A CROSS-SECTIONAL ANALYSIS OF
ASSET HOLDINGS

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1. Introduction

While cross-section data on consumption patterns are very common, similar information about asset holdings are a rarity. The objective of this paper is to introduce and analyse data on the pattern of asset holdings derived from a household survey carried out in Australia. These data exhibit substantially more variability than is found in time series data. Consequently, they may contain more information about the determinants of asset holdings than do the more traditional time series data. In Section 2 of the paper we present the data and then use them in Section 3 to estimate a system of asset Engel curves. Sections 4 and 5 use these estimates to (i) analyse the dispersion of the portfolio and (ii) compute the residual correlations, which can be related to the substitutability among the assets. Concluding comments are given in Section 6.

2. Holdings of 7 Assets by 14 Household Groups

Our data are from the Australian Survey of Consumer Finances and Expenditures 1966-68. This was an Australia-wide sample survey of households carried out jointly by Macquarie University and the University of Queensland (Edwards *et al.*, n.d.), which obtained usable responses from 2,757 households. The data on holdings of 7 assets by 14 household groups are given in Table 1. These data are derived from Podder and Kakwani (1976) who adjusted the raw data for non-response bias.¹

TABLE 1
RELATIVE FREQUENCY, VALUE OF ASSETS, ASSET SHARES AND
STANDARD DEVIATION OF ASSET SHARES: AUSTRALIA, 1966-67

Household group h	Relative frequency $N_h(\%)$	Value of assets W_h	Asset shares s_{ih}							Standard deviation of asset shares $\sqrt{V_h} =$ $\sqrt{\sum_{i=1}^7 (s_{ih} - \frac{1}{7})^2 / 7}$
			Own home	Other property	Cash and bank deposits	Shares and securities	Insurance	Super- annuation	Unincorporated business	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	12.37	1.00	40.2	1.7	37.3	0.8	11.6	8.4	0.0	16.0
2	2.32	2.76	29.0	38.2	19.5	0.0	7.7	2.3	3.2	13.8
3	8.63	5.58	64.1	0.4	21.8	0.7	7.0	5.0	1.0	21.5
4	7.83	12.61	74.9	2.2	10.4	1.4	3.9	6.7	0.5	25.0
5	9.39	16.83	82.3	3.3	6.7	0.3	3.4	3.7	0.3	27.9
6	10.70	19.52	84.6	0.9	5.9	0.9	2.7	4.4	0.4	28.8
7	10.55	22.47	80.8	2.1	6.6	0.9	3.4	5.7	0.5	27.2
8	8.34	25.33	80.6	1.7	7.1	0.9	3.1	6.3	0.3	27.2
9	5.98	28.80	77.5	2.8	7.0	1.1	3.2	7.0	1.3	25.9
10	6.20	33.25	73.9	3.4	7.0	2.7	3.2	8.8	0.9	24.5
11	4.32	37.90	67.9	3.1	9.2	4.5	4.6	7.9	2.8	22.0
12	4.61	45.37	58.7	7.8	9.1	4.7	4.3	11.7	3.7	18.3
13	2.79	55.73	49.6	6.5	9.2	8.7	4.6	15.5	5.9	14.8
14	5.59	106.53	30.7	10.4	8.8	18.2	2.9	18.4	10.6	8.4
Mean										
Unweighted	-	-	63.9	6.0	11.8	3.3	4.7	8.0	2.3	21.5
Weighted	-	-	67.0	3.7	12.7	2.5	4.8	7.3	1.5	24.8

The data are derived from Podder and Kakwani (1976, Tables 4 and 5) who divided the 2,757 households in the Australian Survey of Consumer Finances and Expenditures, 1966-68, into 14 groups. Columns 3-10 are group means, with wealth (column 3) normalized so that the poorest household group = 1. Entries in the last 8 columns are to be divided by 100. There are some minor discrepancies in Podder and Kakwani's Table 5 as some of their asset shares do not have a unit sum; accordingly, these have been renormalized so that they have a unit sum. Two assets of Podder and Kakwani, vehicles and appliances, have been omitted because these were only recorded in the survey if they were purchased during the survey year or were being paid off by hire-purchase; the value of assets W_h and the remaining asset shares were adjusted accordingly. The weighted means in the last row use the relative frequencies $N_h/100$ as weights.

