ON THE RELATIONSHIP BETWEEN
EXPLORATION AND EXTRACTION

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Discussion Paper 83.01

February 1983

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ABSTRACT

This paper examines the behaviour of a risk neutral mining firm facing resource stock uncertainty. The particular emphasis of the paper is on the relationship between the firm's separate investment in the activities of exploration and extraction. It is shown that this relationship is an important determinant of the effects of various taxation policies on the firm's overall investment. It is thus argued that the evaluation of such policies should incorporate this relationship in order to be well-formulated.

analysis), further analysis of the relationship between exploration and extraction is needed in a context where the exploration activity has no such clearly desirable implication for extraction investment. It is also felt that, given the enormous lead times involved in mining operations, results concerning the characteristics of the link between exploration and extraction will be strongest when the two activities are the most temporally proximate.

Having determined to analyse a more intricate aspect of the relationship between exploration and extraction, the problem arises, however, of getting tangible results. This difficulty has necessitated the assumption of a specific functional form for the exploration activity, from which fairly well-defined results can be derived. While some attempt has been made to justify the particular form chosen, policy conclusions must unavoidably be tentative. Since the relevant policy issues include those of exploration incentives and a Resource Rental Tax, further research in this area seems an important priority.

The plan of the paper is as follows. In section II a simple model is set up of a risk neutral mining company making a joint decision on planned exploration and extraction. It is shown that the firm's optimal levels of exploration and extraction depend on the interaction of its expected profit margins with the characteristics of the resource stock uncertainty that it faces. When expected profit margins are relatively large, uncertainty about the resource stock plays a minor role in determining the levels of exploration and extraction; while uncertainty is the dominant factor when expected profit margins are relatively small.

Section III considers the comparative statics of the model with respect to certain policy issues. It is shown that while tax

II THE MODEL

The objective of the firm is assumed to be that of maximising the discounted expected profit made available by exploring for and subsequently extracting an uncertain quantity of resource from a given prior-leased parcel of land. The firm is also assumed to have a prior distribution of resource size which is sufficiently promising to have enticed it to acquire the appropriate lease. The assumption of risk neutrality is made partly to simplify the analysis and partly to abstract from any discussion of the risk-sharing which has been such a prominent feature of resource taxation issues.² This in turn serves to clarify the link between the firm's exploration and its extraction investment.

Formally, the firm's problem can be represented as³:

$$(1) \quad \text{Max}_{X,M} \int_0^{\infty} E(T_a) E(p_t) \cdot M \cdot e^{-rt} - c \cdot M - d \cdot X$$

where:

X = level of exploration investment

M = level of extraction investment and extraction rate
per period⁴

$E(p_t)$ = expected price for sale of the resource in period t

r = rate of discount

c = per unit cost of a unit of extraction investment

d = per unit cost of a unit of exploration investment⁵

2. See Leland (1978), Hyde and Markusen (1982), Emerson and Lloyd (1981), Garnaut and Clunies-Ross (1975), Dowell (1978), Swan (1976), Emerson (1982), Mayo (1979)

3. This model is based on a simpler one used in Fraser (1982).

4. See Campbell (1980) for a justification of this full-capacity assumption.

5. It is assumed that fixed costs are sufficiently small to allow interior solutions for optimal M and X .

$\frac{1}{K}$ = probability of a particular resource size being the actual size⁷

The assumption of a uniform probability distribution may be seen as less restrictive once it is appreciated that much "discovery" exploration is only undertaken to the extent that determines a particular deposit to be commercially viable. For example, no information about the depth of, say, a uranium deposit may exist other than that it is of at least sufficient depth to justify further investment. In these cases a uniform probability distribution would be an appropriate prior. Notice also that:

$$\frac{\partial f(S|X)}{\partial X} = \frac{\alpha X^{\alpha-1}}{K} > 0$$

$$\text{and } \frac{\partial^2 f(S|X)}{\partial X^2} = \frac{\alpha(\alpha-1) X^{\alpha-2}}{K} < 0$$

Diminishing returns to exploration have been assumed in terms of its uncertainty-reducing feature. This assumption implies that there always will be some residual resource stock uncertainty.

