.28 (.17)	-.05 (-)	-.32 (.17)	-.13 (.09)	-.06 (-)	-.08 (.06)	

U = Unrestricted results (from Tables 8.1 and 8.2).

R = Restricted results with relevant income elasticity specified to be positive.

To test the validity of the constraint, we use the likelihood ratio statistic $-2 \log \lambda = -2[\log L_u - \log L_r]$, where L_u and L_r represents the log-likelihood values for the unrestricted and restricted models, respectively. This test statistic has an asymptotic χ^2 distribution with degrees of freedom equal to the number of restrictions. For Denmark, the observed value of the test statistic is .89 [which is to be compared with the critical value of $\chi^2_{.05}(1) = 3.84$]. For Iceland, the observed value is 4.36 [$\chi^2_{.05}(2) = 5.99$]; and for Austria, the observed value is 3.50 [$\chi^2_{.05}(2) = 5.99$]. Therefore, we are unable to reject the restricted models for all three countries. This result agrees with all the negative income elasticities being insignificant.

9. CONCLUDING COMMENTS

In this paper we have presented an extensive application of Theil's (1987b) distribution-free testing procedures to test the hypotheses of demand theory using data for 18 OECD countries. In contrast to most previous results, we found that the hypotheses are reasonably acceptable for all 18 countries. The conclusion is that OECD consumers do indeed behave as if they obey demand theory.

We also introduced a similar distribution-free procedure to test the preference independence hypothesis; i.e., that there is no interaction of commodity groups in the consumer's utility function. In most of the 18 countries, there is at least some tentative support for this hypothesis. The implications of the estimates were explored by tabulating the implied demand elasticities.

APPENDICES

A1. ESTIMATION UNDER PREFERENCE INDEPENDENCE

The Model

For convenience, we reproduce model (7.3).

$$(A1.1) \quad \bar{w}_{it}(Dq_{it} - DQ_t) = \beta_i DQ_t + \phi(\bar{w}_{it} + \beta_i) \left[Dp_{it} - \sum_{j=1}^n (\bar{w}_{jt} + \beta_j) Dp_{jt} \right] + \epsilon_{it}.$$

Since $\sum_{j=1}^n (\bar{w}_{jt} + \beta_j) = 1$, we can eliminate β_n from (A1.1) to give

$$(A1.2) \quad \bar{w}_{it}(Dq_{it} - DQ_t) = \beta_i DQ_t + \phi(\bar{w}_{it} + \beta_i) \left[Dp_{it}^* - \sum_{j=1}^{n-1} (\bar{w}_{jt} + \beta_j) Dp_{jt}^* \right] + \epsilon_{it},$$

where $Dp_{it}^* = Dp_{it} - Dp_{nt}$. Equation (A1.2) can also be written as

$$(A1.3) \quad y_{it} = \beta_i DQ_t + \phi z_{it} + \epsilon_{it},$$

where $y_{it} = \bar{w}_{it}(Dq_{it} - DQ_t)$ and

$$z_{it} = (\bar{w}_{it} + \beta_i) \left[Dp_{it}^* - \sum_{j=1}^{n-1} (\bar{w}_{jt} + \beta_j) Dp_{jt}^* \right].$$

Since z_{it} is a function of the β_i 's, equation (A1.3) is nonlinear in the parameters.

For reasons described in Section 2, we confine ourselves to (A1.3) for $i=1, \dots, n-1$. In vector form (A1.3) can be written as

$$(A1.4) \quad y_t = X_t \theta + \epsilon_t.$$

where $y_t = [y_{it}]$; $X_t = [DQ_t I \quad Z_t]$, I is the identity matrix of order $n-1$; $Z_t = [z_{it}]$; $\epsilon_t = [\epsilon_{it}]$; and $\theta = [\beta' \quad \phi]' = [\beta_1 \dots \beta_{n-1} \quad \phi]'$.

