COMMODITY TAXES UNDER UNCERTAINTY

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ABSTRACT

This paper considers the relative distorting effect on output of commodity taxes, specific and ad valorem, in a situation where a firm faces revenue uncertainty. It is shown generally that this effect depends both on the precise nature of the firm's revenue uncertainty and on the degree of its risk aversion. More specifically, the distorting effect of each tax is characterised for a selection of objective functions and uncertain environments. These characterisations are compared in order to show the influence of alternative specifications on the relative distorting effect.

INTRODUCTION

A firm which faces uncertainty may find the characteristics of this uncertainty altered by the imposition of a tax of some sort. If the firm's decisions are influenced by the presence of the uncertainty then it follows that these decisions may also be altered by the imposition of the tax. Such effects of taxation under uncertainty have long been recognised in the literature, particularly in the context of profits taxation. (See, for example, Domar and Musgrave (1944), Stiglitz (1969), Leland (1978)). More recently, renewed attention has been given to the effects of other sorts of taxation on the decisions of firms. In the context of resource taxation, Conrad and Hool (1981) examine the effects of (among others) specific and ad valorem taxes on a firm's extraction profile when reserves are both endogenous and of heterogeneous quality. Furthermore, in the context of optimal commodity taxation, Kay and Keen (1983) compare the desirability of specific and ad valorem taxes for different market structures and under circumstances of varying product quality.¹ In neither case, however, is attention given to the role of uncertainty in influencing the effects of these two taxes on the behaviour of firms.

The aim of this paper is to show that uncertainty can be an important determinant of the effects of specific and ad valorem taxation on a firm's behaviour, and consequently, that to omit consideration of its role may lead to false conclusions about the relative desirability of these two taxes. In order to emphasise this importance, the approach taken here is to focus on the behaviour of a perfectly competitive firm facing commodity taxation where the standard result of the literature is that "the choice between ad valorem and specific taxation is simply a matter of administrative

1. Although Kay and Keen quite rightly cite Musgrave (1959) (or more specifically, Suits and Musgrave (1953)) as a forerunner in this area.

convenience".^{2,3} The structure of the paper is as follows: Section I examines the behaviour of a risk neutral and a risk averse firm facing price uncertainty alone. Section II examines this behaviour in the context of price and output uncertainty. Section III discusses the paper's policy implications.

SECTION I PRICE UNCERTAINTY

For a firm facing price uncertainty and commodity taxation, after tax profit (π) may be written as:

$$\pi = ((1 - t_v)p - t_s)y - C(y)$$

where: t_v = the rate of ad valorem taxation

t_s = the specific tax

p = uncertain price ($E(p) = \bar{p}$)

y = output

$C(y)$ = cost of producing y ($C(0)=0$, $C'(y) > 0$, $C''(y) > 0$).

A risk averse firm will be concerned to maximise the expected utility of after tax profit ($E(U(\pi))$) by choice of y :

$$E(U(\pi)) = E(U(((1 - t_v)p - t_s)y - C(y))) . \quad (1)$$

Differentiating (1) with respect to y gives the firm's first order condition:⁴

$$E(U'(\pi)((1 - t_v)p - t_s - C'(y))) = 0 . \quad (2)$$

The effect on optimal output of a marginal change in the rate of ad valorem taxation can be determined from (2) using the Implicit Function Theorem:

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2. Kay and Keen (1983) p.342.
 3. This is not to deny that the behaviour of other types of firms facing uncertainty and commodity taxation, such as a quantity or price-setting monopolist is worthy of examination. Rather just that for the purposes of demonstrating the divergent effects of these taxes under uncertainty, the behaviour of the perfectly competitive firm can be more simply characterised (see, for example, Leland (1972)).
 4. The second order condition is satisfied for $U''(\pi) < 0$ and $C''(y) > 0$.

$$\frac{dy}{dt_v} = \frac{-E(U'(\pi) \cdot p) - E(U''(\pi) \cdot p((1 - t_v)p - t_s - C'(y))y)}{-D} \quad (3)$$

where: $D =$ the second order condition (< 0) .

Writing:

$$R^* = \frac{-U''(\pi) \cdot \pi^*}{U'(\pi)} \leq R$$

where: $\pi^* = ((1 - t_v)p - t_s)y - C'(y) \cdot y \leq \pi$

$R =$ the firm's coefficient of relative risk aversion,⁵

equation (3) may be simplified to give:

$$\frac{dy}{dt_v} = \frac{-E(U'(\pi) \cdot p(1 - R^*))}{-D} \quad (4)$$

Turning now to the effect on optimal output of a marginal change in the rate of specific taxation, define:

$$t_s = \alpha \bar{p} \quad (5)$$

where: $\alpha < 1$.

