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IN SECONDARY INDUSTRY

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1. INTRODUCTION

In a previous paper we used recent data to analyse the demand for energy used in transport in the state of Western Australia (Clements, 1982). The objective of this paper is to carry out a similar analysis of energy used in secondary industry in W.A. In particular, our objective is to measure the roles of economic activity and relative prices in determining the demand for four types of energy used in secondary industry. Information about these demand responses is of obvious importance for pricing policy, energy conservation and demand forecasting.

In this paper we use Theil's (1977, 1980) differential approach to factor demand theory, which is set out in Section 2. Section 3 presents the data on energy usage and prices, as well as several Divisia index numbers which provide convenient summary measures. Estimates of the model are presented in Sections 4, 5 and 6, where it is shown that the demand model yields substantially better predictions than does no-change extrapolation of the energy cost shares. Section 7 contains some concluding comments.

2. DIFFERENTIAL FACTOR DEMAND EQUATIONS

In this section we give a brief overview of Theil's differential approach to factor demand theory. For all derivations and further interpretations, see Theil (1977, 1980).

We consider a firm which purchases n inputs $q_1, \dots, q_n$ to minimize total cost $C = \sum_{i=1}^n p_i q_i$ (where p_i is the price of i) for a given output z . We write $f_i = p_i q_i / C$ for the factor share of input i and $d(\log Q) = \sum_{i=1}^n f_i d(\log q_i)$ for the Divisia input volume index. Under appropriate regularity conditions the cost-minimizing factor demand equation for i is

$$(2.1) \quad f_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^n \pi_{ij} d(\log p_j),$$

where $\theta_i = [\partial(p_i q_i) / \partial z] / (\partial C / \partial z)$ is the share of i in marginal cost, which we shall call the marginal share; and π_{ij} is the (i, j) th Slutsky coefficient. The n^2 Slutsky coefficients satisfy demand homogeneity and symmetry,

$$(2.2) \quad \sum_{j=1}^n \pi_{ij} = 0, \quad \pi_{ij} = \pi_{ji} \quad i, j = 1, \dots, n.$$

Now let the inputs be divided into G groups, $S_1, \dots, S_G$, which are separable in the sense that technology describes $\log z$ as the sum of G functions, one for each group. We write $F_g = \sum_{i \in S_g} f_i$ for the factor share of the group S_g and $\theta_g = \sum_{i \in S_g} \theta_i$ for the corresponding marginal share. Then the composite demand equation for S_g as a group is

$$(2.3) \quad F_g d(\log Q_g) = \theta_g d(\log Q) - \psi \theta_g d\left(\log \frac{P'_g}{P'}\right),$$

where $d(\log Q_g) = \sum_{i \in S_g} (f_i / F_g) d(\log q_i)$ is the Divisia input volume index of S_g ; $\psi > 0$ is a curvature measure of the logarithmic cost function; $d\left(\log \frac{P'_g}{P'}\right) = d(\log P'_g) - d(\log P')$ is the change in the relative price of S_g , where $d(\log P'_g) = \sum_{i \in S_g} (\theta_i / \theta_g) d(\log p_i)$ and $d(\log P') = \sum_{i=1}^n \theta_i d(\log p_i)$.

By dividing both sides of (2.3) by F_g it can be seen that the Divisia and price elasticities of the group are θ_g/F_g and $-\psi\theta_g/F_g$, respectively.

The conditional demand equation for $i \in S_g$ is

$$(2.4) \quad f_i d(\log q_i) = \theta'_i F_g d(\log Q_g) + \sum_{j \in S_g} \pi_{ij}^g d(\log p_j),$$

where $\theta'_i = \theta_i/\theta_g$ is the conditional marginal share of i ; and π_{ij}^g is the $(i,j)^{th}$ conditional Slutsky coefficient. Equation (2.4) is the demand equation for $i \in S_g$ given the demand for the group as a whole $F_g d(\log Q_g)$. Note that the variables on the right of this equation are exclusively concerned with the group S_g to which the i^{th} factor belongs. The conditional Slutsky coefficient π_{ij}^g describes the effect on the demand for i of a change in p_j ($i, j \in S_g$) under the condition that the total demand for the group remains unchanged. These conditional coefficients are also subject to homogeneity and symmetry restrictions,

$$(2.5) \quad \sum_{j \in S_g} \pi_{ij}^g = 0, \quad \pi_{ij}^g = \pi_{ji}^g \quad i, j \in S_g.$$

Furthermore, the matrix $[\pi_{ij}^g]$ is negative semidefinite.

By dividing both sides of (2.4) by f_i we find that $\theta'_i F_g / f_i = \frac{\theta_i / f_i}{\theta_g / F_g}$ is the ratio of the Divisia elasticity of i to that of the group to which it belongs. We shall refer to this ratio as the conditional Divisia elasticity of i . We also find that π_{ij}^g / f_i is the conditional price elasticity.

In this paper we analyse the demand for the energy input group. We use data on four energy categories to estimate versions of the conditional demand equations (2.4), as well as the composite demand equation for energy as a whole (2.3).

3. THE ENERGY DATA

Our data are annual for the period 1968/69 to 1979/80 and refer to energy used in secondary industry in W.A. classified into four categories: (i) Fuel oil, (ii) gas, (iii) electricity and (iv) other fuels, comprising primarily of distillate, coal and LPG. Secondary industry is defined as all manufacturing less (i) chemical fertilisers (ASIC 2711), (ii) petroleum refining (ASIC 273), (iii) iron ore pelletising and metallising (ASIC 2911), (iv) alumina (ASIC 2933), (v) smelting and refining of nickel (ASIC 2925) and (vi) smelting and refining of non-ferrous metals not included above (ASIC 2922, 2924, 2926). The source for all the quantity data is State Energy Commission of Western Australia, *Western Australian Fuel Demand 1980-2000* Report No. RP95/1 July 1980. The price data for fuel oil are from Australian Bureau of Statistics, *Census of Manufacturing Establishments: Western Australia* (Cat. No. 8202.5, various issues). The price data for the other categories were supplied by SECWA. For full details of these and the other data used in this paper, see Goldschmidt (1983).

Table 1 gives the basic quantity/price data. For the year ended June 1980, the usage of fuel oil is 6.44 GJ per capita, 5.83 of gas, 3.83 of electricity and 3.20 of other fuels. For the same year, the price is \$3.51 for 1 GJ of fuel oil, \$2.78 for gas, \$13.92 for electricity and \$0.91 for other fuels. In Table 2 these data are converted to log-changes. Here $Dq_{it} = \log q_{it} - \log q_{i,t-1}$ and $Dp_{it} = \log p_{it} - \log p_{i,t-1}$ are the i th quantity and price log-changes, respectively. The most striking feature of these data is the huge increase in gas usage in 1973, which is associated with a large decline in its price. This is due to the availability of the Dongara gas. Looking at the bottom row of Table 2, on average per capita usage of fuel oil declined by about 4.8 percent p.a., gas increased by 36.3 percent, electricity increased by 4.4 percent and other fuels by 11.8 percent. The average annual price changes are 22.7, -1.7, 8.3 and 12.4 (in the same order as before).