Looking at column (3) of Table 1, the wealthiest household has more than 100 times the wealth of the least wealthy. The last row of the table gives the weighted means of the asset shares, with the own home accounting for 67 percent of wealth, other property 4 percent, cash and bank deposits 13 percent, shares and securities 3 percent, insurance 5 percent, superannuation 7 percent and unincorporated business 2 percent. The last column of the table gives the standard deviation of the asset shares $\sqrt{V_h}$. Aside from the first household, $\sqrt{V_h}$ first increases with wealth, reaches a maximum of about 29 percent and then declines; this means that the portfolios of the poor and the rich are less dispersed than that of those in the middle. We shall return to this behavior of the standard deviation in Section 4.

3. Asset Engel Curves

Scatter diagrams of the asset shares (s_{ih}) against the natural logarithm of the wealth ($\log W_h$) indicate that a quadratic would provide a good fit in most cases. Accordingly, let

$$(1) \quad s_{ih} = \alpha_i + \beta_i \log W_h + \gamma_i (\log W_h)^2,$$

$i=1, \dots, 7$ assets; $h=1, \dots, 14$ households.

As $W_h = 1$ for the poorest household, $\alpha_i = s_{ih}$ for that household.

To interpret the β_i 's and γ_i 's, we multiply (1) by W to give

$$a_{ih} = \alpha_i W_h + \beta_i W_h \log W_h + \gamma_i W_h (\log W_h)^2,$$

where $a_{ih} = s_{ih} W_h$ is the value of asset i . We then differentiate this with respect to W ,

$$\begin{aligned}
 (2) \quad \frac{\partial a_{ih}}{\partial w_h} &= \alpha_i + \beta_i (1 + \log w_h) + \gamma_i \log w_h (2 + \log w_h) \\
 &= s_{ih} + \beta_i + 2\gamma_i \log w_h,
 \end{aligned}$$

where the second step is based on (1). Dividing the marginal share $\partial a_i / \partial w$ by the corresponding asset share s_i and using (2) we obtain the wealth elasticity of i ,

$$(3) \quad \eta_{ih} = \frac{\partial(\log a_{ih})}{\partial(\log w_h)} = 1 + \frac{\beta_i + 2\gamma_i \log w_h}{s_{ih}}.$$

Since the data are in grouped form, it is appropriate to use weighted least-squares to estimate (1), the weights being the square roots of the relative frequencies given in column (2) of Table 1. The resulting estimates are given in Table 2. With the exception of those referring to other property, all the β_i 's and γ_i 's are highly significant, which indicates the importance of wealth in determining the pattern of asset holdings. Note that the α_i 's have a unit sum, while those of the β_i 's and γ_i 's are zero; this is a direct consequence of the least-squares procedure. The last column of Table 2 gives the wealth elasticities. Under the usual criterion, cash and bank deposits and insurance are both necessities (wealth elasticities less than unity), the own home has an elasticity of one and the remaining four assets are luxuries. The elasticities for shares and securities and unincorporated business are quite high, but not implausibly so given the nature of these assets. Interestingly, the elasticity for cash and bank deposits being less than unity implies economies of scale in the use of money; this result is implied by the various inventory-based models of money demand (Laidler, 1977).

To measure the goodness-of-fit of the Engel curves we use a concept from information theory, the information inaccuracy (see Theil, 1980, for references). Let $s_{ih}, \dots, s_{7h}$ be the observed asset

TABLE 2

ASSET ENGEL CURVES: AUSTRALIA, 1966-67

$$s_i = \alpha_i + \beta_i \log W + \gamma_i (\log W)^2$$

(Weighted least-squares estimates; standard errors in parentheses)

