A SIMPLE PORTFOLIO ALLOCATION MODEL
OF FINANCIAL WEALTH

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Abstract

This paper introduces a simple but highly flexible portfolio allocation model which provides a convenient framework for testing and imposing considerable structure on the data. The model is estimated with Australian data for four financial assets, fixed deposits, savings bank deposits, building society deposits and holdings of a class of government bonds. In contrast to previous findings, interest rate coefficients are estimated precisely and indicate quite a high degree of substitutability.

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1. Introduction

An understanding of the determinants of holdings of financial assets is of crucial importance for a number of macroeconomic issues. The growth of the non-bank financial intermediaries, the substitutability between money and other assets, the determination of interest rates, and so on are all related to the pattern of financial asset holdings. The most popular approach to modeling holdings of financial assets is the multivariate stock adjustment model due to Brainard and Tobin (1968). This has the distinct advantage over single equation approaches that the balance sheet constraints are built in, so that the asset demands sum to total wealth.

The Brainard-Tobin model does, however, suffer from the problem that there are a large number of unconstrained parameters to be estimated, with the result that many applications are characterized by statistical imprecision of a substantial proportion of the estimates.¹ The source of the problem is that interest rates are highly collinear, so that the data cannot determine such a large number of free parameters. In this paper we introduce a model in the Brainard-Tobin tradition in which it is possible to use economic theory to substantially reduce the number of free parameters. A feature of our model is that homogeneity and Slutsky symmetry can be tested and imposed in a very simple manner. In section 2 of the paper we use a simple investor maximization model to derive homogeneity and symmetry restrictions on the asset demand equations. Section 3 gives the estimation results with Australian data. Our results clearly illustrate the gains to be had by using in estimation the relevant restrictions as additional information. Concluding comments are given in section 4.

1. See Evans (1979) for a recent survey in which this conclusion emerges very clearly.

2. The Model

We write A_i for the nominal value of financial asset i ($i=1, \dots, n$), $a_i = A_i/P$ for the corresponding real value ($P = \text{CPI}$), $W = \sum_{i=1}^n a_i$ for total real wealth and r_i for the interest rate on i . These financial assets are assumed to be held by the investor for the dual purposes of

(i) generating income and (ii) satisfying transactions requirements.

Consequently, we postulate that the portfolio is determined to maximize real interest earnings $\sum_i r_i a_i$ subject to a transactions technology in which the transactions (or liquidity) services of the assets are at a level determined by total real wealth. The transactions technology is described by the transformation function

$$(1) \quad f(a_1, \dots, a_n) = g(W),$$

with $\partial f / \partial a_i > 0$; $[\partial^2 f / \partial a_i \partial a_j]$ a symmetric $n \times n$ positive definite matrix; and $g(W) > 0$. The transformation function describes the ease with which one asset can be substituted against another with the overall portfolio generating the same amount of transactions services. The function $g(\cdot)$ gives the desired amount of transactions services corresponding to the level of wealth.

The problem of maximizing interest earnings subject to (1) generates a demand equation for i of the form $a_i = a_i(W, r_1, \dots, r_n)$. In the Appendix we show that this satisfies the homogeneity and Slutsky symmetry constraints familiar from consumption theory.

We write $s_i = a_i/W$ for the i^{th} portfolio share and postulate that the demand for asset i takes the following form:

$$(2) \quad s_i = \alpha_i + \beta_i \log W + \sum_{j=1}^n \pi_{ij} r_j.$$

The first part of (1), $s_i = \alpha_i + \beta_i \log W$, is the Working (1943) - Leser (1963) Engel curve model from consumption theory. This model is very flexible

and robust in consumer demand applications¹ and there is a presumption that it would probably also be satisfactory for asset demand. The coefficient β_i represents 100 times the effect on s_i of a 1 percent increase in wealth, all interest rates remaining constant. Equation (1) is linear in the levels of interest rate variables, which is a specification corresponding closely to the widely used Cagan (1956) form of the demand for money equation. The coefficient π_{ij} gives the effect on s_i of a 1 percentage point increase in r_j (other things equal). Finally, (1) is very similar to Deaton and Muellbauer's (1980) consumer demand model AIDS (the almost ideal demand system), which possesses a number of desirable properties.