Incorporating this specification of exploration into equation (3) gives:

$$(4) \quad \max_{M, X} E(p_o) \cdot M \left[\int_{\bar{S} - \frac{K}{2X^\alpha}}^{\frac{MT_e}{M} \cdot \frac{X^\alpha}{K}} \frac{S}{M} \cdot \frac{X^\alpha}{K} \cdot dS + \int_{\frac{MT_e}{M} \cdot \frac{X^\alpha}{K}}^{\bar{S} + \frac{K}{2X^\alpha}} T_e \cdot \frac{X^\alpha}{K} \cdot dS \right] - c \cdot M - d \cdot X$$

Equation (4) can be integrated to give:

7. $X = 1$ may be interpreted as no exploration in the sense that only one hole is drilled - the mine itself.

the level of exploration activity is at a maximum when:

$$\frac{c}{E(p_o) \cdot T_e} = \frac{1}{2} \quad (\text{where } M.T_e = \bar{S})$$

It is smaller when expected profit margins are smaller because of the cost restrictions these smaller margins place on the level of exploration activity. Conversely, it is smaller when expected profit margins are larger because the expected benefits from reducing uncertainty that exploration provides seem less important when extraction investment is so profitable. More exploration serves to reduce the probability of seriously over-investing in extraction machinery, which in turn raises the expected actual time period of extraction. This feature of exploration activity is, however, only appreciated by the firm when the cost of over-investing seems more important than the cost of forgone revenue from unextracted resource. With expected profit margins at relatively high levels, exploration activity seems unnecessary and is, to some extent, phased out.

From the foregoing discussion it can be seen that the determination of the firm's optimal levels of exploration and extraction simply involves an appreciation of the relative importance to the firm of the level of uncertainty it faces and the level of profit margins it expects to receive. The role of these two factors will be seen again in the next section which considers the firm's exploration and extraction investment responses to various exogenous changes.

(6) and (9), it can be checked that:

$$\frac{\delta M}{\delta c} = - \frac{K}{X^\alpha E(p_o) \cdot T_e^2} - \left(\frac{1}{2} - \frac{c}{E(p_o) \cdot T_e} \right) \frac{\alpha K}{X^{\alpha+1} \cdot T_e} \cdot \frac{\delta X}{\delta c} < 0$$

That is, the extraction incentive is an incentive to extraction investment regardless of the level of expected profit margins.

(ii) Tax incentives to exploration investment

Similar to the extraction case, it will be assumed that a tax incentive to exploration investment decreases the per unit cost of exploration activity (d). Taking the derivative of equation (8) with respect to d gives:

$$(10) \frac{\delta X^{\alpha+1}}{\delta d} = - \frac{E(p_o) \cdot \alpha K}{2d^2} \left(\frac{1}{4} - \left(\frac{c}{E(p_o) \cdot T_e} - \frac{1}{2} \right)^2 \right) < 0$$

Unlike the extraction incentive, the exploration incentive is an incentive to exploration activity regardless of the level of expected profit margins. This monotonicity does not hold, however, in the case of extraction investment. Combining (10) and (6) gives:

$$\frac{\delta M}{\delta c} = - \left(\frac{1}{2} - \frac{c}{E(p_o) \cdot T_e} \right) \frac{\alpha K}{X^{\alpha+1} \cdot T_e} \cdot \frac{\delta X}{\delta d}$$

From which it follows that:

$$\frac{\delta M}{\delta d} \begin{matrix} < \\ \equiv \\ > \end{matrix} 0 \quad \text{as} \quad \frac{c}{E(p_o) \cdot T_e} \begin{matrix} > \\ \equiv \\ < \end{matrix} \frac{1}{2}$$

The tax incentive to exploration only gives additional incentive to extraction investment if expected profit margins are relatively

$$\begin{aligned}
 (6') \quad \frac{X^\alpha}{K} (\bar{S} - M \cdot T_e) &= \frac{c (1 - t)}{E(p_o) \cdot T_e (1 - t)} - \frac{1}{2} \\
 &= \frac{c}{E(p_o) \cdot T_e} - \frac{1}{2}
 \end{aligned}$$

where: t = rate of resource rent tax.

If, however, the level of exploration is not fixed, then equation (8) must also be recalled:

$$\begin{aligned}
 (8') \quad X^\alpha + 1 &= \frac{E(p_o)(1 - t) \cdot \alpha K}{2d} \left(\frac{1}{4} - \left(\frac{c (1 - t)}{E(p_o) \cdot T_e (1 - t)} - \frac{1}{2} \right)^2 \right) \\
 &= \frac{E(p_o)(1 - t) \cdot \alpha K}{2d} \left(\frac{1}{4} - \left(\frac{c}{E(p_o) \cdot T_e} - \frac{1}{2} \right)^2 \right)
 \end{aligned}$$

Consistent with the findings of Campbell and Lindner (1982), it can be seen that the optimal level of exploration is reduced by a Resource Rental Tax which does not include exploration investment in its full loss offset provision.

