THE DEMAND FOR ENERGY USED IN TRANSPORT

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1. INTRODUCTION

The objective of this paper is to use recent data to measure the role of income and relative prices in determining the demand for four types of energy used in transport in the state of Western Australia. Information about the responsiveness of demand to income and relative prices has important implications for pricing policy, energy conservation and demand forecasting. In a subsequent paper we will analyse the demand for energy used in the non-transport sectors.

Section 2 of the paper presents the data on consumption, prices and income and introduces several Divisia index numbers which provide convenient summary measures. In Section 3 we develop a system-wide demand model which relates consumption to income and relative prices. Here energy demand is formulated as a two-step process:

(i) The demand for total energy, which is concerned with the substitution between energy and all other goods; and (ii) the demand for the various types of energy, given total energy demand. Section 4 discusses the econometric estimation of the model and the empirical results are given in Section 5. Section 6 contains some concluding comments.

2. THE ENERGY DATA

Our data are annual for the period 1966/67 to 1979/80 and refer to energy used in transport in WA classified into four categories: (i) Motor spirit, (ii) automotive diesel oil (ADO) used in land transport, (iii) bunker fuel and (iv) aviation fuel. Land transport ADO comprises of distillate used for (a) passenger road transport, (b) road goods transport and (c) rail and farm activities. For full details of these and all other data used in this paper, see Goldschmidt (1982).

Table 1 gives the basic quantity/price data. Here q_i is the per capita quantity consumed of energy type i ($i=1, \dots, 4$) and p_i is its price. For the year ended June 1980, per capita consumption is 38.8 GJ of motor spirit, 23.6 of land transport ADO, 18.8 of bunker fuel and 7.5 of aviation fuel. For the same year, the price is \$9.04 for 1 GJ of motor spirit, \$7.86 for land transport ADO, \$5.58 for bunker fuel and \$8.58 for aviation fuel. In Table 2 these data are converted to log-changes. Here $Dq_{it} = \log q_{it} - \log q_{i,t-1}$ and $Dp_{it} = \log p_{it} - \log p_{i,t-1}$ are the i^{th} quantity and price log-changes, respectively. Log-changes are approximately percentage changes when multiplied by 100. As can be seen, on average per capita consumption of motor spirit grew by about 2.9 percent p.a., land transport ADO by 6.4 percent, bunker fuel fell by 2.2 percent and aviation fuel rose by 6.1 percent. The average annual price increases are 9.4, 12.8, 15.5 and 10.5 percent (in the same order as before).

Table 3 gives per capita household income in WA (M) and the energy budget shares. Here $w_i = p_i q_i / M$ is the i^{th} budget share, which

TABLE 1

QUANTITIES AND PRICES OF ENERGY USED IN TRANSPORT:

W.A., 1966/67-1979/80

Year	Per capita quantities consumed (GJ)				Prices (cents per GJ)			
	Motor spirit	Land transport ADO	Bunker fuel	Aviation fuel	Motor spirit	Land transport ADO	Bunker fuel	Aviation fuel
	Q ₁	Q ₂	Q ₃	Q ₄	P ₁	P ₂	P ₃	P ₄
1966/67	26.62	10.23	24.91	3.39	267.53	148.72	74.39	218.35
1967/68	28.02	11.35	33.32	4.07	268.10	153.19	77.23	193.82
1968/69	29.90	12.61	29.43	4.62	274.45	150.32	73.24	195.72
1969/70	31.70	14.11	32.68	4.86	282.52	155.66	71.28	206.03
1970/71	31.40	14.89	31.16	5.69	291.17	167.56	80.44	230.24
1971/72	32.22	15.77	35.05	5.85	291.17	173.35	83.55	243.48
1972/73	33.09	18.34	29.91	5.31	337.30	173.43	86.38	258.76
1973/74	34.87	19.80	29.27	6.33	394.96	199.42	126.15	278.06
1974/75	35.26	19.42	25.09	6.72	452.02	231.02	175.68	315.11
1975/76	36.53	21.29	20.64	6.75	464.15	284.57	201.61	362.39
1976/77	38.68	20.83	25.36	7.47	501.62	315.13	224.33	408.24
1977/78	39.31	20.84	24.60	7.66	625.50	367.57	247.20	490.96
1978/79	40.30	22.95	20.49	7.70	804.33	487.71	327.72	656.80
1979/80	38.84	23.59	18.76	7.53	903.50	786.26	557.66	858.46
Mean	34.05	17.57	27.19	6.00	439.88	270.99	171.92	351.17