Asset	α_i × 100	β_i × 100	γ_i × 100	Wealth elasticity η_i
Own home	34.167 (6.969)	38.397 (6.321)	-8.002 (1.447)	.98
Other property	4.798 (4.711)	-2.801 (4.272)	.737 (.978)	1.31
Cash and bank deposits	37.877 (1.860)	-16.448 (1.687)	2.180 (.386)	.69
Shares and securities	1.840 (1.434)	-5.469 (1.300)	1.754 (.298)	2.78
Insurance	11.728 (.559)	-4.398 (.507)	.563 (.116)	.76
Superannuation	8.652 (1.119)	-6.327 (1.015)	1.791 (.232)	1.44
Unincorporated business	.937 (1.081)	-2.954 (.981)	.976 (.225)	1.90

The wealth elasticities are weighted means of (3) with the relative frequencies $N_h/100$ as weights; to evaluate (3) the above estimates of the parameters are used, together with the values of W_h given in Table 1 and the fitted asset shares for s_{ih} .

shares for household h and $\hat{s}_{1h}, \dots, \hat{s}_{7h}$ the predicted shares. The information inaccuracy of these predictions is

$$(4) \quad I_h = \sum_{i=1}^7 s_{ih} \log \frac{s_{ih}}{\hat{s}_{ih}}.$$

When the predictions are perfect ($\hat{s}_{ih} = s_{ih}$ for $i=1, \dots, 7$), $I_h = 0$; otherwise $I_h > 0$. We can also consider the prediction for the i^{th} asset separately by dividing the seven assets into i and all other, so that the observed and predicted asset shares are

$$\begin{aligned} & s_{ih}, 1 - s_{ih} \\ & \hat{s}_{ih}, 1 - \hat{s}_{ih}. \end{aligned}$$

The information inaccuracy for i and its complement is thus

$$(5) \quad I_{ih} = s_{ih} \log \frac{s_{ih}}{\hat{s}_{ih}} + (1 - s_{ih}) \log \frac{1 - s_{ih}}{1 - \hat{s}_{ih}}.$$

Table 3 gives the information inaccuracies with the Engel curves providing the predicted shares. The performance of this model can be judged by a comparison with the following alternative set of predictions. We use as a predictor of s_{ih} the observed asset share of the household whose wealth is immediately below that of household h , $\hat{s}_{ih} = s_{i,h-1}$ ($i=1, \dots, 7$; $h=2, \dots, 14$). We call this no-change extrapolation and the corresponding information inaccuracies are given in Table 4. Comparing the last columns of Tables 3 and 4, it can be seen that for most households the Engel curves yield a lower inaccuracy than does no-change extrapolation. The information inaccuracy based on the Engel curves averaged over all households is about 50 percent lower than that for no-change extrapolation (see the last entries in the last columns of Tables 3 and 4). Similarly, for most assets individually the inaccuracies based on model (1) are substantially lower than that for no-change extrapolation; see the last rows of Tables 3 and 4. The conclusion which emerges is that the fit of the Engel curves is quite satisfactory.

TABLE 3
INFORMATION INACCURACIES BASED ON ENGEL CURVES:
AUSTRALIA, 1966-67

Household Group	Asset							Total
	Own home	Other property	Cash and bank deposits	Shares and securities	Insurance	Superannuation	Unincorporated business	
1	.79	1.38	.01	.38	.00	.00	.93	3.15
2	23.83	74.04	.31	.00	.00	.42	22.77	101.36
3	2.30	1.04	1.39	3.91	.17	.60	5.95	13.43
4	.55	.01	.01	8.77	.01	.74	2.61	12.22
5	.38	.05	.30	.01	.02	.21	.01	.64
6	1.53	1.03	.40	.02	.14	.16	.11	1.89
7	.62	.23	.11	.28	.00	.03	.26	1.03
8	.98	.56	.01	.68	.01	.03	.79	2.28
9	.57	.13	.00	.99	.00	.03	.15	1.32
10	.29	.05	.00	.32	.00	.01	.74	1.08
11	.00	.21	.38	.04	.29	.13	.02	1.00
12	.65	.81	.36	.29	.20	.06	.00	1.87
13	1.88	.10	.34	.02	.30	.48	.11	2.07
14	1.51	.43	.00	.51	.05	.00	.29	2.10
Weighted mean	1.46	2.25	.25	1.28	.07	.19	1.54	5.87

All entries are to be divided by 100. The weighted means use the relative frequencies $N_h/100$ as weights.