More to the point, however, combining (6') and (8') shows that:

$$\frac{\delta M}{\delta t} = - \left(\frac{1}{2} - \frac{c}{E(p_o) \cdot T_e} \right) \frac{\alpha K}{X^\alpha + 1 \cdot T_e} \cdot \frac{\delta X}{\delta t}$$

From which it follows that:

$$\frac{\delta M}{\delta t} \begin{matrix} < \\ > \end{matrix} 0 \quad \text{as} \quad \frac{c}{E(p_o) \cdot T_e} \begin{matrix} > \\ < \end{matrix} \frac{1}{2}$$

Not only is the tax *not* neutral with respect to extraction when the level of exploration can be chosen optimally, but the direction of its bias depends on the expected level of profit margins.

By considering the effects of a Resource Rental Tax in a framework which permits analysis of the interdependence of the exploration and

$$(12) \quad \frac{\delta M}{\delta E(p_o)} = \frac{K}{X^\alpha} \cdot \frac{c}{(E(p_o) \cdot T_e)^2} - \left(\frac{1}{2} - \frac{c}{E(p_o) \cdot T_e}\right) \frac{\alpha K}{X^\alpha + 1 \cdot T_e} \cdot \frac{\delta X}{\delta E(p_o)}$$

From (12) it can be seen both that:

$$\frac{\delta M}{\delta E(p_o)} > 0 \quad \text{if} \quad \frac{c}{E(p_o) \cdot T_e} > \frac{1}{2}$$

and that the response of extraction investment in the situation where:

$$\frac{c}{E(p_o) \cdot T_e} < \frac{1}{2}$$

cannot be determined. Although a Royalty Tax, through its effect on expected revenues, is unambiguously a disincentive to exploration, because of the interdependence of exploration and extraction the same cannot be said for its effect on extraction investment. Consider in particular the situation where the firm is encouraged by relatively large expected profit margins to invest adventurously in extraction machinery ($M \cdot T_e > \bar{S}$). In this case the tax-induced decrease in exploration simultaneously implies an increase in the probability of a very large resource deposit. Therefore, although the tax must be to some extent a disincentive to extraction by its effect on expected profit margins, it is also to some extent an incentive as a result of the indirect effect on the likelihood of a very large deposit. Consequently the possibility exists for the latter effect to dominate and extraction investment to rise following the introduction of a Royalty Tax.

This example shows once again that understanding the effects of a particular taxation policy requires an appreciation of the interdependence of the exploration and extraction activities.

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I INTRODUCTION

Two of the principal issues in resource economics are those of the determinants of exploration and extraction investment. Only recently, however, have economists looked in more detail at the relationship between these two types of investment.¹

The major difficulty in this context is in specifying the relationship between exploration and extraction. In Peterson's analysis, for example, exploration acts as a positive stimulus to extraction by lowering the costs of extraction. While in Hyde and Markusen's analysis, each unit of exploration increases the probability of finding some resource deposit on a given leased parcel of land. Higher planned exploration by a firm is thus associated with a higher planned (prior to exploration) level of extraction investment because the higher planned level of exploration increases the expected level of resource stock which is available for extraction.

The approach of this paper is to focus on exploration as an uncertainty-reducing activity, as distinct from any effect it may have on expected values. In particular it is assumed that the exploration activity serves only to reduce the variance of a prior distribution of resource size for a deposit which has already been located. This sort of exploration activity may be interpreted as the "defining" or "proving" of a previously discovered resource deposit rather than as the discovery activity itself. It is felt that while it seems intuitively plausible for an exploration activity which increases the expected resource stock to be a positive stimulus to the level of planned extraction investment (as exemplified by the Hyde and Markusen

1. See Peterson (1977), Emerson and Lloyd (1981), Hyde and Markusen (1982) and Campbell and Lindner (1982).

incentives which are specific to either exploration or extraction will generally stimulate that particular activity; they also affect the non-tax-specific activity. Furthermore, the direction of this influence depends on other characteristics of the firm's situation, particularly its expected profit margins. This type of spill-over effect appears again in the case of a Resource Rental Tax where the "neutrality" feature of this tax is shown to be invalid when the exploration and extraction activities are considered jointly. Finally, it is shown that, because of its effect on the level of exploration activity, a reduction in Royalty levels may not always stimulate investment in extraction.