Wealth and interest rate elasticities for asset i are

$$\partial \log a_i / \partial \log W = 1 + \beta_i / s_i \quad \partial \log a_i / \partial \log r_j = r_j \pi_{ij} / s_i.$$

Thus an asset with positive (negative) β_i has a wealth elasticity larger (smaller) than unity. The β_i 's will be nonzero when (1) is nonhomothetic.

The balance sheet restrictions can be derived by summing equation (2) over $i=1, \dots, n$. Since the asset shares sum to unity this gives

$$1 = \sum_i \alpha_i + \sum_i \beta_i \log W + \sum_i \sum_j \pi_{ij} r_j.$$

For this equality to hold for all values of the variables on the right, the coefficients must satisfy the following constraints:

$$\sum_i \alpha_i = 1 \quad \sum_i \beta_i = 0 \quad \sum_i \pi_{ij} = 0.$$

With these restrictions, equation (2) satisfies the balance sheet constraint ($\sum_i a_i = W$ or $\sum_i s_i = 1$) globally.

The interest rate coefficients of (2) are subject to the homogeneity and Slutsky symmetry restrictions. Homogeneity means that it is only

1. See, e.g. Clements and Theil (1979), Izan and Clements (1979) and Leser (1963).

changes in relative rates of return which matter.¹ With specification (2), the homogeneity condition is formulated in terms of equal percentage point changes, not equi-proportionate changes.² Homogeneity implies

$$\sum_j \pi_{ij} = 0 \quad i=1, \dots, n.$$

Slutsky symmetry means that interest rate substitution effects are symmetric,³

$$\pi_{ij} = \pi_{ji} \quad i, j=1, \dots, n.$$

These restrictions are linear in the coefficients and can thus be easily tested and then imposed.

3. Results

We use quarterly Australian data from March 1970-June 1979 and distinguish $n=4$ assets, (i) fixed deposits, (ii) total savings bank deposits, (iii) permanent building society deposits and (iv) special bonds plus Australian savings bonds (hereafter, bonds). Financial wealth is defined as the sum of the holdings of these assets. Full definitions, sources and data are given in a separate appendix, available on request.

Initial results suggest that savings bonds are qualitatively different to special bonds. We model this quality change by allowing the wealth coefficients to be different before and after the introduction of savings bonds. This allows the wealth elasticities to change with the introduction of the new asset. Our estimating equation becomes

$$s_i = \alpha_i + \beta_i D \cdot \log W + \beta'_i (1-D) \cdot \log W + \sum_{j=1}^n \pi_{ij} r_j,$$

where $D = 1$ before the introduction of savings bonds in 1976, 0 afterwards.

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1. Perrin (1980) argues that homogeneity is a property of any optimizing portfolio model.
 2. The homogeneity condition in Parkin's (1970) model is also in terms of equal percentage point changes.
 3. For an analysis of the application of symmetry to asset demands, see for example Feige (1964) and Royama and Hamada (1967).

Table 1 gives the FIML estimates for the unrestricted model,¹ together with wealth and interest rate elasticities. For all assets, both before and after the introduction of savings bonds, the wealth elasticities are significantly different from unity. The effects of own interest rates on fixed deposits, deposits with building societies and bonds are all positive and significant. For deposits with savings banks the own interest rate is negative but not significant. Of the 12 off-diagonals, 3 are negative and significant, indicating substitutability. Four off-diagonal elements are positive (indicating complementarity) but only one is significantly so, this being the effect of the interest rate on deposits with savings banks on fixed deposits.