TABLE 1
QUANTITIES AND PRICES OF ENERGY USED IN SECONDARY INDUSTRY:
W.A., 1968/69-1979/80

Year	Quantities (GJ per capita)					Prices (cents per GJ)			
	Fuel oil	Gas	Electricity	Other fuels	Total energy	Fuel oil	Gas	Electricity	Other fuels
	q_1	q_2	q_3	q_4	$\sum_{i=1}^4 q_i$	p_1	p_2	p_3	p_4
1968/69	10.86	.11	2.37	.88	14.21	29.1	336.1	558.3	23.1
1969/70	12.32	.13	2.60	1.12	16.18	30.7	316.7	541.7	23.7
1970/71	10.86	.14	2.71	.89	14.61	32.6	305.6	550.0	24.5
1971/72	9.81	.29	2.75	.99	13.84	39.1	263.9	625.0	25.4
1972/73	8.88	3.91	3.01	1.00	16.80	40.7	169.4	663.9	28.7
1973/74	8.47	4.34	3.08	1.56	17.44	50.9	155.6	663.9	30.7
1974/75	7.68	5.21	3.05	2.93	18.88	110.2	166.7	822.2	33.8
1975/76	7.33	5.42	3.22	4.00	19.98	140.2	191.7	1083.0	41.4
1976/77	6.91	5.68	3.24	3.10	18.93	153.0	194.4	1092.0	47.4
1977/78	6.56	5.09	3.51	2.93	18.09	173.0	233.3	1289.0	51.8
1978/79	6.20	5.48	3.62	3.61	18.92	301.4	252.8	1328.0	72.8
1979/80	6.44	5.83	3.83	3.20	19.31	351.1	277.8	1392.0	90.8
Mean	8.53	3.47	3.08	2.18	17.27	121.0	238.7	884.1	41.2

GJ = Joules $\times 10^9$.

TABLE 2
QUANTITY AND PRICE LOG-CHANGES OF ENERGY USED IN SECONDARY INDUSTRY:
W.A., 1968/69-1979/80

Year	Quantity log-changes				Price log-changes			
	Fuel oil	Gas	Electricity	Other fuels	Fuel oil	Gas	Electricity	Other fuels
	Dq ₁	Dq ₂	Dq ₃	Dq ₄	Dp ₁	Dp ₂	Dp ₃	Dp ₄
1969/70	12.62	22.20	9.49	24.43	5.46	-5.95	-3.02	2.56
1970/71	-12.63	7.34	4.24	-22.84	5.88	-3.57	1.52	3.32
1971/72	-10.14	69.96	1.18	10.45	18.23	-14.67	12.78	3.61
1972/73	-9.98	260.30	9.19	1.04	4.09	-44.33	6.04	12.21
1973/74	-4.80	10.47	2.26	44.27	22.36	-8.50	.00	6.74
1974/75	-9.71	18.36	-.86	63.22	77.24	6.89	21.39	9.62
1975/76	-4.64	3.85	5.44	31.14	24.08	13.97	27.55	20.28
1976/77	-5.99	4.70	.57	-25.53	8.74	1.40	.83	13.53
1977/78	-5.15	-11.00	7.96	-5.65	12.29	18.24	16.59	8.88
1978/79	-5.61	7.40	3.12	20.99	55.51	8.03	2.98	34.03
1979/80	3.80	6.16	5.74	-12.06	15.26	9.43	4.71	22.09
Mean	-4.75	36.34	4.39	11.77	22.65	-1.73	8.31	12.44

The year 1969/70, for example, refers to the transition from 1968/69 to 1969/70.
All entries are to be divided by 100.

We use value added in secondary industry as a measure of the dollar size of that industry, so that it plays the role of total cost. This value added is given in Table 3, together with the factor shares and the corresponding conditional shares $f_1/F_g, \dots, f_4/F_g$. Looking at the last four columns, it can be seen that there the conditional shares vary substantially over this period. One of the objectives of this paper is to analyse the extent to which this variability can be accounted for by changes in the size of secondary industry and by changes in relative energy prices.

Table 4 gives the Divisia moments of the data. To explain these we write C_g for expenditure on the energy group,

$$\sum_{i=1}^4 p_i q_i = C_g$$

and take differentials to give $\sum_i p_i dq_i + \sum_i q_i dp_i = dC_g$. Dividing this by C_g and using $dx/x = d(\log x)$, we obtain $\sum_i \frac{p_i q_i}{C_g} d(\log q_i) + \sum_i \frac{p_i q_i}{C_g} d(\log p_i) = d(\log C_g)$ or

$$\sum_{i=1}^4 \frac{f_i}{F_g} d(\log q_i) + \sum_{i=1}^4 \frac{f_i}{F_g} d(\log p_i) = d(\log C_g),$$

where $f_i = p_i q_i / C$ is the factor share of i and $F_g = \sum_{i=1}^4 f_i$ is the factor share of the energy group (as before). We write this as

$$(3.1) \quad d(\log Q_g) + d(\log P_g) = d(\log C_g),$$

where

$$(3.2) \quad d(\log Q_g) = \sum_{i=1}^4 \frac{f_i}{F_g} d(\log q_i)$$

is the Divisia volume index of the group, which was introduced in the previous section; and

TABLE 3

VALUE ADDED IN SECONDARY INDUSTRY, FACTOR SHARES AND CONDITIONAL FACTOR SHARES OF ENERGY USED IN SECONDARY INDUSTRY:

W.A., 1968/69-1979/80

Year	Value added (current dollars per capita) C	Factor shares × 100					Conditional factor shares × 100			
		Fuel oil	Gas	Electricity	Other fuels	Total energy	Fuel oil	Gas	Electricity	Other fuels
		f_1	f_2	f_3	f_4	F_g	f_1/F_g	f_2/F_g	f_3/F_g	f_4/F_g
1968/69	354.00	.8920	.1016	3.7301	.0573	4.7810	18.66	2.13	78.02	1.20
1969/70	368.68	1.0262	.1148	3.8209	.0720	5.0339	20.39	2.28	75.90	1.43
1970/71	369.65	.9567	.1189	4.0368	.0591	5.1715	18.50	2.30	78.06	1.14
1971/72	391.83	.9786	.1950	4.3791	.0642	5.6168	17.42	3.47	77.96	1.14
1972/73	481.67	.7505	1.3748	4.1484	.0596	6.3333	11.85	21.71	65.50	.94
1973/74	555.04	.7764	1.2168	3.6824	.0861	5.7617	13.47	21.12	63.91	1.49
1974/75	640.81	1.3212	1.3566	3.9162	.1546	6.7486	19.58	20.10	58.03	2.29
1975/76	733.53	1.4019	1.4164	4.7582	.2258	7.8022	17.97	18.15	60.98	2.89
1976/77	795.99	1.3279	1.3873	4.4465	.1846	7.3462	18.08	18.88	60.53	2.51
1977/78	819.97	1.3843	1.4479	5.5174	.1850	8.5346	16.22	16.96	64.65	2.17
1978/79	848.87	2.2027	1.6319	5.6646	.3098	9.8090	22.46	16.64	57.75	3.16
1979/80	953.63	2.3726	1.6976	5.5978	.3049	9.9728	23.79	17.02	56.13	3.06
Mean	609.47	1.2826	1.0050	4.4749	.1469	6.9093	18.20	13.40	66.45	1.95

The factor share of an energy category is expenditure on the category as a proportion of value added.
The conditional factor share is expenditure as a proportion of total cost of energy used in secondary industry."