GJ = Joules $\times 10^9$.

TABLE 2

QUANTITY AND PRICE LOG-CHANGES OF ENERGY USED IN TRANSPORT:

W.A., 1966/67-1979/80

Year	Per capita quantity log-changes $\times 100$				Price log-changes $\times 100$			
	Motor spirit	Land transport ADO	Bunker fuel	Aviation fuel	Motor spirit	Land transport ADO	Bunker fuel	Aviation fuel
	Dq ₁	Dq ₂	Dq ₃	Dq ₄	Dp ₁	Dp ₂	Dp ₃	Dp ₄
1967/68	5.13	10.39	29.09	18.28	.21	2.96	3.75	-11.92
1968/69	6.49	10.53	-12.41	12.68	2.34	-1.89	-5.31	.98
1969/70	5.85	11.24	10.48	5.06	2.90	3.49	-2.71	5.13
1970/71	-.95	5.38	-4.76	15.77	3.02	7.37	12.09	11.11
1971/72	2.58	5.74	11.76	2.77	0.00	3.40	3.79	5.59
1972/73	2.66	15.10	-15.86	-9.69	14.71	.05	3.33	6.09
1973/74	5.24	7.66	-2.16	17.57	15.78	13.96	37.87	7.19
1974/75	1.11	-1.94	-15.41	5.98	13.49	14.71	33.12	12.51
1975/76	3.54	9.19	-19.52	.45	2.65	20.85	13.77	13.98
1976/77	5.72	-2.18	20.59	10.14	7.76	10.20	10.68	11.91
1977/78	1.62	.05	-3.04	2.51	22.07	15.39	9.71	18.45
1978/79	2.49	9.64	-18.28	.52	25.15	28.28	28.20	29.11
1979/80	-3.69	2.75	-8.82	-2.23	11.63	47.76	53.16	26.77
Mean	2.91	6.43	-2.18	6.14	9.36	12.81	15.50	10.53

The year 1967/68, for example, refers to the transition from 1966/67 to 1967/68.