ML Estimation With Unknown Covariance Matrix

In this subsection we describe the ML estimation procedure which is based on Theil (Section 2.9, 1987a). Assuming that the ϵ_t 's are independent normal vectors with zero mean and (unknown) nonsingular covariance matrix Σ , the log-likelihood function of the y_t 's is given by

$$(A1.5) \quad L(\theta, \Sigma; y) = C + \frac{T}{2} \log |\Sigma^{-1}| - \frac{1}{2} \sum_{t=1}^T (y_t - X_t \theta)' \Sigma^{-1} (y_t - X_t \theta),$$

where C is a constant; and T is the sample size. The first-order conditions for a maximum of (A1.5) are

$$(A1.6) \quad \frac{\partial L}{\partial \Sigma^{-1}} = \frac{T}{2} \Sigma - \frac{1}{2} \sum_{t=1}^T (y_t - X_t \theta)(y_t - X_t \theta)' = 0$$

and

$$(A1.7) \quad \frac{\partial L}{\partial \theta'} = \sum_{t=1}^T (y_t - X_t \theta)' \Sigma^{-1} \frac{\partial X_t \theta}{\partial \theta'} = 0,$$

where

$$(A1.8) \quad \frac{\partial X_t \theta}{\partial \theta'} = \left[DQ_t I + \phi \frac{\partial Z_t}{\partial \beta'} \quad Z_t \right];$$

$$\frac{\partial Z_t}{\partial \beta'} = \left[\frac{\partial z_{it}}{\partial \beta_j} \right];$$

and

$$\frac{\partial z_{it}}{\partial \beta_j} = -(\bar{w}_{it} + \beta_i) Dp_{jt}^* + \delta_{ij} \left[Dp_{it}^* - \sum_{k=1}^{n-1} (\bar{w}_{kt} + \beta_k) Dp_{kt}^* \right].$$

From (A1.7),

$$(A1.9) \quad \frac{\partial^2 L}{\partial \Sigma^{-1} \partial \theta'} = \sum_{t=1}^T (y_t - X_t \theta) \left[\frac{\partial X_t \theta}{\partial \theta'} \right]'$$

Since $E[(y_t - X_t \theta)] = E[\epsilon_t] = 0$, the right-hand side of (A1.9) vanishes and therefore the information matrix of the ML procedure is block-diagonal with respect to θ and Σ^{-1} . From (A1.7) we also have

$$\frac{\partial^2 L}{\partial \theta \partial \theta'} = - \sum_{t=1}^T \frac{\partial (X_t \theta)'}{\partial \theta} \Sigma^{-1} \frac{\partial (X_t \theta)}{\partial \theta'} + \sum_{t=1}^T (y_t - X_t \theta)' \Sigma^{-1} \frac{\partial^2 (X_t \theta)}{\partial \theta \partial \theta'}.$$

The second term on the right-hand side has zero expectation. Therefore, the asymptotic covariance matrix of the ML estimator of θ is

$$(A1.10) \quad V = - \left[E \left[\frac{\partial^2 L}{\partial \theta \partial \theta'} \right] \right]^{-1} = \left[\sum_{t=1}^T \frac{\partial (X_t \theta)'}{\partial \theta} \Sigma^{-1} \frac{\partial (X_t \theta)}{\partial \theta'} \right]^{-1}.$$

The ML estimator of θ is obtained by means of a Newton's iterative scheme based on successive estimates of V and Σ . The asymptotic standard errors are the square roots of the diagonal elements of V with ML estimates substituted for the unknown parameters in V :