TABLE 4

DIVISIA MOMENTS OF ENERGY USED IN SECONDARY INDUSTRY, CPI AND REAL VALUE ADDED LOG-CHANGES:

W.A., 1968/69-1979/80

Year	Divisia indexes of energy used in secondary industry		Divisia variances of energy used in secondary industry		Divisia price-quantity covariance of energy used in secondary industry	Perth-CPI log-change	Real value added log-change
	Volume	Price	Quantity	Price			
	$DQ_g \times 100$	$DP_g \times 100$	$K_g \times 10000$	$\Pi_g \times 10000$	$\Gamma_g \times 10000$	$DP^* \times 100$	$DQ = (DC - DP^*) \times 100$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1969/70	10.58	-1.35	7.24	11.86	3.66	3.63	.43
1970/71	.68	2.28	52.29	3.76	-12.60	4.21	-3.95
1971/72	1.26	12.86	161.61	28.20	-66.97	5.62	.21
1972/73	39.32	-.80	7428.64	287.49	-1450.58	5.32	15.32
1973/74	3.64	1.08	40.11	77.98	-32.96	9.94	4.24
1974/75	2.84	27.54	154.36	531.12	-177.46	16.67	-2.30
1975/76	3.92	24.12	34.45	27.02	.57	13.23	.28
1976/77	-5.56	2.70	28.09	12.19	-15.78	14.60	-6.43
1977/78	2.02	15.97	62.57	4.65	2.80	10.26	-7.29
1978/79	2.61	14.94	26.31	426.97	-64.96	7.79	-4.33
1979/80	4.81	8.48	9.88	24.66	-10.51	8.95	2.69
Mean	6.47	9.80	727.78	130.54	-165.89	9.11	-.10

The year 1969/70, for example, refers to the transition from 1968/69 to 1969/70.

$$(3.3) \quad d(\log P_g) = \sum_{i=1}^4 \frac{f_i}{F_g} d(\log p_i)$$

is the corresponding Divisia price index.

Equation (3.1) decomposes the change in the cost of the energy into a volume and price index. The volume index defined in (3.2) is a weighted average of the four quantity changes $d(\log q_1), \dots, d(\log q_4)$, the weights being the conditional factor shares which are positive and have a unit sum. Similarly, the price index (3.3) is a weighted average of the price changes. The Divisia indexes are thus summary measures in the sense that they are averages.

Equations (3.2) and (3.3) are formulated in terms of infinitesimal changes. To apply them to finite-change data requires two adjustments. First, we replace f_i and F_g with their arithmetic averages over the years $t-1$ and t , $\bar{f}_{it} = \frac{1}{2}(f_{it} + f_{i,t-1})$, $\bar{F}_{gt} = \frac{1}{2}(F_{gt} + F_{g,t-1})$. This is a natural choice when considering the change from $t-1$ to t . Second, we replace $d(\log q_i)$ with its log-change $Dq_{it} = \log q_{it} - \log q_{i,t-1}$; and similarly for the prices. Accordingly, the finite-change Divisia volume and price indexes are

$$(3.4) \quad DQ_{gt} = \sum_{i=1}^4 \frac{\bar{f}_{it}}{\bar{F}_{gt}} Dq_{it},$$

$$(3.5) \quad DP_{gt} = \sum_{i=1}^4 \frac{\bar{f}_{it}}{\bar{F}_{gt}} Dp_{it}.$$

These are given in the first two columns of Table 4. On average the volume of energy per capita used in secondary industry increased by about 6.5 percent p.a., while prices rose by 9.8 percent.

Equations (3.4) and (3.5) are first-order moments of the quantity and price log-changes. The corresponding second-order moments are the Divisia quantity and price variances,

$$(3.6) \quad K_{gt} = \sum_{i=1}^4 \frac{\bar{f}_{it}}{\bar{F}_{gt}} (Dq_{it} - DQ_{gt})^2,$$

$$(3.7) \quad \Pi_{gt} = \sum_{i=1}^4 \frac{\bar{f}_{it}}{\bar{F}_{gt}} (Dp_{it} - DP_{gt})^2.$$

These measure the degree to which the quantities and prices move disproportionately. The Divisia variances are given in columns (3) and (4) of Table 4.

Finally, there is the Divisia price-quantity covariance,

$$(3.8) \quad \Gamma_g = \sum_{i=1}^4 \frac{\bar{f}_{it}}{\bar{F}_{gt}} (Dp_{it} - DP_{gt}) (Dq_{it} - DQ_{gt}),$$

which measures the co-movement of prices and quantities in the energy group. This covariance is given in column (5) of Table 4 and it is negative in 8 out of 11 cases, with the 3 positive covariances being small. The negative covariances reflect the tendency to substitute away from high-cost energy categories, toward lower cost alternatives.

The last two columns of Table 4 contain the CPI and real value added log-changes, which will be used subsequently.

4. ESTIMATES OF THE CONDITIONAL DEMAND EQUATIONS

In this section we present estimates of the finite-change version of the four conditional demand equations

$$(4.1) \quad \bar{f}_{it} Dq_{it} = \theta_i \bar{f}_{gt} DQ_{gt} + \sum_{j=1}^4 \pi_{ij}^g Dp_{jt} + \epsilon_{it}, \quad i=1, \dots, 4,$$

where $\bar{f}_{it} = \frac{1}{2}(f_{it} + f_{i,t-1})$ is the arithmetic average of the i^{th} factor share; $Dq_{it} = \log q_{it} - \log q_{i,t-1}$ is the log-change of q_i ; $\bar{F}_{gt} = \frac{1}{2}(F_{gt} + F_{g,t-1})$ is the arithmetic average of the factor share of the energy group; DQ_{gt} is the Divisia volume index of the group, defined in equation (3.4);

$\Delta p_{jt} = \log p_{jt} - \log p_{j,t-1}$ is the j^{th} price log-change; and ϵ_{it} is a normally distributed disturbance with a zero mean and constant variance. The coefficients of (4.1), the θ_i' 's and π_{ij}^g 's, are taken to be constants; the coefficient θ_i' is the conditional marginal share of i , while π_{ij}^g is the $(i,j)^{\text{th}}$ conditional Slutsky coefficient. Equation (4.1) is a finite-change version of (2.4).

We estimate (4.1) by maximum likelihood subject to the homogeneity and symmetry restrictions (2.5) under the assumption that the disturbances are serially independent, but correlated across equations. The prices and $\bar{P}_{gt}^{DQ_{gt}}$ are taken to be predetermined. Throughout we use Wymer's (1977) maximum likelihood estimation program RESIMUL. Below, we estimate three versions of (4.1), (i) without constant (intercept) terms; (ii) with constants added to each equation to take account of trend-like changes in technology, etc; and (iii) with negativity of the matrix $[\pi_{ij}^g]$ imposed.