TABLE 3

HOUSEHOLD INCOME, BUDGET SHARES AND CONDITIONAL BUDGET SHARES OF ENERGY

USED IN TRANSPORT: W.A., 1966/67-1979/80

Year	Per capita household income (current dollars) M	Budget shares × 100					Conditional budget shares × 100			
		Motor spirit	Land transport ADO	Bunker fuel	Aviation fuel	Total energy	Motor spirit	Land transport ADO	Bunker fuel	Aviation fuel
		w_1	w_2	w_3	w_4	w_g	w_1/w_g	w_2/w_g	w_3/w_g	w_4/w_g
1966/67	1438.83	4.9499	1.0571	1.2879	.5143	7.8091	63.39	13.54	16.49	6.59
1967/68	1563.97	4.8032	1.1119	1.6452	.5045	8.0647	59.56	13.79	20.40	6.26
1968/69	1721.96	4.7655	1.1011	1.2515	.5250	7.6430	62.35	14.41	16.37	6.87
1969/70	1861.45	4.8113	1.1797	1.2512	.5378	7.7800	61.84	15.16	16.08	6.91
1970/71	2017.06	4.5328	1.2370	1.2429	.6495	7.6621	59.16	16.14	16.22	8.48
1971/72	2217.76	4.2299	1.2328	1.3203	.6421	7.4251	56.97	16.60	17.78	8.65
1972/73	2424.30	4.6038	1.3121	1.0659	.5668	7.5486	60.99	17.38	14.12	7.51
1973/74	3130.48	4.3993	1.2615	1.1794	.5622	7.4024	59.43	17.04	15.93	7.60
1974/75	3719.37	4.2851	1.2061	1.1852	.5695	7.2459	59.14	16.65	16.36	7.86
1975/76	4333.22	3.9128	1.3980	.9603	.5645	6.8356	57.24	20.45	14.05	8.26
1976/77	4818.31	4.0269	1.3623	1.1807	.6330	7.2029	55.91	18.91	16.39	8.79
1977/78	5213.94	4.7158	1.4691	1.1663	.7213	8.0726	58.42	18.20	14.45	8.94
1978/79	5639.68	5.7475	1.9847	1.1907	.8967	9.8195	58.53	20.21	12.13	9.13
1979/80	6285.29	5.5832	2.9510	1.6645	1.0284	11.2272	49.73	26.28	14.83	9.16
Mean	3313.26	4.6691	1.4189	1.2566	.6368	7.9813	58.76	17.48	15.83	7.93

The budget share of an energy category is expenditure on the category as a proportion of household income.
The conditional budget share is expenditure as a proportion of total expenditure on energy used in transport.

is expenditure on i as a proportion of M . Mean income is \$3313 per capita p.a., and 4.7 percent of that is devoted to motor spirit, 1.4 percent to land transport ADO, 1.3 to bunker fuel and .6 to aviation fuel. Summing the four budget shares gives the share for total energy used in transport $W_g = \sum_{i=1}^4 w_i$, which is given in column 6 of Table 3. On average 8.0 percent of income is devoted to energy used in transport. Dividing the budget share by the total gives the conditional budget share w_i/W_g , which is expenditure on i as a proportion of expenditure on the group (with $\sum_{i=1}^4 w_i/W_g = 1$). These are given in the last four columns of Table 3. On average motor spirit accounts for 58.8 percent of the group, land transport ADO 17.5, bunker fuel 15.8 and aviation fuel 7.9.

Table 4 gives the Divisia moments of the data. To explain these we write M_g for expenditure on the group,

$$\sum_{i=1}^4 p_i q_i = M_g$$

and take differentials to give $\sum_i p_i dq_i + \sum_i q_i dp_i = dM_g$. Dividing this by M_g and using $dx/x = d(\log x)$, we obtain $\sum_i \frac{p_i q_i}{M_g} d(\log q_i) + \sum_i \frac{p_i q_i}{M_g} d(\log p_i) = d(\log M_g)$ or

$$\sum_{i=1}^4 \frac{w_i}{W_g} d(\log q_i) + \sum_{i=1}^4 \frac{w_i}{W_g} d(\log p_i) = d(\log M_g).$$