ML Estimation With Specified Covariance Matrix

When the covariance matrix is specified as $\Sigma^* = \lambda^2 \Omega$, the

log-likelihood function is

$$(A1.11) \quad L(\theta, \lambda^2; y) = C + \frac{T}{2} \log \lambda^{-2(n-1)} - \frac{1}{2\lambda^2} \sum_{t=1}^T (y_t - X_t \theta)' \Omega^{-1} (y_t - X_t \theta),$$

where C is a constant; and T is the sample size. The first-order conditions for a maximum of (A1.11) are

$$(A1.12) \quad \frac{\partial L}{\partial \lambda^2} = -\frac{T(n-1)}{2\lambda^2} + \frac{1}{2\lambda^4} \sum_{t=1}^T (y_t - X_t \theta)' \Omega^{-1} (y_t - X_t \theta) = 0$$

and

$$(A1.13) \quad \frac{\partial L}{\partial \theta'} = \frac{1}{\lambda^2} \sum_{t=1}^T (y_t - X_t \theta)' \Omega^{-1} \frac{\partial X_t \theta}{\partial \theta'} = 0,$$

where $\partial(X_t \theta)/\partial \theta'$ is given by (A1.8).

From (A1.13),

$$(A1.14) \quad \frac{\partial^2 L}{\partial \lambda^2 \partial \theta'} = -\frac{1}{\lambda^4} \sum_{t=1}^T (y_t - X_t \theta)' \Omega^{-1} \frac{\partial X_t \theta}{\partial \theta'}.$$

Since $E[(y_t - X_t \theta)] = E[\epsilon_t] = 0$, from (A1.14), the information matrix of the ML procedure is block-diagonal with respect to θ and λ^2 . From (A1.13) we also have

$$\frac{\partial^2 L}{\partial \theta \partial \theta'} = -\frac{1}{\lambda^2} \sum_{t=1}^T \frac{\partial(X_t \theta)'}{\partial \theta} \Omega^{-1} \frac{\partial(X_t \theta)}{\partial \theta'} + \frac{1}{\lambda^2} \sum_{t=1}^T (y_t - X_t \theta)' \Omega^{-1} \frac{\partial^2(X_t \theta)}{\partial \theta \partial \theta'}.$$

The second term on the right-hand side has zero expectation. Therefore,

the asymptotic covariance matrix of the ML estimator of θ is

$$V^* = - \left[E \left[\frac{\partial^2 L}{\partial \theta \partial \theta'} \right] \right]^{-1} = \left[\frac{1}{\lambda^2} \sum_{t=1}^T \frac{\partial(X_t \theta)'}{\partial \theta} \Omega^{-1} \frac{\partial(X_t \theta)}{\partial \theta'} \right]^{-1}.$$

The ML estimator of θ is obtained by means of a Newton's iterative scheme based on successive estimates of V^* and λ^2 . The asymptotic standard errors are the square roots of the diagonal elements of V^* with ML estimates substituted for the unknown parameters in V^* .

A2. FURTHER RESULTS ON TESTING PREFERENCE INDEPENDENCE

As noted in Section 7, the results of testing preference independence based on S and Σ^* are different for the seven countries with non-singular covariance matrix.

In the testing procedure described in Section 7, the covariance matrix of the error terms is used at 3 stages:

1. To obtain the data-based homogeneity- and symmetry-constrained parameter estimates, where we use the covariance matrix of the error terms given homogeneity ($S_h; \Sigma_h^*$).
2. To obtain the parameter estimates under preference independence (the null hypothesis) and to generate error terms under the null hypothesis. Here we use the covariance matrix of the error terms under preference independence ($S_p; \Sigma_p^*$). At this stage we also compute the data-based value of the test statistic τ .
3. To obtain simulated homogeneity- and symmetry-constrained parameter estimates, where we use the covariance matrix of the error terms under homogeneity ($S_h; \Sigma_h^*$). At this stage we also compute the simulated value of the test statistic τ .

To see whether the effect of S is different from that of Σ^* at a particular stage, we replace S by Σ^* (or vice versa) at various stages of our testing procedure. Table A2.1 summarizes the various possible combinations of S and Σ^* . For each of these combinations, we test the preference independence hypothesis for the seven countries with nonsingular covariance matrix. In Table A2.2, we present the ranks of the data-based test statistics in 99 simulations for the six combinations of S and Σ^* . The upper half of columns 2 and 7 are the relevant parts of columns 2 and 4 of Table 7.1; and the lower half of these columns are the relevant parts of columns 3 and 5 of Table 7.1. As can be seen from the last row of both the upper and lower parts of the table, there is a tendency towards acceptance when we use Σ^* instead of S at all stages. That is, as we move across the columns of Table A2.2, the number of acceptances increases.