Section IV concludes the paper. It is argued that although this analysis of the relationship between exploration and extraction is both limited in scope and specific in form, it nonetheless raises considerable doubts about the validity of the continued separate treatment of these two activities, particularly with respect to taxation issues.

and $E(T_a)$ = expected actual time period of extraction

$$(2) \quad = \int_0^{MT_e} \frac{S}{M} f(S|X) dS + \int_{MT_e}^{\infty} T_e f(S|X) dS$$

where:

T_e = length of lease⁶

S = actual size of resource

$f(S|X)$ = probability of a resource stock of size S given a
level of exploration X .

Assuming that the expected price for sale of the resource is independent of the resource stock uncertainty, and that it rises at the rate of discount:

$E(p_t) = E(p_0) \cdot e^{rt}$ where $E(p_0)$ is the expected price for the next period, equation (1) can be simplified to give:

$$(3) \quad \text{Max}_{M, X} \quad E(p_0) \cdot M \cdot E(T_a) - c \cdot M - d \cdot X.$$

In order to introduce more specifically the role of exploration investment, assume that this activity has the following functional form:

$$f(S|X) = \frac{X^\alpha}{K} \quad \text{where } \alpha < 1 \\ X \geq 1$$

When no exploration activity is undertaken, the probability distribution of resource size is uniform over the range:

$$\bar{S} - \frac{1}{2}K \leq S \leq \bar{S} + \frac{1}{2}K$$

where:

$\bar{S}$ = expected size of resource deposit

K = extent of spread of probable resource sizes

6. T_e may be interpreted alternatively as the lifetime of extraction machinery.

$$(5) \quad \text{Max}_{M,X} E(p_o) \cdot M \left[\frac{1}{2} T_e + \frac{\bar{S}}{2M} - \frac{X^\alpha}{K} \frac{(\bar{S} - MT_e)^2}{2M} - \frac{1}{8M} \cdot \frac{K}{X^\alpha} \right] \\ - c \cdot M - d \cdot X$$

Taking the derivative of (5) with respect to M gives:

$$(6) \quad \frac{X^\alpha}{K} (\bar{S} - MT_e) = \frac{c}{E(p_o) \cdot T_e} - \frac{1}{2}$$

From (6) it can be seen that, independent of the level of exploration undertaken:

$$MT_e \begin{matrix} \geq \\ < \end{matrix} \bar{S} \quad \text{as} \quad \frac{c}{E(p_o) \cdot T_e} \begin{matrix} \leq \\ > \end{matrix} \frac{1}{2}$$

If the expected return to a unit of extraction investment over the duration of the lease is more than double its cost, then the firm will plan to extract more than the expected level of resource. Expected profitability is so high that the firm is prepared to gamble on the actual deposit being relatively large. Smaller expected profit margins lead, however, to a more cautious extraction investment level ($M \cdot T_e < \bar{S}$).

Taking the derivative of (5) with respect to X gives:

$$(7) \quad \frac{E(p_o)}{8} \cdot \frac{\alpha K}{X^\alpha + 1} - \frac{E(p_o)}{2} \cdot \frac{\alpha X^{\alpha-1}}{K} (\bar{S} - MT_e)^2 = d$$

Substituting equation (6) into (7) and rearranging gives⁸:

$$(8) \quad X^\alpha + 1 = \frac{E(p_o) \cdot \alpha K}{2d} \left[\frac{1}{4} - \left(\frac{c}{E(p_o) \cdot T_e} - \frac{1}{2} \right)^2 \right]$$

From equation (8) two things should be noted. Firstly, the analytical solution to the optimal level of M can be found by solving (8) for the optimal level of X and substituting this into equation (6). Secondly,

8. From (6) and (8) it can be checked that the second order conditions are satisfied.

III COMPARATIVE STATICS

This section examines the effect on the firm's optimal levels of exploration and extraction of changes in its parameters. All the changes considered relate to various resource taxation issues. The intention here is not so much to provide policy conclusions on resource taxation questions, as to emphasise the interdependence of exploration and extraction in determining the investment effects of different types of resource taxation. The taxation issues considered are:

- (i) Tax incentives to extraction investment
- (ii) Tax incentives to exploration investment
- (iii) A "neutral" Resource Rental Tax
- (iv) A Royalty Tax
- (i) Tax incentives to extraction investment