We write this as

$$(2.1) \quad d(\log Q_g) + d(\log P_g) = d(\log M_g),$$

where

$$(2.2) \quad d(\log Q_g) = \sum_{i=1}^4 \frac{w_i}{W_g} d(\log q_i)$$

is the Divisia volume index of the group; and

TABLE 4
DIVISIA MOMENTS OF ENERGY USED IN TRANSPORT, CPI AND REAL INCOME
LOG-CHANGES: W.A., 1966/67-1979/80

Year	Divisia indexes of energy used in transport		Divisia variances of energy used in transport		Perth CPI log-change	Real income log-change
	Volume $DQ_g \times 100$	Price $DP_g \times 100$	Quantity $K_g \times 10000$	Price $\Pi_g \times 10000$	$DP^* \times 100$	$DQ \times 100 = (DM - DP^*) \times 100$
1967/68	11.12	.46	85.10	12.72	2.86	5.48
1968/69	3.98	.25	64.41	9.03	2.50	7.13
1969/70	7.34	2.23	5.59	5.06	3.63	4.16
1970/71	.71	5.78	27.35	13.63	4.21	3.82
1971/72	4.67	1.68	11.59	4.19	5.62	3.86
1972/73	.83	9.71	89.89	38.15	5.32	3.58
1973/74	5.48	18.14	20.66	73.87	9.94	15.63
1974/75	-1.69	16.79	39.59	51.63	16.67	.57
1975/76	.82	8.62	80.34	54.76	13.23	2.04
1976/77	6.81	9.04	46.50	2.30	14.60	-3.99
1977/78	.69	18.61	3.00	20.95	10.26	-2.37
1978/79	.96	26.51	64.71	2.68	7.79	.06
1979/80	-2.74	27.12	12.60	321.09	8.95	1.89
Mean	3.00	11.15	42.41	46.93	8.12	3.22

The year 1967/68, for example, refers to the transition from 1966/67 to 1967/68.

$$(2.3) \quad d(\log P_g) = \sum_{i=1}^4 \frac{w_i}{W_g} d(\log p_i)$$

is the corresponding Divisia price index.

Equation (2.1) decomposes the change in expenditure on the group into a volume and price index. The volume index defined in (2.2) is a weighted average of the four quantity changes $d(\log q_1), \dots, d(\log q_4)$, the weights being the conditional budget shares which are positive and have a unit sum. Similarly, the price index (2.3) is a weighted average of the price changes. The Divisia indexes are thus summary measures in the sense that they are averages.

Equations (2.2) and (2.3) are formulated in terms of infinitesimal changes. To apply them to finite-change data requires two adjustments. First, we replace w_i and W_g with their arithmetic averages over the periods $t-1$ and t , $\bar{w}_{it} = \frac{1}{2}(w_{it} + w_{i,t-1})$, $\bar{W}_{gt} = \frac{1}{2}(W_{gt} + W_{g,t-1})$. This is a natural choice when considering the change from $t-1$ to t . Second, we replace $d(\log q_i)$ with its log-change $Dq_{it} = \log q_{it} - \log q_{i,t-1}$; and similarly for the prices. Accordingly, the finite-change Divisia volume and price indexes are

$$(2.4) \quad DQ_{gt} = \sum_{i=1}^4 \frac{\bar{w}_{it}}{\bar{W}_{gt}} Dq_{it},$$

$$(2.5) \quad DP_{gt} = \sum_{i=1}^4 \frac{\bar{w}_{it}}{\bar{W}_{gt}} Dp_{it}.$$

These are given in the first two columns of Table 4. On average the per capita volume of consumption of energy used in transport increased by about 3.0 percent p.a., while prices rose by 11.2 percent. The effects of the first and second oil price shocks can be seen quite clearly by the large increases in DP_g in the mid- and late-1970s.

Equations (2.4) and (2.5) are first-order moments of the quantity and price log-changes. The corresponding second-order moments are the Divisia quantity and price variances,

$$(2.6) \quad K_{gt} = \sum_{i=1}^4 \frac{\bar{w}_{it}}{\bar{w}_{gt}} (Dq_{it} - DQ_{gt})^2,$$

$$(2.7) \quad \Pi_{gt} = \sum_{i=1}^4 \frac{\bar{w}_{it}}{\bar{w}_{gt}} (Dp_{it} - DP_{gt})^2.$$

These measure the degree to which the quantities and prices move disproportionately. The Divisia variances are given in columns (3) and (4) of Table 4. In 9 out of the 13 cases the quantity variance K_g exceeds the price variance Π_g , which agrees with the results of Clements (1982), Meisner (1979) and Suhm (1979).

Finally, Table 4 also gives the CPI and real income log-changes, which will be used subsequently.

3. THE ENERGY DEMAND MODEL

In this section we model the demand for the four energy categories in two steps. First, we formulate a demand equation for energy as a group. This is known as a composite demand equation and is concerned with the allocation of income between energy as a whole and all other goods. The composite equation thus determines the total demand for energy. Then, given total energy demand, as the second step we specify a system of conditional demand equations, one for each category of energy. These serve to allocate total energy demand to the four categories. This two-step procedure is intuitively plausible and can be justified formally in terms of a utility (or production) function which is separable in energy and all other goods. This formal justification is given in the Appendix. This section is based on Theil (1975/76, 1980).