TABLE A2.1
COMBINATIONS OF S AND Σ^* AT DIFFERENT STAGES

Combination	Stage		
	Data-based Estimation with Homogeneity and Symmetry	Data-based Estimation with Preference Independence and Data-based Value of τ	Simulated Value of τ
(1)	(2)	(3)	(4)
1	S	S	S
2	S	S	Σ^*
3	Σ^*	S	S
4	Σ^*	Σ^*	S
5	S	Σ^*	Σ^*
6	Σ^*	Σ^*	Σ^*

TABLE A2.2
RANK OF THE DATA-BASED TEST STATISTIC FOR PREFERENCE INDEPENDENCE
WITH DIFFERENT COMBINATIONS OF S AND Σ^*
IN 99 SIMULATIONS FOR 7 COUNTRIES

Country (1)	Combination					
	1 (2)	2 (3)	3 (4)	4 (5)	5 (6)	6 (7)
No constant						
1. U.S.	100	96	81	9	45	58
2. Canada	98	89	81	2	5	23
3. Switzerland	91	44	66	74	76	95
4. Australia	86	80	75	47	55	93
5. Germany	95	74	99	51	22	84
6. Belgium	99	99	50	12	79	43
7. Netherlands	98	97	69	11	58	39
Percentage significance at 5%	57	43	14	0	0	0
With constant						
1. U.S.	100	99	94	17	47	70
2. Canada	96	94	30	1	7	15
3. Switzerland	86	90	94	46	37	82
4. Australia	70	57	66	44	46	92
5. Germany	50	32	64	14	2	30
6. Belgium	100	100	78	14	86	49
7. Netherlands	99	100	43	5	86	25
Percentage significance at 5%	57	43	0	0	0	0

A3. THE ASYMPTOTIC STANDARD ERRORS OF THE PRICE ELASTICITIES

To obtain the asymptotic standard error of the i^{th} own-price elasticity η_{ii} we write

$$\eta_{ii} = \eta_{ii}(\hat{\delta}_i),$$

where $\hat{\delta}_i$ is the ML estimator of $\delta_i = [\phi \ \beta_i]'$. That is, η_{ii} is a nonlinear function of $\hat{\delta}_i$ which is totally differentiable. Therefore $\sqrt{T}[\eta_{ii}(\hat{\delta}_i) - \eta_{ii}(\delta_i)]$ converges in distribution to a normal variate with mean zero and asymptotic variance $[\partial\eta_{ii}/\partial\hat{\delta}_i']\hat{\Sigma}_i[\partial\eta_{ii}/\partial\hat{\delta}_i]$, where $\hat{\Sigma}_i = \text{var } \hat{\delta}_i$. Therefore the asymptotic standard error of η_{ii} is given by

$$\begin{aligned} \text{a.s.e.}[\eta_{ii}] &= \left[\frac{\partial\eta_{ii}}{\partial\hat{\delta}_i'} \hat{\Sigma}_i \frac{\partial\eta_{ii}}{\partial\hat{\delta}_i} \right]^{1/2} \\ &= \left[\left(\frac{\partial\eta_{ii}}{\partial\hat{\phi}} \right)^2 \text{var } \hat{\phi} + \left(\frac{\partial\eta_{ii}}{\partial\hat{\beta}_i} \right)^2 \text{var } \hat{\beta}_i + 2 \left(\frac{\partial\eta_{ii}}{\partial\hat{\phi}} \right) \left(\frac{\partial\eta_{ii}}{\partial\hat{\beta}_i} \right) \text{cov}[\hat{\phi}, \hat{\beta}_i] \right]^{1/2}. \end{aligned}$$

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