A tax incentive to extraction investment is assumed to decrease the per unit cost of extraction machinery (c). It remains to consider the effect of such a decrease on the optimal levels of exploration and extraction. From equation (8) it can be seen that:

$$\frac{\delta X^{\alpha+1}}{\delta c} = -2 \left(\frac{c}{E(p_o) \cdot T_e} - \frac{1}{2} \right) \cdot \frac{\alpha K}{2d \cdot T_e}$$

It follows that:

$$(9) \quad \frac{\delta X^{\alpha+1}}{\delta c} \begin{matrix} < \\ > \end{matrix} 0 \quad \text{as} \quad \frac{c}{E(p_o) \cdot T_e} \begin{matrix} > \\ < \end{matrix} \frac{1}{2}$$

From equation (9) it is clear that a taxation incentive to extraction investment is also an incentive to exploration investment if expected profit margins are relatively low. When these margins are relatively high, however, increasing them further means that the uncertainty-reducing benefits of exploration are even less appreciated, and the extraction incentive becomes an exploration disincentive. Combining

low. As has already been seen, when these margins are relatively high additional exploration serves principally to reduce the likelihood of a very large deposit, which in turn discourages adventurous levels of extraction investment. It follows that for this case a tax incentive to exploration must be a disincentive to extraction.

A consideration of the firm's responses to (i) and (ii) shows in particular the interdependence of the exploration and extraction activities with respect to tax incentive issues. Although specific incentives to exploration and extraction are seen to have the desired effects, these specific incentives are shown also to have spill-over effects on the other activity. Furthermore, the direction of the spill-over effects is shown to depend once again on the relative importance to the firm of the resource uncertainty it faces and the profit margins it expects to receive.

(iii) A "neutral" Resource Rent Tax.

The version of a Resource Rent Tax used here is "neutral" in the sense that it includes a full loss offset provision. This means that every possible profit outcome from extracting a resource deposit is reduced by a percentage equal to the tax rate. Since all magnitudes are diminished by the same proportion, such a tax is considered not to affect a risk neutral firm's decision with respect to extraction investment.⁹

Recalling equation (6) and assuming a fixed level of exploration, it can be seen that the addition of such a tax *is* neutral in the sense that it has no effect on the optimal level of extraction investment:

9. This sense of neutrality is the same as that used by Mayo (1979) and Campbell and Lindner (1982).

extraction activities, it has been possible to show that a supposedly-neutral tax on extraction profits affects the level of exploration undertaken by a risk-neutral firm. Moreover, because of the interdependence of these activities, the tax has been shown to be distortionary even with respect to the level of extraction investment. Only if the full loss offset provision in the tax includes exploration investment, or if the firm has no choice over its level of exploration is the neutrality feature preserved.

(iv) A Royalty Tax

A Royalty Tax is one designed to collect some proportion of the revenues a firm receives from sale of extracted resource. As a result of its effect on expected revenues, such a tax is generally thought to bias downwards the level of extraction investment.

For analytical purposes it is assumed here that the introduction of the tax is perceived as a decrease in the expected price from resource sales. Recalling equation (8), it follows that:

$$\begin{aligned} \frac{\delta X^{\alpha+1}}{\delta E(p_o)} &= \frac{\alpha K}{2d} \left(\frac{1}{4} - \left(\frac{c}{E(p_o) \cdot T_e} - \frac{1}{2} \right)^2 \right) \\ &+ \frac{\alpha K}{d} \left(\frac{c}{E(p_o) \cdot T_e} - \frac{1}{2} \right) \frac{c}{E(p_o) \cdot T_e} \end{aligned}$$

which may be simplified to give:

$$(11) \quad \frac{\delta X^{\alpha+1}}{\delta E(p_o)} = \frac{\alpha K}{2d} \cdot \frac{c^2}{(E(p_o) \cdot T_e)^2} > 0$$

The introduction of a Royalty Tax will decrease the level of exploration activity that the firm undertakes. Furthermore, recalling equation (6) and making use of (11), it follows that:

IV CONCLUSION

An aim of this paper has been to examine both the relationship between the activities of exploration and extraction and the implications of the relationship for several resource taxation issues. It has been shown that even in the case of a risk neutral firm these activities are linked in a complicated manner. Furthermore, by taking the features of this relationship into account, it has been shown that new and conflicting intuitions can be derived concerning the investment effects of various tax policies.

A further aim, however, has been not to draw policy conclusions from the analysis, particularly in view of the specific forms used. Rather, the intention is to emphasise the interdependence of the exploration and extraction activities. Only once this interdependence has been adequately incorporated into resource policy discussions can such policies be well-formulated.