A Composite Energy Demand Equation

The equation we use to describe the demand for energy as a group takes the following form:

$$(3.1) \quad W_g d(\log Q_g) = \Theta_g d(\log Q) + \phi \Theta_g d \left(\log \frac{P_g}{P} \right),$$

where $W_g = \sum_{i=1}^4 w_i$ is the budget share of the energy group; $d(\log Q_g)$ is the Divisia volume index of the group (2.2); $\Theta_g = \partial M_g / \partial M$ is the marginal share of the group, with M_g group expenditure and M income; $d(\log Q) = d(\log M) - d(\log P)$ is the change in real income, with $d(\log P)$ the Divisia price index of all n goods consumed $\sum_{i=1}^n w_i d(\log p_i)$; ϕ is the income flexibility, defined as $(\partial \log \lambda / \partial \log M)^{-1} < 0$, with λ the marginal utility of income; and $d \left(\log \frac{P_g}{P} \right)$ is the change in the relative price of the group $d(\log P_g) - d(\log P)$, where $d(\log P_g)$ is the Divisia price index of the group (2.3). The derivation of (3.1) is given in the Appendix.

To interpret the variable on the left of (3.1) we take the differential of $w_i = p_i q_i / M$ to give $dw_i = w_i d(\log p_i) + w_i d(\log q_i) - w_i d(\log M)$. Summing this over $i=1, \dots, 4$ gives

$$(3.2) \quad dW_g = W_g d(\log P_g) + W_g d(\log Q_g) - W_g d(\log M).$$

This states that the change in the budget share of the group is made up of price, quantity and income components. As demand theory postulates that income and prices are given, the quantity component of the budget share change $W_g d(\log Q_g)$ is the endogenous component of dW_g . This is the dependent variable of (3.1). It can be shown that $W_g d(\log Q_g)$ also has the interpretation as the contribution of the group to the change in real income $d(\log Q)$.

Equation (3.1) thus describes the quantity component of the change in the energy budget share in terms of the change in real income

$d(\log Q)$ and the relative price of energy $d\left(\log \frac{P_g}{P}\right)$. The marginal share θ_g tells us what fraction of an additional dollar of income is spent on energy. By dividing both sides of (3.1) by W_g , we find that θ_g/W_g and $\phi\theta_g/W_g$ are the income and price elasticities, respectively. This price elasticity is the elasticity of the Divisia volume index of the group with respect to the relative price of the group.

Conditional Energy Demand Equations

As we are dealing with energy used in the transport sector, it is highly unlikely that our 14 years of data will reveal much substitution within the energy group. The reason is that much energy usage in transport is highly dependent on the existing capital stock of that sector, a stock which is adjusted only slowly in response to changes in relative energy prices. Accordingly, we shall confine the role of price substitution to the composite demand equation (energy versus all other goods) and ignore it within the group. Hence, we use a system of Engel curves for the conditional demand equations,

$$(3.3) \quad w_i d(\log q_i) = \frac{\theta_i}{\theta_g} W_g d(\log Q_g) \quad i=1, \dots, 4,$$

where $\theta_i = \partial(p_i q_i)/\partial M$ is the i^{th} marginal share and $\theta_i/\theta_g = \partial(p_i q_i)/\partial M_g$ is the conditional marginal share of i within the energy group, with $\sum_{i=1}^4 \theta_i/\theta_g = 1$. This conditional marginal share answers the question, if income increases by one dollar, resulting in a certain additional amount spent on the energy group, what is the proportion of this additional amount that is allocated to i ?