TABLE 4
 INFORMATION INACCURACIES BASED ON NO-CHANGE EXTRAPOLATION:
 AUSTRALIA, 1966-67

Household Group	Asset							Total
	Own home	Other property	Cash and bank deposits	Shares and securities	Insurance	Superannuation	Unincorporated business	
1	-	-	-	-	-	-	-	-
2	2.72	90.20	7.47	.80	.83	3.32	22.68	116.45
3	26.36	45.71	.16	3.89	.04	1.22	1.06	58.09
4	2.68	1.97	4.50	.27	.87	.28	.15	8.02
5	1.57	.24	.83	.64	.03	.85	.05	2.87
6	.19	1.26	.05	.39	.08	.06	.02	1.66
7	.52	.59	.04	.00	.09	.18	.01	1.18
8	.00	.04	.02	.00	.01	.03	.05	.15
9	.30	.30	.00	.02	.00	.04	.91	1.22
10	.36	.06	.00	.84	.00	.23	.07	1.25
11	.89	.01	.34	.52	.28	.05	1.30	2.77
12	1.86	2.61	.00	.00	.01	.87	.14	4.09
13	1.68	.12	.00	1.44	.01	.64	.58	3.34
14	7.34	1.07	.01	4.45	.38	.31	1.63	11.23
Weighted mean	3.89	7.59	.74	.97	.17	.47	1.00	11.58

All entries are to be divided by 100. See notes to Table 3.

4. Explaining the Variance of the Asset Shares

In this section we analyse the extent to which variations in wealth across households account for the inverted U-shape of the standard deviation of the asset shares $\sqrt{V_h}$, given in the last column of Table 1. The variance of the shares is defined as

$$(6) \quad V_h = \frac{1}{7} \sum_{i=1}^7 \left(s_{ih} - \frac{1}{7} \right)^2.$$

This is the observed variance and is given in column (2) of Table 5.

The corresponding variance implied by model (1) is

$$(7) \quad \hat{V}_h = \frac{1}{7} \sum_{i=1}^7 \left(\hat{s}_{ih} - \frac{1}{7} \right)^2,$$

where $\hat{s}_{ih}$ is the fitted value of the i^{th} asset share. We call (7) the fitted variance and it is given in column (3) of Table 5. As $\hat{V}_h$ is the variance resulting solely from differences in wealth across households, the difference between V_h and $\hat{V}_h$ is the variance corrected for changes in wealth,

$$(8) \quad R_h = V_h - \hat{V}_h.$$

For brevity we call R_h the wealth-corrected variance and it is given in the last column of Table 5. In Figure 1 we plot the observed and fitted standard deviations against the logarithm of wealth.

As can be seen, for most households the fitted variance is quite close to the observed value. The only major exceptions are household numbers 2 and 14, which are not serious as there are not many observations in these cells (the relative frequencies are 2.3 and 5.6 percent, respectively). One way of measuring the extent to which differences in wealth account for the differences in V_h is by means of weighted sums of squares. Writing $N_h/100$ for the relative frequency

TABLE 5

OBSERVED, FITTED AND WEALTH-CORRECTED VARIANCE OF ASSET SHARES:

AUSTRALIA, 1966-67

Household group h (1)	Variance of asset shares		
	Observed V_h (2)	Fitted $\hat{V}_h$ (3)	Wealth-corrected R_h (4)
1	256.0	201.9	54.1
2	190.4	488.9	-298.5
3	462.3	677.3	-215.0
4	625.0	732.8	-107.8
5	778.4	700.8	77.6
6	829.4	674.1	155.4
7	739.8	642.9	97.0
8	739.8	612.1	127.8
9	670.8	575.2	95.6
10	600.3	529.8	70.4
11	484.0	485.2	-1.2
12	334.9	420.3	-85.4
13	219.0	343.3	-124.2
14	70.6	121.8	-51.3
Weighted sum of squares	357761.0		14594.5

All entries in columns (2)-(4), rows 1-14 are to be divided by 100^2 . The weighted sums of squares are to be divided by 100^4 ; these use the relative frequencies $N_h/100$ as weights.

