ON THE DESIRABILITY OF AN
INCOME STABILISATION SCHEME

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ABSTRACT

This paper considers the desirability for producers of a scheme which stabilises incomes by introducing a negative covariance between price and production and which is to be funded by producers. It shows that this desirability depends primarily on the cost of the scheme, the risk aversion of producers and the extent of their initial income instability. Furthermore, "reasonable" parameter values suggest that only an inexpensive scheme will benefit the majority of producers.

INTRODUCTION

The understanding of commodity price stabilisation has been greatly enhanced by the recent contribution of Newbery and Stiglitz (1981), a work which examines comprehensively the case for price stabilisation schemes for agricultural commodities. In doing so it emphasizes that the intention of a price stabilisation scheme should be to stabilise producers' incomes and evaluates such schemes on these grounds. Price stabilisation schemes are shown to have two effects on a producer's income: a "transfer" effect representing the change in mean income and an "efficiency" effect representing the change in income variability. After both theoretical and empirical consideration of these effects: "Our main conclusion is that price stabilisation schemes have limited efficacy in stabilising the real spendable income of producing countries and that most of the other benefits associated with the stabilisation schemes are transfer benefits which, in many cases, seem to benefit the consuming countries at the expense of the producing countries".¹ In considering alternative proposals Newbery and Stiglitz argue that "direct income stabilisation appears very much more attractive" and suggest a scheme in which a regulatory body introduces an inverse relationship between producer prices and production and which is funded by producers.²

The intention of this paper is to examine further the desirability of an income stabilisation scheme. The scheme considered here is one whereby a regulatory body attempts to stabilise the income of producers by introducing a negative covariance between prices and production. Such a negative covariance could be achieved by pricing below the average price in "good" years and above the average price in "bad" years, so that although the individual variation of prices and production would not be altered, the negative correlation of their values would decrease

the overall variation of incomes. However, not only will such a scheme have a cost, but also the size of benefits will depend both on the risk aversion of producers and on the initial extent of income instability. This paper investigates how the parameters of cost, risk aversion and income instability interact in determining the overall desirability of this type of income stabilisation scheme.

The plan of the paper is as follows: Section II outlines the general structure of the income stabilisation scheme and its potential benefits and costs for a risk averse producer. The producer is assumed to have a quadratic utility function as this particular concave function allows a clear characterisation of the relevant costs and benefits, unlike more general forms. Section III introduces a simple numerical example and examines how these factors interact in determining the desirability of the scheme. Section IV concludes the paper.

SECTION II: THE MODEL

A risk averse producer is assumed to have a quadratic utility function of profits (Π) given by:

$$U(\Pi) = a + b.\Pi - d.\Pi^2$$

and to face uncertainty about both price and production:

$$\tilde{\Pi} = \tilde{p} \cdot \tilde{x} - c(y)$$

where:

$\tilde{\Pi}$ = uncertain profit

$\tilde{p}$ = uncertain price ($E(\tilde{p}) = \bar{p}$)

$\tilde{x}$ = uncertain production ($E(\tilde{x}) = y$)

y = planned production

$c(y)$ = cost function.

Prior to the introduction of the income stabilisation scheme, the producer's expected utility ($E_0(U(\tilde{\Pi}))$) is given by:

$$E_0(U(\tilde{\Pi})) = a + b.E(\tilde{\Pi}) - d.E(\tilde{\Pi}^2)$$

It will be assumed that price and production are initially uncorrelated so that:

$$E_0(U(\tilde{\Pi})) = a + b(\bar{p}.y - c(y)) - d [(\text{Var}(\tilde{p}) + \bar{p}^2)(\text{Var}(\tilde{x}) + y^2) - 2\bar{p}.y.c(y) + (c(y))^2] \quad (1)$$

where:

$\text{Var}(\tilde{p})$ = variance of $\tilde{p}$

$\text{Var}(\tilde{x})$ = variance of $\tilde{x}$

After the introduction of the scheme, represented by the appearance of a negative correlation between price and production, expected utility ($E_1(U(\tilde{\Pi}))$) is given by:

$$E_1(U(\tilde{\Pi})) = a + b(\bar{p}.y - k - c(y)) - d [(\text{Var}(\tilde{p}) + \bar{p}^2)(\text{Var}(\tilde{x}) + y^2) + \text{cov}(\tilde{p}^2, \tilde{x}^2) - 2\bar{p}.y.c(y) - 2\text{cov}(\tilde{p}, \tilde{x}).c(y) + (c(y))^2] \quad (2)$$

where:

$$\begin{aligned} k &= \text{the cost of the scheme to the individual producer}^3 \\ \text{cov}(\tilde{p}, \tilde{x}) &= \text{covariance of } \tilde{p} \text{ and } \tilde{x} \\ \text{cov}(\tilde{p}^2, \tilde{x}^2) &= \text{covariance of } \tilde{p}^2 \text{ and } \tilde{x}^2. \end{aligned}$$