Equation (3.3) is the demand equation for i within the energy group, given the demand for energy as a whole $W_g d(\log Q_g)$. As before, the variable on the left is (i) the quantity component of the change in w_i and (ii) the contribution of i to the change in real income $d(\log Q)$.

Dividing both sides of (3.3) by w_i , we see that $\frac{\theta_i}{\theta_g} \frac{W_g}{w_i} = \frac{\theta_i/w_i}{\theta_g/W_g}$ is the ratio of the income elasticity of i to that of the energy group. We shall refer to this ratio as the conditional income elasticity of i .

Equations (3.1) and (3.3) form a recursive system: Changes in real income and the relative price of energy determine from (3.1) the change in the demand for energy as a whole $W_g d(\log Q_g)$. Equation (3.3) then allocates this total energy demand to the four categories, $w_1 d(\log q_1), \dots, w_4 d(\log q_4)$ with $\sum_{i=1}^4 w_i d(\log q_i) = W_g d(\log Q_g)$ as $\sum_{i=1}^4 \theta_i / \theta_g = 1$.

Unconditional Energy Demand Equations

To obtain the effects of income and prices on the demand for category i , we need the unconditional demand equation. To derive this we substitute the right hand side of (3.1) for $W_g d(\log Q_g)$ into (3.3) to give

$$(3.4) \quad w_i d(\log q_i) = \theta_i d(\log Q) + \phi \theta_i d\left(\log \frac{P_g}{P}\right).$$

To simplify the price substitution term we number goods such that the energy categories are the first four and write the overall Divisia index as

$$d(\log P) = \sum_{j=1}^n w_j d(\log p_j) = W_g \sum_{j=1}^4 \frac{w_j}{W_g} d(\log p_j) + (1 - W_g) \sum_{k=5}^n \frac{w_k}{\sum_{\ell=5}^n w_\ell} d(\log p_k)$$

as $W_g = \sum_{i=1}^4 w_i = 1 - \sum_{\ell=5}^n w_\ell$, or

$$(3.5) \quad d(\log P) = W_g d(\log P_g) + (1 - W_g) d(\log P_0),$$

where $d(\log P_g)$ is the Divisia index of the group and

$$d(\log P_0) = \sum_{k=5}^n \frac{w_k}{\sum_{\ell=5}^n w_\ell} d(\log p_k) \text{ is the Divisia index of all other goods.}$$

In words, the overall Divisia price index is a weighted average of the Divisia indexes for energy and all other, the weights being the group budget shares.

Using (3.5) the substitution term in (3.4) becomes

$$\phi \theta_i d \left(\log \frac{P_g}{P} \right) = \phi (1 - W_g) \theta_i [d(\log P_g) - d(\log P_0)].$$

Substituting $\sum_{j=1}^4 \frac{w_j}{W_g} d(\log p_j)$ for $d(\log P_g)$, the above becomes

$$(3.6) \quad \phi \theta_i d \left(\log \frac{P_g}{P} \right) = \sum_{j=1}^4 \pi_{ij} d(\log p_j) + \pi_{i0} d(\log P_0)$$

where

$$(3.7) \quad \pi_{ij} = \phi (1 - W_g) \theta_i \frac{w_j}{W_g} \quad \pi_{i0} = -\phi (1 - W_g) \theta_i$$

are unconditional Slutsky coefficients.

Using (3.6), (3.4) becomes

$$(3.8) \quad w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^4 \pi_{ij} d(\log p_j) + \pi_{i0} d(\log P_0),$$

from which it can be seen that the unconditional income and price elasticities are θ_i/w_i , π_{ij}/w_i and π_{i0}/w_i .