No Constants

The estimates with no constant terms in the demand equations are given in Table 5; and the implied elasticities are given in Table 6. As the estimates of the π_{ij}^g 's are symmetry constrained, we give only the upper triangle of $[\pi_{ij}^g]$. The conditional marginal shares for gas and electricity are significant (having t -values greater than 2), while those for oil and other are not. All the π_{ii}^g 's are negative as they should be, but only the oil coefficient is significant. Furthermore, the only significant off-diagonal π_{ij}^g is that for oil-other (π_{14}^g). The characteristic roots of $[\pi_{ij}^g]$ are all nonpositive, which verifies that this matrix is negative semidefinite. The χ^2 value given at the bottom of the table gives the result of a likelihood ratio test of the homogeneity and symmetry restrictions (2.5). The χ^2 value of 13.45 is greater than the critical value at the 5 percent level of 12.59. This indicates that the restrictions should be rejected. All in all, these results are not satisfactory.

TABLE 5

CONDITIONAL DEMAND EQUATIONS FOR ENERGY:

HOMOGENEITY AND SYMMETRY IMPOSED, W.A., 1968/69-1979/80

$$\bar{F}_{it}^{Dq_{it}} = \theta_i^g \bar{F}_{gt}^{DQ_{gt}} + \sum_{j=1}^4 \pi_{ij}^g Dp_{jt} + \varepsilon_{it}$$

(Asymptotic standard errors in parentheses)

Energy category	Conditional marginal share θ_i^g	Conditional Slutsky coefficients				R^2	DW
		π_{i1}^g $\times 100$	π_{i2}^g $\times 100$	π_{i3}^g $\times 100$	π_{i4}^g $\times 100$		
Fuel oil	.010 (.042)	-.198 (.085)	.174 (.148)	-.096 (.157)	.120 (.037)	.20	1.68
Gas	.651 (.126)		-.655 (.499)	.501 (.480)	-.021 (.098)	.96	1.51
Electricity	.337 (.119)			-.438 (.521)	.034 (.079)	.53	1.09
Other	.003 (.024)				-.132 (.091)	.56	2.77

Log-likelihood value = -63.022; $\chi^2 = 13.45$ with 6 d.f.Characteristic roots of $[\pi_{ij}^g] \times 100 = 0, -.065, -.253, -1.106$

TABLE 6

CONDITIONAL DEMAND ELASTICITIES FOR ENERGY

BASED ON TABLE 5 ESTIMATES:

W.A., 1968/69-1979/80

Energy category	Conditional Divisia elasticity $\theta_i^g \bar{F}_{gt} / \bar{F}_{it}$	Conditional price elasticities			
		$\pi_{i1}^g / \bar{F}_{it}$	$\pi_{i2}^g / \bar{F}_{it}$	$\pi_{i3}^g / \bar{F}_{it}$	$\pi_{i4}^g / \bar{F}_{it}$
Fuel oil	.06	-.16	.14	-.08	.10
Gas	4.72	.17	-.65	.50	-.02
Electricity	.51	-.02	.11	-.10	.01
Other	.16	.86	-.15	.24	-.94

Evaluated at sample means.

With Constants

We give in Table 7 the estimates with a constant term added to each equation. As can be seen, there is a significant autonomous trend out of gas into electricity. Now six π_{ij}^g 's are significant. The χ^2 value of 7.75 indicates that the restrictions cannot be rejected. Although these results are a considerable improvement, the problem is that the matrix of conditional Slutsky coefficients $[\pi_{ij}^g]$ has a positive root, so that it is not negative semidefinite.

Imposing Negativity

To estimate the model subject to the constraint that $\pi^g = [\pi_{ij}^g]$ is negative semidefinite, we express this matrix as $\pi^g = -\Phi\Phi'$, where Φ is a lower triangular matrix. We substitute $-\Phi\Phi'$ for π^g in the demand equations and estimate the elements of Φ directly as parameters. Estimates obtained in this way by definition satisfy the negativity restriction (Clements, 1980).

Homogeneity, symmetry and negativity are enforced when

$$\Phi = \begin{bmatrix} \phi_{11} & 0 & 0 & 0 \\ \phi_{21} & \phi_{22} & 0 & 0 \\ \phi_{31} & \phi_{32} & \phi_{33} & 0 \\ -(\phi_{11} + \phi_{21} + \phi_{31}) & -(\phi_{22} + \phi_{32}) & -\phi_{33} & 0 \end{bmatrix}$$

so that the reparametrization of π^g takes the form

$$\pi^g = - \begin{bmatrix} \phi_{11}^2 & \phi_{11}\phi_{21} & \phi_{11}\phi_{31} & -\phi_{11}(\phi_{11} + \phi_{21} + \phi_{31}) \\ & \phi_{21}^2 + \phi_{22}^2 & \phi_{21}\phi_{31} + \phi_{22}\phi_{32} & -\phi_{21}(\phi_{11} + \phi_{21} + \phi_{31}) \\ & & \phi_{31}^2 + \phi_{32}^2 + \phi_{33}^2 & -\phi_{31}(\phi_{11} + \phi_{21} + \phi_{31}) \\ & & & -\phi_{32}(\phi_{22} + \phi_{32}) \\ & & & -\phi_{33}^2 \\ & & & (\phi_{11} + \phi_{21} + \phi_{31})^2 \\ & & & + (\phi_{22} + \phi_{32})^2 \\ & & & + \phi_{33}^2 \end{bmatrix},$$

which satisfies (2.5).

TABLE 7

CONDITIONAL DEMAND EQUATIONS FOR ENERGY WITH CONSTANTS:

HOMOGENEITY AND SYMMETRY IMPOSED, W.A. 1968/69-1979/80

$$\bar{F}_{it} Dq_{it} = \alpha_i + \theta'_i \bar{F}_{gt} DQ_{gt} + \sum_{j=1}^4 \pi_{ij}^g Dp_{jt} + \epsilon_{it}$$

(Asymptotic standard errors in parentheses)

Energy category	Constant α_i $\times 1000$	Conditional marginal share θ'_i	Conditional Slutsky coefficients				R^2	DW
			π_{i1}^g $\times 100$	π_{i2}^g $\times 100$	π_{i3}^g $\times 100$	π_{i4}^g $\times 100$		
Fuel oil	-.321 (.294)	.057 (.038)	-.106 (.090)	.322 (.140)	-.351 (.088)	.135 (.041)	.30	2.09
Gas	-1.873 (.518)	.684 (.080)		-1.123 (.331)	.851 (.215)	-.050 (.103)	.96	2.68
Electricity	2.299 (.289)	.258 (.051)			-.468 (.182)	-.032 (.073)	.80	2.79
Other	-.106 (.121)	.001 (.024)				-.053 (.079)	.52	2.77

Log-likelihood value = -49.399; $\chi^2 = 7.75$ with 6 d.f.Characteristic roots of $[\pi_{ij}^g] \times 100 = .232, 0, -.147, -1.834$

TABLE 8
 CONDITIONAL DEMAND ELASTICITIES FOR ENERGY

BASED ON TABLE 7 ESTIMATES:

W.A., 1968/69-1979/80

Energy category	Conditional Divisia elasticity $\theta'_{i\text{gt}} \bar{F}_{it} / \bar{F}_{it}$	Conditional price elasticities			
		$\pi_{i1}^g / \bar{F}_{it}$	$\pi_{i2}^g / \bar{F}_{it}$	$\pi_{i3}^g / \bar{F}_{it}$	$\pi_{i4}^g / \bar{F}_{it}$
Fuel oil	.32	-.09	.26	-.28	.11
Gas	4.96	.32	-1.11	.84	-.05
Electricity	.39	-.08	.19	-.11	-.01
Other	.05	.96	-.36	-.23	-.38

Evaluated at sample means.