If it is assumed that the scheme alters only the relationship between uncertain price and production, thereby leaving the expectation and variance of these parameters unchanged, then expected utility before and after the scheme is introduced can be directly compared by subtracting (1) from (2)^{4,5}:

$$E_1(U(\tilde{\Pi})) - E_0(U(\tilde{\Pi})) = -b.k - d [(\text{cov}(\tilde{p}^2, \tilde{x}^2) - 2\text{cov}(\tilde{p}, \tilde{x}).c(y)] \quad (3)$$

From (3) it can be seen that the desirability of the scheme from the producer's point of view depends on:

- (i) its cost (k)
- (ii) the extent of his risk aversion (represented by d)
- (iii) the extent of income instability (the magnitude of $\text{cov}(\tilde{p}^2, \tilde{x}^2)$ increases with the variance of price and production)

In the next section the interaction of these parameters is considered in the context of a numerical example.

SECTION III: NUMERICAL ANALYSIS

This section develops a simple numerical example of the desirability of an income stabilisation scheme from a producer's point of view.

The following assumptions about the distribution of price are made:

$$\begin{aligned} \bar{p} &= 20 \\ \tilde{p} &= 20 \pm \mu \text{ with probability } \frac{1}{2} \end{aligned}$$

Three cases of price uncertainty are considered:

$$\begin{aligned} \mu &= 1 \text{ so that } \text{Var}(\tilde{p}) = 1 \\ \mu &= 3 \text{ so that } \text{Var}(\tilde{p}) = 9 \\ \mu &= 5 \text{ so that } \text{Var}(\tilde{p}) = 25 \end{aligned}$$

The producer's cost function is assumed to have the form:

$$c(y) = y^2$$

and the uncertainty of production has the form⁶:

$$x = y \pm \varepsilon \text{ with probability } \frac{1}{2}$$

where

$$\varepsilon = 1 \text{ so that } \text{Var}(\tilde{x}) = 1$$

A risk neutral firm would therefore find:

$$E(\tilde{\Pi}) = 20.y - y^2$$

which is maximised for $y = 10$, $E(\tilde{\Pi}) = 100$.

Using:

$$\text{Var}(\tilde{\Pi}) = \bar{p}^2 \cdot \text{Var}(\tilde{x}) + y^2 \cdot \text{Var}(\tilde{p}) + \text{Var}(\tilde{p}) \cdot \text{Var}(\tilde{x})$$

a measure of the extent of income instability associated with each price uncertainty can be found using the coefficient of variation (CV) of the risk neutral firm:

$\text{Var}(\tilde{p}) = 1$	$\text{CV} = 22.4\%$
$\text{Var}(\tilde{p}) = 9$	$\text{CV} = 36.2\%$
$\text{Var}(\tilde{p}) = 25$	$\text{CV} = 54.1\%$

Three alternative degrees of risk aversion are assumed:

$$d = .1, .25 \text{ and } .5$$

Using the definition of relative risk aversion (R):

$$R = - \frac{U''(\Pi) \cdot \Pi}{U'(\Pi)}$$

evaluated at the expected profit associated with the optimal planned production of the risk averse producer, and assuming $\text{Var}(\tilde{p}) = 1$, gives^{7,8}:

$d = .1$	$R = .245$
$d = .25$	$R = .794$
$d = .5$	$R = 1.985$

Given the simple characterisation of the producer's price and production uncertainties, the income stabilisation scheme can be

represented by the regulatory body's coupling of the high price with the low production and the low price with the high production. This gives:

$$\begin{array}{ll} \text{Var}(\tilde{p}) = 1 & \text{cov}(\tilde{p}, \tilde{x}) = -1 \\ \text{Var}(\tilde{p}) = 9 & \text{cov}(\tilde{p}, \tilde{x}) = -3 \\ \text{Var}(\tilde{p}) = 25 & \text{cov}(\tilde{p}, \tilde{x}) = -5 \end{array}$$

Bearing in mind that the benefits from such income stabilisation are given by:

$$-d [\text{cov}(\tilde{p}^2, \tilde{x}^2) - 2 \text{cov}(\tilde{p}, \tilde{x}) \cdot y^2]$$

and that:

$$\text{cov}(\tilde{p}^2, \tilde{x}^2) = E [(\tilde{p}^2 - (\bar{p}^2 + \text{Var}(\tilde{p}))) (\tilde{x}^2 - (y^2 + \text{Var}(\tilde{x})))]$$

the following table of (util) benefits can be calculated:

Util Benefits From Stabilisation

	d = .1 (R = .245)	d = .25 (R = .794)	d = .5 (R = 1.985)
Var($\tilde{p}$) = 1 (CV = 22.4%) (cov($\tilde{p}, \tilde{x}$) = -1)	55.07	110.22	150.36
Var($\tilde{p}$) = 9 (CV = 36.2%) (cov($\tilde{p}, \tilde{x}$) = -3)	163.39	326.63	441.57
Var($\tilde{p}$) = 25 (CV = 54.1%) (cov($\tilde{p}, \tilde{x}$) = -5)	270.04	530.77	703.95