4. ESTIMATION

For estimation we use the Rotterdam parametrization of treating the coefficients as constants; replace budget shares with their arithmetic averages and infinitesimal logarithmic changes with finite log-changes, as before; and approximate the overall Divisia price index by the CPI log-change DP^* . After adding disturbance terms E_{gt} and ε_{it} , the estimating equations are:

$$(4.1) \quad \bar{w}_{gt} DQ_{gt} = \theta_g DQ_t + \phi \theta_g [DP_{gt} - DP_t^*] + E_{gt},$$

$$(4.2) \quad \bar{w}_{it} Dq_{it} = \frac{\theta_i}{\theta_g} \bar{w}_{gt} DQ_{gt} + \varepsilon_{it} \quad i=1, \dots, 4.$$

The only variable not previously discussed is DQ , which is the change in real income. We measure DQ as the difference between the log-change in per capita money income M and the CPI log-change; DP^* and DQ are given in the last two columns of Table 4.

The disturbances E_{gt} and ϵ_{it} are assumed to be serially independent and normally distributed with zero means and a constant contemporaneous covariance matrix. Equations (4.1) and (4.2) form a system of five simultaneous equations with endogenous variables $\bar{W}_{gt} DQ_{gt}$ and $\bar{W}_{it} DQ_{it}$ ($i=1, \dots, 4$). The prices and DQ_t are taken to be predetermined. Note that (4.1) implies that the only random component of $\bar{W}_{gt} DQ_{gt}$ is the disturbance E_{gt} . Hence, we can treat $\bar{W}_{gt} DQ_{gt}$ as predetermined in (4.2) if E_{gt} is independent of the disturbance of that equation ϵ_{it} . Remarkably, Theil's (1975/76, 1980) theory of rational random behavior implies that these disturbances are independent. Accordingly, we shall estimate (4.1) and (4.2) by OLS; these estimates are consistent under rational random behavior.¹

5. EMPIRICAL RESULTS

In this section we first give the estimates of the composite energy demand equation and the conditional equations. Second, we derive the unconditional demand responses and then use the information inaccuracy as a measure of the goodness-of-fit of the models in order to establish the preferred model.

The Composite Energy Demand Equation

The estimates of the composite equation are given in Table 5. In the first row we have included a constant term in the equation to take account of trend-like changes in tastes, etc. This constant term is significantly positive, indicating that there is a trend into energy over this period. The estimate indicates that per capita consumption of energy is growing autonomously at an exponential rate of $\lambda/\bar{W}_{gt} = .2181/.0786 = 2.8$ percent p.a. (at sample means). The marginal share in row 1 is .021, which states that when income increases by one dollar, expenditure on energy rises by 2 cents. This, however, is not estimated very precisely. The price coefficient $\phi\theta_g$ is $-.020$, which is significantly different from zero, so that the income flexibility ϕ is $-.020/.021 = -.985$. This value of ϕ is reasonable, but somewhat higher (in absolute value) than previous estimates.²

The last two entries of row 1 of Table 5 give the corresponding elasticities (evaluated at sample means). The income elasticity is .3, while the price elasticity is $-.3$.

Row 2 of Table 5 gives the results when the constant term in the demand equation is suppressed. As can be seen, the results change quite a bit. The marginal share almost doubles to .041 and becomes significant, while the price coefficient falls by about 30 percent and becomes insignificant. The reason is that the three "variables" of the first equation (the constant, DQ and $DP_g - DP^*$) are all on average positive, so that when we set $\lambda=0$, the effects of the autonomous trend are absorbed into θ_g and $\phi\theta_g$, which increase algebraically. The income and price elasticities implied by this equation are .5 and $-.2$ respectively.

Although the choice between the two equations is not clear-cut, in the subsequent computations we shall use the equation with a constant term. The constant and price coefficient in this equation are both significant and the Durbin-Watson statistic is near 2.³

TABLE 5

COMPOSITE ENERGY DEMAND EQUATIONS WITH AND WITHOUT A CONSTANT TERM:

W.A., 1966/67-1979/80

$$\bar{W}_{gt} DQ_{gt} = \lambda + \theta_g DQ_t + \phi \theta_g [DP_{gt} - DP_t^*] + E_{gt}$$

(Standard errors in parentheses)

	Constant $\lambda \times 100$	Marginal share θ_g	Price coefficient $\phi \theta_g$	DW	Income elasticity $\theta_g / \bar{W}_g$	Price elasticity $\phi \theta_g / \bar{W}_g$
1. With constant	.2181 (.0964)	.0206 (.0163)	-.0203 (.0096)	1.92	.26	-.26
2. Without constant	-	.0407 (.0160)	-.0137 (.0108)	1.41	.52	-.17

The elasticities are evaluated at the mean of the arithmetic average of the group budget share $\bar{W}_g$ of .0786.