Observed and fitted weighted
standard deviation of asset shares,
 $\sqrt{V_h \cdot N_h} / 100$, $\sqrt{\hat{V}_h \cdot N_h} / 100 \times 100$

Figure 1

of group h , the expression to be explained is then the weighted sum of the squares of (6), $\sum_{h=1}^{14} (N_h/100) V_h^2$, which is equal to $357,761 \times 100^{-4}$ (see the last row of Table 5). The analogous weighted sum of squares of (8) equals $14,595 \times 100^{-4}$, which shows that the wealth differences among the 14 households account for $1 - 14,595/357,761 = 96$ percent of the observed differences in the variance of the asset shares. Accordingly, model (1) performs rather well in explaining V_h .

5. Asset Substitutability and Residual Correlations

Suppose that wealth of the representative household is held constant and that holdings of asset i increase on account of random factors of one form or another. Suppose also that the same random factors cause holdings of asset j to simultaneously fall. What can be said about the substitutability/complementarity relationship between assets i and j ? As more of asset i has been compensated by less of j , this would seem to indicate that both assets satisfy the same type of want or play the same type of role in the household's portfolio. In other words, these assets are competitive, or substitutes. This line of argument therefore implies that assets whose random components are negatively correlated are substitutes. Conversely, complements have positive correlations.

This idea has a long history in economics, going back to Allen and Bowley (1935). Theil and his associates have elaborated this idea to provide an elegant general theory for any decision-maker under uncertainty, known as rational random behavior; see Theil (1980) for details and references. In this section we use the residuals from the asset Engel curves to compute the correlation matrix and then use its sign pattern to identify substitutes and complements, as well as the strength of the relationships.²

The residual correlation matrix is given in Table 6. A number of interesting results emerge. First, the own home is a substitute for each of the other assets as the off-diagonal correlations involving housing are all negative. This makes sense as housing on average absorbs more than 60 percent of the total portfolio.³ The own home is a strong substitute for (i) other property, (ii) shares and securities and (iii) unincorporated business. Second, other property is a complement with (i) shares and securities and (ii) unincorporated business, unrelated to insurance and a substitute for the remaining three other assets. Third, the last five assets (cash and bank deposits through unincorporated business) form a group whose members are all pairwise complements. This can be understood in terms of the nature of these assets. They are business or business-related assets which tend to satisfy differing roles in business; hence, on average they move in the same direction, making them complements.

6. Concluding Comments

In this paper we have used cross-section data exhibiting substantial variability to analyse the pattern of asset holdings by households. We estimated asset Engel curves which were shown to fit the data well. The Engel curves were then used to explain differences in the dispersion of the portfolio across households and to provide the residual correlation measure of asset substitutability. We found that (i) housing was a substitute for all other assets and (ii) the business-related assets are all pairwise complements.

TABLE 6
RESIDUAL CORRELATION COEFFICIENTS:
AUSTRALIA, 1966-67

	Own home	Other property	Cash and bank deposits	Shares and securities	Insurance	Superannuation	Unincorporated business
Own home	1	-.73	-.42	-.69	-.45	-.46	-.91
Other property		1	-.23	.24	.01	-.12	.57
Cash and bank deposits			1	.39	.83	.68	.37
Shares and securities				1	.10	.49	.88
Insurance					1	.48	.27
Superannuation						1	.39
Unincorporated business							1

The residuals are from the weighted least-squares regressions described in Section 3.

FOOTNOTES

1. For further details and analysis of these data, see Yates (1981).
2. For a previous application of rational random behavior to asset demand, see O'Brien (1974). See also Black (1976, Sec. IIIC).
3. It should be noted that because wealth is held constant, on average the off-diagonal elements in a given row of the matrix must be negative. While this doesn't rule out complementarity, on average the assets must be substitutes.

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