Table 2 presents the results of imposing zero coefficients on those variables (interest rates) whose coefficients in Table 1 were either the wrong sign or insignificant.² This involves setting the following parameters to zero: π_{12} , π_{21} , π_{22} , π_{23} , π_{24} , π_{31} , and π_{34} . A consequence of this is that to satisfy the balance sheet restriction π_{42} remains positive. Testing the appropriateness of these restrictions via a likelihood ratio test leads to their rejection at the 5 percent level (see Table 5).

Table 3 presents the results when homogeneity is imposed. Homogeneity leads to the own interest rate on deposits with savings banks switching to the correct sign but not significantly so. A likelihood ratio test also rejects the homogeneity restrictions (see Table 5).

If for the moment homogeneity is accepted, then Slutsky symmetry can be imposed on the matrix of interest rate coefficients. The data also reject this restriction relative to the totally unrestricted model.

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1. We use Wymer's (1978) RESIMUL program.
 2. On *a priori* considerations diagonal elements should be positive and off-diagonals negative, that is, we expect the four assets to be substitutes.

However, if the model with homogeneity is used as the alternative hypothesis, then symmetry is accepted. Importantly, with symmetry imposed the own interest rate in the equation for savings bank deposits is not only the correct sign but significant. Results are given in Tables 4 and 5.

Whilst the restrictions are formally rejected, it can be argued that the restricted model of Table 4 should be accepted on economic grounds. In the unrestricted model the coefficient on the savings banks' own rate of return has the wrong sign. When homogeneity and symmetry are imposed this parameter is the correct sign and significant. With these restrictions all assets are substitutes, except for some weak evidence that savings bank deposits and bonds are complements. Another reason for accepting the restricted model is that our tests are asymptotic and Laitinen (1978) and Meisner (1979) have shown that such tests are seriously biased toward rejecting the null hypothesis in small samples. Furthermore, all but one of the interest rate coefficients in Table 4 are significant.

Although homogeneity and symmetry lead to a significant reduction in the likelihood value ($\chi^2 = 19.1$), this reduction is less than that when coefficients were set to zero which were the wrong sign and/or insignificant ($\chi^2 = 36.8$). This indicates that the data are more compatible with homogeneity and symmetry than with the popular practice of dropping variables from the demand equations. Accordingly, these restrictions are a better way of improving the precision of estimates.

We should mention that we also estimated a multivariate stock adjustment model. Very few of the adjustment parameters were of the correct sign and significant, while the interest rate coefficients were poorly determined. These results agree with those of Parkin *et al.* (1975) and Purvis (1975) and suggest that the multivariate stock adjustment model is overparameterized.

4. Concluding Comments

In this paper we have developed a simple portfolio model to analyse how the household allocates a given level of financial wealth amongst four assets. Estimation proceeded in a nested fashion, starting with a general model and progressively adding restrictions with the aim of reducing the number of parameters. In contrast to most previous findings, interest rate coefficients are estimated precisely and indicate quite a high degree of substitutability between assets.

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TABLE 1: PARAMETER ESTIMATES FOR UNRESTRICTED MODEL

Equation	Parameters							Elasticities						R^2
	α_i	β_i	β'_i	π_{i1}	π_{i2}	π_{i3}	π_{i4}	W	W'	r_1	r_2	r_3	r_4	
1. Fixed deposits	.3831 (3.1)	-.0348 (2.4)	-.0354 (2.4)	.0121 (4.6)	.0271 (3.3)	-.0114 (4.4)	-.0071 (3.1)	.71	.71	.73	1.02	-.75	-.46	.82
2. Savings Bank Deposits	2.9789 (14.5)	-.2420 (9.9)	-.2449 (10.1)	-.0059 (1.4)	-.0231 (1.7)	.0031 (0.7)	.0054 (1.4)	.62	.61	-.07	-.17	.04	.07	.97
3. Permanent Building Society Deposits	-2.8825 (26.1)	.3277 (25.02)	.3279 (25.2)	-.0002 (0.1)	-.0142 (1.9)	.0125 (5.3)	-.0040 (1.9)	2.85	2.86	-.01	-.37	.56	-.18	.99
4. Special/Australian Savings Bonds	.5205 (3.9)	-.0508 (3.2)	-.0476 (3.04)	-.0060 (2.1)	.0102 (1.1)	-.0041 (1.5)	.0057 (2.3)	.28	.33	-.61	.65	-.46	.63	.83

Asymptotic t-values (derived from the information matrix) are given in parentheses. R^2 is the squared correlation coefficient between the actual and predicted values of the dependent variables s_i .