Using this reparametrization, the parameter ϕ_{33} is locally almost non-identified; its asymptotic standard error is very large. To avoid the problem we set $\phi_{33} = 0$. The estimates are given in Table 9. The results are broadly consistent with those of Table 7, where negativity is not imposed. The constant for gas indicates that per capita consumption is falling autonomously at a rate of $2.004/(\bar{f}_{2t} \times 1000) = 2.004 \times 100/(1.665 \times 1000) = 12.04$ percent p.a. in 1979/80; the positive autonomous trend for electricity is $2.137 \times 100/(5.631 \times 1000) = 3.80$ percent p.a. in 1979/80. These are the residual trends after taking out the effects of (i) changes in the overall scale of energy usage as measured by DQ_g and (ii) changes in relative energy prices. The conditional marginal shares for gas and electricity are both highly significant, with the gas share being quite large. Three of the four π_{ii}^g 's are significant, as are four off-diagonal π_{ij}^g 's. The χ^2 value indicates that we are unable to reject the restrictions; and the roots of $[\pi_{ij}^g]$ are all nonpositive, which verifies that this matrix is negative semidefinite.

We give in Table 10 the conditional demand elasticities implied by the Table 9 estimates. The most notable features here are the high Divisia and own-price elasticities for gas. It is also interesting to note that oil and electricity are significant complements; and that the elasticity of gas usage with respect to the price of electricity is quite high at .8, indicating that these two categories are quite substitutable for one another.

It is worth mentioning that we re-estimated the models given in Tables 5, 7 and 9 with the additional constraint that the conditional Divisia elasticity for gas is unity; this restriction is enforced by setting the conditional marginal share for gas equal to the corresponding factor share. In all cases, this restriction was decisively rejected and the results inferior to those reported above.

TABLE 9

CONDITIONAL DEMAND EQUATIONS FOR ENERGY WITH CONSTANTS:

HOMOGENEITY, SYMMETRY AND NEGATIVITY IMPOSED, W.A., 1968/69-1979/80

$$\bar{F}_{it} Dq_{it} = \alpha_i + \theta_i' \bar{F}_{gt} DQ_{gt} + \sum_{j=1}^4 \pi_{ij}^g Dp_{jt} + \epsilon_{it}$$

(Asymptotic standard errors in parentheses)

Energy category	Constant α_i $\times 1000$	Conditional marginal share θ_i'	Conditional Slutsky coefficients				R^2	DW
			π_{i1}^g $\times 100$	π_{i2}^g $\times 100$	π_{i3}^g $\times 100$	π_{i4}^g $\times 100$		
Fuel oil	-.082 (.290)	.059 (.040)	-.232 (.074)	.378 (.140)	-.268 (.086)	.122 (.040)	.28	1.95
Gas	-2.004 (.513)	.673 (.079)		-1.206 (.322)	.840 (.225)	-.013 (.100)	.96	2.59
Electricity	2.137 (.300)	.261 (.054)			-.586 (.194)	.014 (.070)	.77	2.99
Other	-.051 (.116)	.007 (.024)				-.123 (.080)	.57	2.80

Log-likelihood value = -51.844; $\chi^2 = 12.64$ with 7 d.f.Characteristic roots of $[\pi_{ij}^g] \times 100 = 0, -.001, -.226, -1.920$

TABLE 10
 CONDITIONAL DEMAND ELASTICITIES FOR ENERGY

BASED ON TABLE 9 ESTIMATES:

W.A., 1968/69-1979/80

Energy category	Conditional Divisia elasticity $\theta_{i\bar{F}_{gt}}/\bar{F}_{it}$	Conditional price elasticities			
		$\pi_{i1}^g/\bar{F}_{it}$	$\pi_{i2}^g/\bar{F}_{it}$	$\pi_{i3}^g/\bar{F}_{it}$	$\pi_{i4}^g/\bar{F}_{it}$
Fuel oil	.33	-.19	.30	-.21	.10
Gas	4.88	.37	-1.19	.83	-.01
Electricity	.39	-.06	.19	-.13	.00
Other	.37	.87	-.09	.10	-.88

Evaluated at sample means.

The Information Inaccuracy

To measure the fit of the demand equations we use the information inaccuracy (see, e.g., Theil, 1980). Let $f_{1t}/F_{gt}, \dots, f_{4t}/F_{gt}$ be the observed conditional factor shares of the four energy categories in year t and $\hat{f}_{1t}/\hat{F}_{gt}, \dots, \hat{f}_{4t}/\hat{F}_{gt}$ the predicted shares. The conditional information inaccuracy of these predictions is

$$(4.2) \quad I_t = \sum_{i=1}^4 \frac{f_{it}}{F_{gt}} \log \frac{f_{it}/F_{gt}}{\hat{f}_{it}/\hat{F}_{gt}}.$$

When the predictions are perfect ($\hat{f}_{it}/\hat{F}_{gt} = f_{it}/F_{gt}$ for $i=1, \dots, 4$) $I_t = 0$; otherwise $I_t > 0$.

We can also consider the prediction for the i^{th} category separately by dividing the energy group into i and all other within the group, so that the observed and predicted conditional factor shares are

$$\frac{f_{it}}{F_{gt}}, \quad 1 - \frac{f_{it}}{F_{gt}}$$

$$\frac{\hat{f}_{it}}{\hat{F}_{gt}}, \quad 1 - \frac{\hat{f}_{it}}{\hat{F}_{gt}}.$$

The conditional information inaccuracy for the i^{th} category and its complement is thus

$$(4.3) \quad I_{it} = \frac{f_{it}}{F_{gt}} \log \frac{f_{it}/F_{gt}}{\hat{f}_{it}/\hat{F}_{gt}} + \left(1 - \frac{f_{it}}{F_{gt}}\right) \log \frac{1 - f_{it}/F_{gt}}{1 - \hat{f}_{it}/\hat{F}_{gt}}.$$

We shall evaluate (4.2) and (4.3) with the demand equations providing the predicted factor shares. As the model determines the change in the share we write the prediction as

$$(4.4) \quad \frac{\hat{f}_{it}}{\hat{F}_{gt}} = \frac{f_{i,t-1}}{F_{g,t-1}} + \text{prediction of } \Delta \left(\frac{f_{it}}{F_{gt}} \right).$$

To derive an expression for the last term of (4.4) we take the differential of $f_i/F_g = p_i q_i / C_g$ (where $C_g = \sum_{i=1}^4 p_i q_i$) to give $d(f_i/F_g) = (f_i/F_g)d(\log p_i) + (f_i/F_g)d(\log q_i) - (f_i/F_g)d(\log C_g)$. A finite-change version of this is

$$(4.5) \quad \Delta \left(\frac{f_{it}}{F_{gt}} \right) = \frac{\bar{f}_{it}}{\bar{F}_{gt}} Dp_{it} + \frac{\bar{f}_{it}}{\bar{F}_{gt}} Dq_{it} - \frac{\bar{f}_{it}}{\bar{F}_{gt}} DC_{gt}.$$