A marginal change in α (holding $\bar{p}$ constant) can be interpreted as a marginal change in the rate of specific taxation. Substituting (5) into (2) and again using the Implicit Function Theorem gives:

$$\frac{dy}{d\alpha} = \frac{-E(U'(\pi) \cdot \bar{p}(1 - R^*))}{-D} \quad (6)$$

To compare the distorting effects on optimal output of a marginal change in ad valorem and specific taxation, combine equations (4) and (6) to give:

$$\frac{dy}{dt_v} = \frac{dy}{d\alpha} \cdot \frac{E(U'(\pi) \cdot p(1 - R^*))}{E(U'(\pi) \cdot \bar{p}(1 - R^*))} \quad (7)$$

Further analysis of (7) is difficult without additional assumptions about the characteristics of the firm's risk aversion. It is clear, however,

5. These inequalities follow from the assumptions that there are no fixed costs ($C(0)=0$) and that marginal cost is non-decreasing ($C'(y) \geq 0$).

that if the firm is risk neutral (7) simplifies to:

$$\frac{dy}{dt_v} = \frac{dy}{d\alpha}.$$

This shows that the standard result of the literature concerning the equivalence of the distorting effects of ad valorem and specific taxation on a competitive firm's output continues to hold in the presence of price uncertainty if the firm is risk neutral.

If the firm is risk averse, however, this equivalence is no longer guaranteed. Consider, for example the special case where both marginal cost and the firm's coefficient of relative risk aversion are constant. In this case (7) simplifies to:

$$\frac{dy}{dt_v} = \frac{dy}{d\alpha} \cdot \left[1 + \frac{\text{Cov}(U'(\pi), p)}{E(U'(\pi)) \cdot \bar{p}} \right] \quad (8)$$

where: $\text{Cov}(U'(\pi), p)$ = covariance of $U'(\pi)$ and p .

Since this covariance is negative, it follows from (8) that a marginal change in the rate of ad valorem taxation will be less distorting to output than a marginal change in the rate of specific taxation.⁶ Although this is a result derived from a special case, the intuition behind it would suggest it is more generally valid. In particular, the less distorting effect of a marginal change in ad valorem taxation can be seen to derive from the greater risk-sharing qualities of such taxation. Simply put, even though the two taxes can be seen to have an equivalent impact on expected profit, the amount of tax actually paid under a regime of ad valorem taxation will be higher when prices (and consequently profit) are higher and lower when prices are lower. Therefore, the overall variation of profit is reduced by comparison with a regime of specific taxation where no such risk-sharing takes place.⁷

SECTION II PRICE AND OUTPUT UNCERTAINTY

Consider now a firm facing commodity taxation and uncertainty of both

6. Alternatively, a given (expected) amount of extra tax revenue may be raised more efficiently using ad valorem taxation.
7. This argument is based on that used by Stiglitz (1969) (and earlier, Domar and Musgrave (1944)) to explain why the corporation tax may encourage risk-taking.

price and output, such as is likely to prevail in the case of agricultural producers. For this purpose two further assumptions are made. First, it is assumed that production uncertainty is multiplicative:

$$x = (1 + \theta)y$$

where: x = actual output

y = planned production and expected output ($E(\theta)=0$).

This assumption simplifies the analysis of production uncertainty but retains the desirable characteristic of the variance of output being an increasing function of planned production. Second, it is assumed that the firm's utility of profit is separable in after tax revenue (I) and costs (C). This assumption also simplifies the analysis by allowing it to focus specifically on the relationship between revenue uncertainty and taxation. Such an assumption, while uncommon in the standard literature on the theory of the firm under uncertainty, is more commonly made in the context of agricultural decision analysis.^{8,9}

Given these assumptions, the risk averse firm will be concerned to maximise by choice of y :

$$E(U(((1 - t_v)p - t_s)x)) - C(y)) . \quad (9)$$

Differentiating (9) with respect to y gives¹⁰:

$$E(U'(I)((1 - t_v)p - t_s)(1 + \theta)) - C'(y) = 0 . \quad (10)$$

From (10), once again using the Implicit Function Theorem:

$$\frac{dy}{dt_v} = \frac{-E(U'(I)(p(1 + \theta))) - E(U''(I)((1 - t_v)p - t_s)(1 + \theta)px)}{-D} . \quad (11)$$

Equation (11) may be simplified to give:¹¹

$$\frac{dy}{dt_v} = \frac{-E(U'(I)(p(1 + \theta))(1 - R))}{-D} . \quad (12)$$

Similarly:

8. See, for example, Newbery and Stiglitz (1981), Ch.6.

9. Notice that this assumption also implies that the firm's costs depend only on planned production, thus omitting consideration of, for example, marketing costs associated with unplanned output.