Elasticities are evaluated at sample means; $\bar{s}_1 = .1210$, $\bar{s}_2 = .6313$, $\bar{s}_3 = .1768$, $\bar{s}_4 = .0710$,

$\bar{W} = \$11249m$, $\bar{r}_1 = 7.27$, $\bar{r}_2 = 4.57$, $\bar{r}_3 = 7.96$, and $\bar{r}_4 = 7.79$. Wealth elasticities before the introduction of savings bonds in 1976 are given in the column headed W, and those after the introduction in the column headed W'.

TABLE 2: PARAMETER ESTIMATES WITH INSIGNIFICANT AND WRONG SIGNED PARAMETERS SET TO ZERO

Equation	Parameters							Elasticities						R ²
	α_i	β_i	β'_i	π_{i1}	π_{i2}	π_{i3}	π_{i4}	W	W'	r ₁	r ₂	r ₃	r ₄	
1. Fixed deposits	.0647 (0.7)	.0106 (0.7)	.0107 (0.7)	.0120 (6.5)	0	-.0086 (4.0)	-.0036 (2.1)	1.09	1.09	.72	0	-.57	-.23	.72
2. Savings Bank Deposits	2.6193 (18.3)	-.3135 (13.9)	-.3171 (14.3)	0	0	0	0	.50	.50	0	0	0	0	.95
3. Permanent Building Society Deposits	-2.0031 (27.2)	.3491 (28.2)	.3494 (28.5)	0	-.0258 (5.5)	.0105 (5.9)	0	2.98	2.98	0	-.67	.47	0	.98
4. Special/ Australian Savings Bonds	.3191 (3.5)	-.0462 (3.0)	-.0430 (2.9)	-.0120 (6.5)	.0258 (5.5)	-.0019 (0.8)	.0036 (2.1)	.35	.39	-1.22	1.66	-.21	.40	.79

See notes to Table 1

TABLE 3: PARAMETER ESTIMATES WITH HOMOGENEITY IMPOSED

Equation	Parameters							Elasticities						R ²
	α_i	β_i	β'_i	π_{i1}	π_{i2}	π_{i3}	π_{i4}	W	W'	r ₁	r ₂	r ₃	r ₄	
1. Fixed Deposits	.1976 (1.6)	-.0072 (0.5)	-.0074 (0.5)	.0176 (7.2)	-.0011 (0.6)	-.0116 (3.8)	-.0049 (1.9)	.94	.94	1.06	-.04	-.76	-.31	.76
2. Savings Bank Deposits	3.1634 (16.2)	-.2695 (12.5)	-.2728 (12.9)	-.0113 (3.0)	.0050 (1.8)	.0032 (0.7)	.0032 (0.8)	.57	.57	-.13	.04	.04	.03	.97
3. Permanent Building Society Deposits	-2.8298 (28.1)	.3198 (28.7)	.3199 (29.2)	-.0017 (0.9)	-.0062 (4.2)	.0125 (5.3)	-.0046 (2.3)	2.81	2.81	-.07	-.16	.57	-.20	.99
4. Special/Australian Savings Bonds	.4688 (3.9)	-.0431 (3.2)	-.0398 (3.0)	-.0045 (1.9)	.0023 (1.3)	-.0042 (1.5)	.0063 (2.6)	.39	.44	-.46	.15	-.47	.69	.83