As the demand model provides the prediction of $\bar{f}_{it} Dq_{it}$, we obtain the prediction of the second term on the right of (4.5) by dividing this by $\bar{F}_{gt}$,

$$(4.6) \quad \text{prediction of } \frac{\bar{f}_{it}}{\bar{F}_{gt}} Dq_{it} = \frac{\bar{f}_{it}}{\bar{F}_{gt}} Dq_{it} - \frac{\hat{\epsilon}_{it}}{\bar{F}_{gt}},$$

where $\hat{\epsilon}_{it}$ is the residual of the i^{th} conditional demand equation. Using (4.6) and recognizing that the first and last terms in (4.5) are given, we obtain

$$(4.7) \quad \begin{aligned} \text{prediction of } \Delta \left(\frac{f_{it}}{F_{gt}} \right) &= \frac{\bar{f}_{it}}{\bar{F}_{gt}} Dp_{it} + \frac{\bar{f}_{it}}{\bar{F}_{gt}} Dq_{it} - \frac{\hat{\epsilon}_{it}}{\bar{F}_{gt}} - \frac{\bar{f}_{it}}{\bar{F}_{gt}} DC_{gt} \\ &= \Delta \left(\frac{f_{it}}{F_{gt}} \right) - \frac{\hat{\epsilon}_{it}}{\bar{F}_{gt}}. \end{aligned}$$

Finally, using (4.7) in (4.4) gives

$$(4.8) \quad \frac{\hat{f}_{it}}{\hat{F}_{gt}} = \frac{f_{it}}{F_{gt}} - \frac{\hat{\epsilon}_{it}}{\bar{F}_{gt}}.$$

We use (4.8) and the demand equations of Table 9 to compute the predicted share. The results are given in Table 11. With the exception of the first two years for fuel oil and gas, the predictions are reasonably close to the actual shares. The results of applying the information inaccuracies (4.2) and (4.3) to these predictions are given in Table 12. The performance of the demand model can be judged by a comparison with no-change extrapolation, $\hat{f}_{it}/\hat{F}_{gt} = f_{i,t-1}/F_{g,t-1}$. Table 13 gives the inaccuracies based on this alternative set of predictions. As can be seen, in most years the demand model yields a lower inaccuracy than does no-change extrapolation. Looking at the mean of the total column, the information inaccuracy for the demand model is about 75 percent below that for the no-change extrapolation.

TABLE 11

ACTUAL AND PREDICTED CONDITIONAL FACTOR SHARES OF ENERGY USED IN SECONDARY INDUSTRY:

W.A., 1968/69-1979/80

Year	Energy category							
	Fuel oil		Gas		Electricity		Other fuels	
	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted
1969/70	20.39	17.88	2.28	6.18	75.90	74.78	1.43	1.16
1970/71	18.50	20.30	2.30	.19	78.06	78.10	1.14	1.42
1971/72	17.42	16.80	3.47	5.11	77.96	76.76	1.14	1.34
1972/73	11.85	12.47	21.71	20.64	65.50	65.81	.94	1.08
1973/74	13.47	12.91	21.12	21.09	63.91	64.77	1.49	1.24
1974/75	19.58	18.07	20.10	21.23	58.03	58.34	2.29	2.36
1975/76	17.97	18.24	18.15	19.39	60.98	60.25	2.89	2.13
1976/77	18.08	19.00	18.88	15.28	60.53	62.67	2.51	3.05
1977/78	16.22	17.20	16.96	17.32	64.65	63.18	2.17	2.30
1978/79	22.46	22.91	16.64	16.43	57.75	57.83	3.16	2.83
1979/80	23.79	23.26	17.02	17.00	56.13	56.42	3.06	3.32
Mean	18.16	18.09	14.42	14.53	65.40	65.36	2.02	2.02

The predictions are based on the demand equations of Table 9. All entries are to be divided by 100.

TABLE 12

CONDITIONAL INFORMATION INACCURACIES FOR ENERGY USED IN SECONDARY INDUSTRY BASED ON DEMAND MODEL:

W.A., 1968/69-1979/80

Year	Energy category				Total
	Fuel oil	Gas	Electricity	Other fuels	
1969/70	2059.7	17057.2	340.4	293.5	18307.0
1970/71	1024.8	36989.9	.5	284.7	37868.5
1971/72	138.4	3098.8	411.3	147.6	3312.9
1972/73	179.6	342.5	21.3	88.8	523.3
1973/74	142.5	.3	162.1	258.3	436.5
1974/75	754.9	385.3	19.9	11.4	939.1
1975/76	24.1	493.5	112.4	1269.2	1703.1
1976/77	283.6	4733.4	968.8	524.3	5063.5
1977/78	342.7	44.2	466.4	40.4	530.5
1978/79	58.5	16.1	1.5	188.7	242.4
1979/80	79.4	.1	16.8	113.7	178.2
Mean	462.6	5741.9	229.2	292.8	6282.3

The demand model is given in Table 9. All entries are to be divided by 10^6 .

TABLE 13

CONDITIONAL INFORMATION INACCURACIES FOR ENERGY USED IN SECONDARY INDUSTRY BASED ON NO-CHANGE EXTRAPOLATION:

W.A., 1968/69-1979/80

Year	Energy category				Total
	Fuel oil	Gas	Electricity	Other fuels	
1969/70	962.2	56.4	1277.6	215.9	1335.2
1970/71	1122.0	.8	1296.9	315.9	1515.9
1971/72	391.2	2651.2	2.6	0	2901.4
1972/73	11887.5	233988.8	40548.8	191.0	236339.2
1973/74	1216.0	102.4	555.5	1396.0	2720.8
1974/75	14311.9	313.9	7345.0	1848.6	16965.6
1975/76	841.3	1212.2	1805.5	752.8	3131.7
1976/77	4.0	177.9	43.9	271.2	429.3
1977/78	1195.2	1236.2	3599.7	253.5	3615.9
1978/79	13071.2	38.4	10149.1	2031.5	16522.7
1979/80	504.4	53.3	535.4	17.0	678.8
Mean	4137.0	21802.9	6105.4	663.0	26014.2

All entries are to be divided by 10^6 .

5. ESTIMATES OF THE COMPOSITE DEMAND EQUATION

In this section we estimate a finite-change version of (2.3), the demand equation for energy as a group. As before, we replace F_g with $\bar{F}_{gt} = \frac{1}{2}(F_{gt} + F_{g,t-1})$ and infinitesimal logarithmic changes with finite log-changes. We also replace $d(\log Q)$ with the real value-added log-change DQ , given in column (7) of Table 4. Numbering inputs such that those pertaining to energy are the first four, the composite demand equation in terms of finite changes is thus

$$(5.1) \quad \bar{F}_{gt} DQ_{gt} = \theta_g DQ_t - \psi \theta_g [DP'_{gt} - DP'_t] + E_{gt},$$

where $DP'_{gt} = \sum_{i=1}^4 \theta_i' DP_{it}$ is the Frisch price index of energy; $DP'_t = \sum_{i=1}^n \theta_i DP_{it}$ is the Frisch price index of all inputs; and E_{gt} is a disturbance with the usual classical properties. To form DP'_{gt} we use the previous estimates of the conditional marginal shares given in Table 9.