This table of benefits should be compared with the (util) cost of stabilisation:

$$100.k$$

where $k = i$ represents $i\%$ of the expected profit of the risk neutral producer.⁹

In doing so it can be seen that the benefits from income stabilisation are uniformly larger when the initial instability of income is greater or the producer's risk aversion is greater. Furthermore, the magnitude of benefits is such that if the cost of the scheme is about 3% of $E(\tilde{\Pi})$ then only those producers with both typical risk aversion ($R = .794$) or greater and moderate income instability ($CV = 36.2\%$) or greater will benefit from the scheme. Finally, a scheme with a cost of about 5.5% of $E(\tilde{\Pi})$ will only benefit those with extreme levels of both risk aversion and income instability; a scheme which costs about 2.5% of $E(\tilde{\Pi})$ benefits any producer with very unstable income; while a scheme which costs about 1% of $E(\tilde{\Pi})$ benefits all but the least risk averse with the most stable income.

SECTION IV: CONCLUSION

This paper has considered the desirability for producers of a scheme which stabilises income by introducing a negative covariance between price and production and which is funded by the contributions of producers. Using a numerical example it has examined the interaction of the main factors which determine this desirability: the cost of the scheme, the extent of initial income instability and the risk aversion of producers. Although empirical research is needed to estimate the magnitude of the relevant parameters, the main conclusion of the numerical example is that for "reasonable" values of these parameters the majority of producers may well not benefit from such a scheme.

ENDNOTES

1. Newbery and Stiglitz (1981) pp. 39-40.
2. Newbery and Stiglitz (1981) p. 299.
3. k may be interpreted as some fraction or multiple of $\text{cov}(\tilde{p}, \tilde{x})$, i.e. $E(\tilde{p}, \tilde{x}) = \bar{p} \cdot y + \text{cov}(\tilde{p}, \tilde{x})$. If $k < \text{cov}(\tilde{p}, \tilde{x})$ then producers would receive a refund from the regulatory body, whereas if $k > \text{cov}(\tilde{p}, \tilde{x})$ then extra contributions would be required from producers.
4. The assumption of prior independence of price and production, which is maintained throughout the paper, means that the benefits of an income stabilisation scheme are understated to the extent that price and production are initially positively correlated, or overstated to the extent that they are initially negatively correlated.
5. This comparison is made on the basis of a constant level of planned production. Newbery and Stiglitz (1981) examine the conditions under which the sign of the long run impact of a price stabilisation scheme (i.e. with a supply response) will be the same as that of the short run impact.
6. The assumption of additive production uncertainty seems less appropriate than that of multiplicative uncertainty. However, it is used as a simplifying assumption to make the producer's variance of production independent of his planned production. Multiplicative uncertainty would mean that the value of $\text{cov}(\tilde{p}, \tilde{x})$ is positively related to y which, on the basis of (3), suggests that the additive form understates to some extent the benefits from stabilisation for large producers compared with smaller producers.
7. The optimal planned production levels are calculated in the appendix. The value of R is virtually uniform with respect to the various distributions of price uncertainty considered.
8. Newbery and Stiglitz (1981) cite values of R between .5 and 1.2 as being typical of empirical estimates, while values of R greater than 2 are uncommon.
9. $E(\tilde{\Pi}) = 100$ for a risk neutral firm. Although the $E(\tilde{\Pi})$ of a risk averse producer is typically somewhat less than this, the risk neutral producer's $E(\Pi)$ is used as a benchmark.

APPENDIX

Differentiating (1) with respect to y gives the first order condition for optimal planned production:

$$b(\bar{p} - c'(y)) - d [2(\text{Var}(\tilde{p}) + \bar{p}^2)y - \bar{p}.c(y) - \bar{p}.y.c'(y) + 2c'(y).c(y)] = 0$$

Substituting the assumed numerical values from Section III and, in addition, assuming $b = 100$ gives:

$$\text{Var}(\tilde{p}) = 1 \quad 2000 - 200y - 804dy + 60dy^2 = 0$$

$$\text{Var}(\tilde{p}) = 9 \quad 2000 - 200y - 820dy + 60dy^2 = 0$$

$$\text{Var}(\tilde{p}) = 25 \quad 2000 - 200y - 852dy + 60dy^2 = 0$$

Based on these equations the following table of values of optimal planned production can be calculated:

Optimal values of y			
	$d = .1$	$d = .25$	$d = .5$
$\text{Var}(\tilde{p}) = 1$	8.8	6.6	4.2
$\text{Var}(\tilde{p}) = 9$	8.7	6.5	4.1
$\text{Var}(\tilde{p}) = 25$	8.6	6.3	3.9
$R = \frac{2dE(\tilde{\Pi})}{b-2dE(\tilde{\Pi})}$	.245	.794	1.985

REFERENCES

Newbery, D.M.G. and J.E. Stiglitz (1981) The Theory of Commodity Price Stabilisation (Clarendon Press, Oxford).