See notes to Table 1

TABLE 4: PARAMETER ESTIMATES WITH HOMOGENEITY AND SLUTSKY SYMMETRY IMPOSED

Equation	Parameters							Elasticities						R ²
	α_i	β_i	β'_i	π_{i1}	π_{i2}	π_{i3}	π_{i4}	W	W'	r ₁	r ₂	r ₃	r ₄	
1. Fixed Deposits	.2720 (2.0)	-.0169 (1.2)	-.0165 (1.2)	.0135 (8.2)	-.0044 (3.2)	-.0042 (2.8)	-.0049 (3.3)	.86	.86	.81	-.17	-.28	-.32	.73
2. Savings Bank Deposits	3.0722 (15.0)	-.2572 (11.5)	-.2612 (11.8)		.0097 (4.6)	-.0072 (5.3)	.0019 (1.4)	.59	.59	-.05	.07	-.09	.02	.96
3. Permanent Building Society Deposits	-2.8247 (27.6)	.3187 (28.3)	.3189 (28.8)			.0146 (6.3)	-.0032 (2.0)	2.80	2.80	-.17	-.18	.66	-.14	.99
4. Special/Austral- ian Savings Bonds	.4805 (4.1)	-.0446 (3.4)	-.0412 (3.2)				.0062 (3.0)	.37	.42	-.50	.12	-.35	.68	.83

See notes to Table 1

TABLE 5: LIKELIHOOD RATIO TEST STATISTICS

Null Hypothesis	χ^2
Zero Coefficients	36.8 (14.1)
Homogeneity	11.8 (7.8)
Homogeneity and Symmetry	19.1 (12.6)

The test statistic is $-2 \log \lambda = -2(L_r - L_u)$, where L_r is the log-likelihood value under the null hypothesis and L_u is the value for the unrestricted model. Critical values of the χ^2 distribution at the .05 level are given in parentheses.

APPENDIX

We write $a = [a_i]$ and $r = [r_i]$. The first-order conditions to maximize $r'a$ subject to (1) are $r = \lambda \partial f / \partial a$, where λ is a Lagrange multiplier, and (1). Differentiating these conditions with respect to r and W gives

$$(A1) \quad \begin{bmatrix} \lambda F & \partial f / \partial a \\ \partial f / \partial a' & 0 \end{bmatrix} \begin{bmatrix} \partial a / \partial r' & \partial a / \partial W \\ \partial \lambda / \partial r' & \partial \lambda / \partial W \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0' & g' \end{bmatrix}$$

where $F = \partial^2 f / \partial a \partial a'$ (symmetric positive definite); $\partial a / \partial r'$ is a matrix of interest rate derivatives of the demand functions; $\partial a / \partial W$ is a vector of derivatives with respect to wealth; $\partial \lambda / \partial r'$ is a vector of interest rate derivatives of the Lagrange multiplier; $\partial \lambda / \partial W$ is the wealth derivative; and $g' = \partial g / \partial W$. Writing $b = \partial f / \partial a$, (A1) can be solved to give

$$(A2) \quad \frac{\partial a}{\partial r'} = \lambda^{-1} F^{-1} - (b' \lambda^{-1} F^{-1} b)^{-1} \lambda^{-1} F^{-1} b (\lambda^{-1} F^{-1} b)'.$$

In view of the symmetry of the Hessian F , this shows that $\partial a / \partial r'$ is symmetric (Slutsky symmetry). Post-multiplying both sides of (A2) by r and using the first-order condition $r = \lambda b$, we obtain

$$(A3) \quad \frac{\partial a}{\partial r'} r = F^{-1} b - (b' \lambda^{-1} F^{-1} b)^{-1} \lambda^{-1} F^{-1} b (b' \lambda^{-1} F^{-1} b) \lambda = 0.$$

This shows that the demand equations are homogeneous of degree zero in r , implying that an equi-proportionate change in interest rates does not affect the demand for any asset. In the model equi-proportionate changes are approximated by equal percentage point changes.