10. The second order condition is satisfied for $U''(I) < 0$ and $C''(y) \geq 0$.

11. Notice that R is now defined on after tax revenue (I).

$$\frac{dy}{d\alpha} = \frac{-E(U'(I)(\bar{p}(1+\theta))(1-R))}{-D} \quad (13)$$

Combining equations (12) and (13) gives:

$$\frac{dy}{dt_v} = \frac{dy}{d\alpha} \cdot \frac{E(U'(I)(p(1+\theta))(1-R))}{E(U'(I)(\bar{p}(1+\theta))(1-R))} \quad (14)$$

As in the previous section, further analysis is difficult without additional assumptions about the characteristics of the firm's risk aversion. In the special case of risk neutrality, however, (14) simplifies to:

$$\frac{dy}{dt_v} = \frac{dy}{d\alpha} \left(1 + \frac{\text{Cov}(p, \theta)}{\bar{p}} \right) \quad (15)$$

If the firm's uncertain price and output are uncorrelated, (15) shows that the distorting effects on planned production of a marginal change in ad valorem and specific taxation will be equivalent, as was the case for price uncertainty alone. If, on the other hand, uncertain price and output are correlated, then the relative distorting effect of the two taxes depends on whether this covariance is positive or negative:

$$\left| \frac{dy}{dt_v} \right| < \left| \frac{dy}{d\alpha} \right| \quad \text{as} \quad \text{Cov}(p, \theta) < 0 \quad (16)$$

Therefore, in this case of revenue uncertainty it is possible to distinguish between the distorting effect on planned production of ad valorem and specific taxation even for a risk neutral firm. The intuition for the results summarised in (16) derives once again from the effect of ad valorem taxation on the variation of profits. In the case of risk neutrality, however, this effect is only of concern to the firm through the associated effect of ad valorem taxation on the after tax value of the covariance between price and output, and consequently on expected profit:

$$E(\pi) = ((\bar{p} + \text{Cov}(p, \theta))(1 - t_v) - t_s)y - C(y) \quad .$$

A marginal increase in the rate of ad valorem taxation collects more of the expected benefits of a positive covariance between price and output which a marginal increase in the rate of specific taxation does not. Similarly, ad valorem taxation shares in the expected profit burden of a negative

covariance which specific taxation does not.¹²

If the firm is risk averse, it would seem likely that the greater risk-sharing qualities of ad valorem taxation will again be relevant in determining the relative distorting effect of the two taxes. This creates no problem in the case of a negative covariance between price and output, where ad valorem taxation was seen to be less distorting even for a risk neutral firm. In the case of a positive covariance, however, ad valorem taxation was seen to be more distorting to planned production than specific taxation for a risk neutral firm. Therefore, for a risk averse firm there will be a conflict between the risk neutral and risk averse aspects of the relative distorting effect of the two taxes. To see this more clearly, consider (14) for the special case of constant relative risk aversion:

$$\frac{dy}{dt_v} = \frac{dy}{d\alpha} \cdot \frac{E(U'(I)(p(1 + \theta)))}{E(U'(I)(\bar{p}(1 + \theta)))} \quad (14)'$$

which may be rewritten as:

$$\frac{dy}{dt_v} \bigg/ \frac{dy}{d\alpha} = \frac{E(U'(I)(\bar{p} + \text{Cov}(p, \theta))) + \text{Cov}(U'(I), p(1 + \theta))}{E(U'(I) \cdot \bar{p}) + \text{Cov}(U'(I), \bar{p}(1 + \theta))} \cdot (17)$$

A positive covariance between price and output influences the value of the right-hand-side of (17) in two ways. It increases the (positive) magnitude of the first term in the numerator relative to the first term in the denominator (the risk neutral effect), and it increases the (negative) magnitude of the second term in the numerator relative to that term in the denominator (the risk averse effect). For a more risk averse firm, marginal utility declines more quickly with increases in revenue and so the impact of the positive covariance on the second term may dominate the impact on the first term, resulting in:

12. Notice, however, it does not follow from (16) that in the case of a risk neutral firm and, say, a negative covariance, a given (expected) amount of extra tax revenue may be raised more efficiently using ad valorem taxation. This is because expected tax revenue is also dependent on the relationship between uncertain price and output.

$$\left| \frac{dy}{dt_v} \right| < \left| \frac{dy}{d\alpha} \right| .$$

For a firm which is less risk averse, however, the first term effect may dominate:

$$\left| \frac{dy}{dt_v} \right| > \left| \frac{dy}{d\alpha} \right| .$$

In this case, the question of which tax is more distorting to planned production thus depends on the extent of the firm's risk aversion.

More generally, it can be seen that the relative distorting effect of ad valorem and specific taxation may depend both on the relationship between the firm's uncertainties and on the extent of its risk aversion. This contrasts markedly with the simple equivalence suggested by the certainty approach.

SECTION III CONCLUSION

The implications of this paper for tax policy are three-fold, each stemming from the recognition that the relevance of uncertainty in influencing a firm's behaviour destroys the substitutability of specific and ad valorem taxes as means of raising revenue. First, in all but the simplest of uncertain environments, each tax was shown to have an identifiably separate influence on this environment. Therefore, the choice of which tax to employ in an industry will require an understanding of the microfoundations of uncertainty in that industry. Second, the presence of risk aversion among firms introduces an extra dimension to the effects of each tax: its risk-sharing characteristics. An appreciation of the attitudes to risk of producers in an industry will thus be important in determining the choice of commodity tax for that industry. Finally, in the specific context of agricultural production, the likely co-existence of a negative covariance between uncertain price and output and risk averse producers represents strong grounds for the exclusive use of ad valorem commodity taxation.

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