To simplify the estimation procedure we combine the non-energy prices in DP' into one price index. To do this we write the overall Frisch price index as

$$(5.2) \quad DP'_t = \sum_{i=1}^n \theta_i DP_{it} = \theta_g \sum_{i=1}^4 \theta_i' DP_{it} + (1 - \theta_g) \sum_{j=5}^n \frac{\theta_j}{\sum_{k=5}^n \theta_k} DP_{jt},$$

where the second step follows from $\theta_i' = \theta_i / \theta_g$ and $1 - \theta_g = 1 - \sum_{i=1}^4 \theta_i = \sum_{i=5}^n \theta_i$ (as $\sum_{i=1}^n \theta_i = 1$). In words, the overall Frisch price index is a weighted average of the Frisch indexes for energy and all other, the weights being the group marginal shares. We eliminate the marginal shares not involving energy by approximating the Frisch index of all other in (5.2), $\sum_{j=5}^n (\theta_j / \sum_{k=5}^n \theta_k) DP_{jt}$, by the change in the consumer price index excluding energy (see Appendix) DP_{ot} . That is

$$(5.3) \quad DP'_t \approx \theta_g DP'_{gt} + (1 - \theta_g) DP_{ot}.$$

Substituting (5.3) in (5.1) we obtain the final estimating equation,

$$(5.4) \quad \bar{F}_{gt} DQ_{gt} = \theta_g DQ_t - \psi \theta_g (1 - \theta_g) [DP'_{gt} - DP_{gt}] + E_{gt}.$$

The variables in (5.4) taken to be predetermined are the prices and DQ . It should be noted that the only random component of $\bar{F}_{gt} DQ_{gt}$ is the disturbance of (5.4) E_{gt} . It thus follows that our previous procedure of treating $\bar{F}_{gt} DQ_{gt}$ as predetermined in the conditional demand equation (4.1) yields consistent estimates provided E_g and ϵ_{it} , the disturbance in (4.1), are independent. Remarkably, Theil's (1975/76, 1980) theory of rational random behavior implies that these disturbances are in fact independent. Accordingly, the estimates of the conditional demand equations are consistent under rational random behavior.

The estimates of the composite equation are given in Table 14. In the first row there is no constant and the estimates of the marginal share and the price coefficient are both significant. At the sample mean, the Divisia and price elasticities are .7 and -.5, respectively.

In the second row of Table 14 a constant has been added to the equation to take account of trend-like changes in technology, etc. This constant term is significantly positive and indicates that per capita energy usage is growing autonomously at a rate of $.3758/(\bar{F}_{gt} \times 100) = .3758/9.8909 = 3.79$ percent p.a. in 1979/80. As can be seen, the estimates of the marginal share and price coefficient change substantially. The estimate of θ_g almost doubles to .0841 while the absolute value of the price coefficient falls substantially. Note, however, that this price coefficient is now not significant. As the choice between the two equations is not clear-cut, in the subsequent computations we use both equations.

To evaluate the fit of the composite demand equation we apply the information inaccuracy to the energy group. To do this we divide the n inputs into two groups, energy and all other, so that (4.2) becomes

TABLE 14

COMPOSITE ENERGY DEMAND EQUATIONS WITH AND WITHOUT A CONSTANT TERM:

W.A., 1968/69-1979/80

$$\bar{F}_{gt} DQ_{gt} = \lambda + \theta_g DQ_{gt} - \psi \theta_g (1 - \theta_g) [DP'_{gt} - DP_{0t}] + E_{gt}$$

(Standard errors in parentheses)

	Constant $\lambda \times 100$	Marginal share θ_g	Price coefficient $-\psi \theta_g (1 - \theta_g)$	R^2	DW	Divisia elasticity $\theta_g / \bar{F}_g$	Price elasticity $-\psi \theta_g / \bar{F}_g$
1. No constant	-	.0496 (.0297)	-.0308 (.0137)	.57	1.91	.72	-.47
2. With constant	.3758 (.1450)	.0841 (.0268)	-.0062 (.0143)	.77	2.68	1.22	-.10

The elasticities are evaluated at the sample mean.

$$(5.5) \quad I_{gt} = F_{gt} \log \frac{F_{gt}}{\hat{F}_{gt}} + (1 - F_{gt}) \log \frac{1 - F_{gt}}{1 - \hat{F}_{gt}},$$

where $\hat{F}_{gt}$ is the predicted energy factor share. Applying a similar argument that led to (4.8), the prediction of F_{gt} from the demand equation (5.4) is

$$(5.6) \quad \hat{F}_{gt} = F_{gt} - \hat{E}_{gt},$$

where $\hat{E}_{gt}$ is the composite demand equation residual. In addition to (5.6), we also use no-change extrapolation $\hat{F}_{gt} = F_{g,t-1}$.

Table 15 gives the actual factor shares of the energy group, as well as three sets of predicted shares. Two are based on the demand equations given in Table 14 (no constant term and with constant) and the third is based on no-change. The results of applying the information inaccuracy (5.5) to these predictions are given in the last three columns of Table 15. As can be seen, in most years both demand models yield substantially lower inaccuracies than does no-change extrapolation. In addition, the with constant demand model gives a lower inaccuracy in most years than does the no constant model.

6. COMBINING THE CONDITIONAL AND COMPOSITE DEMAND EQUATIONS

The conditional demand equation (4.1) depends on total energy usage and the prices of the four energy categories. As stated previously, the conditional Slutsky coefficient π_{ij}^g measures the effect of a change in the price of energy category j on the demand for category i with total energy usage held constant. From the composite demand equation (5.4), this total also depends on the price of j as DP_g' contains Dp_j [see (5.2),

TABLE 15

ACTUAL AND PREDICTED FACTOR SHARES OF ENERGY USED IN SECONDARY INDUSTRY AND INFORMATION INACCURACIES:

W.A., 1968/69-1979/80

Year	Actual energy factor share × 100	Predicted energy factor shares × 100			Information inaccuracies × 10 ⁶		
		Demand model with no constant	Demand model with constant	No-change	Demand model with no constant	Demand model with constant	No-change
1969/70	5.034	4.793	4.979	4.781	62.6	3.2	69.1
1970/71	5.171	5.124	5.217	5.034	2.3	2.1	19.6
1971/72	5.617	5.887	6.007	5.171	66.7	138.0	196.9
1972/73	6.333	5.779	5.856	5.617	274.0	201.9	466.0
1973/74	5.762	6.209	6.366	6.333	176.1	315.8	283.5
1974/75	6.749	6.491	6.760	5.762	54.2	.1	852.6
1975/76	7.802	7.353	7.881	6.749	145.4	4.3	842.5
1976/77	7.346	7.494	7.309	7.802	16.0	1.0	147.2
1977/78	8.535	7.778	8.090	7.346	387.7	130.9	989.8
1978/79	9.809	9.274	9.565	8.535	167.2	34.1	996.5
1979/80	9.973	9.642	10.101	9.809	62.3	9.1	15.1
Mean	7.103	6.893	7.103	6.631	128.6	76.4	443.5

The demand model is given in Table 14.

TABLE 16

UNCONDITIONAL DEMAND EQUATIONS FOR ENERGY:

W.A., 1968/69-1979/80

$$\bar{F}_{it} Dq_{it} = \delta_i + \theta_i DQ_t + \sum_{j=1}^4 \pi_{ij} Dp_{jt} + \pi_{i0} DP_{0t}$$

Energy category	Constant δ_i $\times 1000$	Marginal share θ_i	Slutsky coefficients				
			π_{i1} $\times 100$	π_{i2} $\times 100$	π_{i3} $\times 100$	π_{i4} $\times 100$	π_{i0} $\times 100$
<u>No constant in composite demand equation</u>							
Fuel oil	-.082	.0029	-.243	.256	-.315	.121	.182
Gas	-2.004	.0334		-2.601	.299	-.028	2.073
Electricity	2.137	.0129			-.796	.008	.804
Other	-.051	.0003				-.123	.022
<u>With constant in composite demand equation</u>							
Fuel oil	.140	.0050	-.234	.353	-.278	.122	.037
Gas	.525	.0566		-1.487	.731	-.016	.417
Electricity	3.118	.0220			-.628	.013	.162
Other	-.025	.0006				-.123	.004

Derived from estimates given in Tables 9 and 14.

TABLE 17

UNCONDITIONAL DEMAND ELASTICITIES FOR ENERGY BASED ON TABLE 16 ESTIMATES:

W.A., 1968/69-1979/80

Energy category	Divisia elasticity $\theta_i/\bar{F}_{it}$	Price elasticities				
		$\pi_{i1}/\bar{F}_{it}$	$\pi_{i2}/\bar{F}_{it}$	$\pi_{i3}/\bar{F}_{it}$	$\pi_{i4}/\bar{F}_{it}$	$\pi_{i0}/\bar{F}_{it}$
<u>No constant in composite demand equation</u>						
Fuel oil	.23	-.19	.20	-.25	.10	.15
Gas	3.29	.25	-2.56	.29	-.03	2.04
Electricity	.29	-.07	.07	-.18	.00	.18
Other	.21	.84	-.19	.06	-.86	.15
<u>With constant in composite demand equation</u>						
Fuel oil	.40	-.19	.28	-.22	.10	.03
Gas	5.58	.35	-1.47	.72	-.02	.41
Electricity	.49	-.06	.16	-.14	.00	.04
Other	.42	.85	-.11	.09	-.86	.03

Evaluated at sample means.

and (5.3)]. Thus a change in p_j has a direct effect on the usage of energy category i , via the conditional demand equation, and an indirect effect via the composite demand equation. These two effects can be combined to give the total effect by substituting the right side of (5.4) for $\bar{F}_{gt}^{DQ_{gt}}$ into (4.1). This yields

$$(6.1) \quad \bar{F}_{it}^{DQ_{it}} = \theta_i^{DQ_t} + \sum_{j=1}^4 \pi_{ij}^{DP_{jt}} + \pi_{io}^{DP_o},$$

where

$$(6.2) \quad \theta_i = \theta_g \theta'_i$$

is the unconditional marginal share of i ; and

$$(6.3) \quad \pi_{ij} = \pi_{ij}^g - \psi \theta_g (1 - \theta_g) \theta'_i \theta'_j \quad \pi_{io} = \psi \theta_g (1 - \theta_g) \theta'_i$$

are unconditional Slutsky coefficients. In deriving (6.1) we have set the disturbances at their expected values of zero.

It is to be noted that the π_{ij} 's defined in (6.3) satisfy the homogeneity and symmetry restrictions (2.2). As equation (6.1) does not hold total energy usage constant, it is known as the unconditional demand equation for i . By dividing this equation by $\bar{F}_{it}$, it can be seen that $\theta_i/\bar{F}_{it}$, $\pi_{ij}/\bar{F}_{it}$ and $\pi_{io}/\bar{F}_{it}$ are the unconditional Divisia and price elasticities.

We use the estimates given in Tables 9 and 14 to evaluate (6.2) and (6.3) for $i, j=1, \dots, 4$. The results are given in Table 16. When there is no constant in the composite equation, the constants in the unconditional equations are identical to those of the conditional equations. When the composite equation does contain a constant, the unconditional constants take the form $\delta_i = \alpha_i + \theta'_i \lambda$, where α_i is the conditional constant, θ'_i is the conditional marginal share and λ is the constant from the composite demand equation.

We give in Table 17 the elasticities implied by the Table 16 estimates. The (unconditional) Divisia and own-price elasticities are:

	Divisia		Own-price	
	No constant	With constant	No constant	With constant
Fuel oil	.2	.4	-.19	-.19
Gas	3.3	5.6	-2.56	-1.47
Electricity	.3	.5	-.18	-.14
Other	.2	.4	-.86	-.86

As before, the notable results are the high elasticities for gas. Also, oil and electricity are unconditional complements.

7. CONCLUDING COMMENTS

In this paper we have analysed the demand for energy used in secondary industry using Theil's (1977, 1980) differential factor demand approach. The model assumes that the energy and other inputs are separable in the production function. This leads to a composite demand equation for energy as a group and conditional demand equations for the various energy categories. The composite equation is concerned with the demand for energy as a whole vis-à-vis other inputs, while the conditional demand equations deal with the competition between the energy categories, given the total.

We estimated conditional demand equations for energy usage in Western Australian secondary industry with energy classified into four categories, (i) fuel oil, (ii) gas, (iii) electricity and (iv) other fuels. The preferred model is given in Table 9 and we are unable to reject the homogeneity, symmetry and negativity restrictions. The predictions from this model dominate no-change extrapolation of the energy cost shares.

We also estimated the composite demand equation, which was also shown to give substantially better predictions than no-change extrapolation. Finally, we combined the conditional and composite demand equations to give the unconditional demand responses. The implied Divisia and price elasticities are given in Table 17.

APPENDIX

The Construction of DP_0

As an approximation, we write the log-change in the consumer price index (DP^*) as a weighted average of the n price log-changes,

$$(A1) \quad DP_t^* = \sum_{i=1}^4 \bar{f}_{it} DP_{it} + \sum_{j=5}^n \bar{f}_{jt} DP_{jt} = \bar{F}_{gt} DP_{gt} + (1 - \bar{F}_{gt}) DP_{ot},$$

where $\bar{F}_{gt} = \sum_{i=1}^4 \bar{f}_{it}$; $DP_{gt} = \sum_{i=1}^4 (\bar{f}_{it} / \bar{F}_{gt}) DP_{it}$ is the Divisia price index of energy; and $DP_{ot} = \sum_{i=5}^n [\bar{f}_{it} / (1 - \bar{F}_{gt})] DP_{it}$ is the Divisia price index of all other. We express DP_0 in terms of observables by rearranging (A1) to give

$$DP_{ot} = \frac{1}{1 - \bar{F}_{gt}} (DP_t^* - \bar{F}_{gt} DP_{gt}),$$

which is the equation used to define DP_0 .

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