The Conditional Energy Demand Equations

Table 6 gives the conditional equations with constant terms included. These results are not satisfactory as the conditional income elasticity of bunker fuel is implausibly high; also, the elasticity of land transport ADO is quite low.

The results improve when the constant terms are suppressed; see Table 7. Here the conditional marginal shares are all significantly positive and indicate that a one dollar rise in energy expenditure causes expenditure on motor spirit to increase by 44 cents, land transport ADO by 15 cents, bunker fuel 30 cents and aviation fuel 11 cents. The conditional income elasticities are .7, .9, 1.9 and 1.4 (in the same order). However, it could be argued that the elasticity of bunker fuel is still on the high side.

To analyse further this issue we examine the sensitivity of the estimates to the inclusion of outlying observations. After some experimentation, we found that the first observation (1966/67-1967/68) is an outlier. Accordingly, we re-estimate the demand equations with a dummy variable which takes the value 1 for that observation, 0 otherwise; this is equivalent to omitting the observation. The results are given in Table 8. Relative to Table 7, the main changes are that conditional income elasticity of motor spirit rises from .7 to .9, while that of bunker fuel falls from 1.9 to 1.1. In the subsequent computations we shall use both the Table 7 and Table 8 sets of estimates.⁴

The Unconditional Demand Responses

We use the estimates of the composite and conditional demand equations to compute the unconditional demand elasticities.

The expressions for these unconditional elasticities are set out in the

TABLE 6

CONDITIONAL ENERGY DEMAND EQUATIONS WITH CONSTANT TERMS:

W.A., 1966/67-1979/80

$$\bar{w}_{it} Dq_{it} = \alpha_i + \frac{\theta_i}{\theta_g} \bar{w}_{gt} DQ_{gt} + \epsilon_{it}$$

(Standard errors in parentheses)

Energy category	Constant $\alpha_i \times 1000$	Conditional marginal share $\frac{\theta_i}{\theta_g}$	DW	Conditional income elasticity $\frac{\theta_i}{\theta_g} \frac{\bar{w}_g}{\bar{w}_i}$
Motor spirit	.481 (.314)	.362 (.083)	1.87	.61
Land transport ADO	.764 (.244)	.031 (.065)	2.25	.18
Bunker fuel	-1.389 (.407)	.521 (.108)	2.39	3.30
Aviation fuel	.143 (.141)	.086 (.038)	2.66	1.09

The elasticities are evaluated at the means of the ratios of the arithmetic average of the budget shares to that of the group $\bar{w}_i/\bar{w}_g$. These are .589, .173, .158, .079 (in the same order as above).

TABLE 7

CONDITIONAL ENERGY DEMAND EQUATIONS WITHOUT CONSTANT TERMS:

W.A., 1966/67-1979/80

$$\bar{w}_{it} Dq_{it} = \frac{\theta_i}{\theta_g} \bar{w}_{gt} DQ_{gt} + \epsilon_{it}$$

(Standard errors in parentheses)

Energy Category	Conditional marginal share $\frac{\theta_i}{\theta_g}$	DW	Conditional income elasticity $\frac{\theta_i}{\theta_g} \frac{\bar{w}_g}{\bar{w}_i}$
Motor spirit	.438 (.071)	1.74	.74
Land transport ADO	.152 (.069)	1.51	.88
Bunker fuel	.302 (.120)	1.64	1.91
Aviation fuel	.108 (.030)	2.57	1.37

See notes to Table 6.

TABLE 8

CONDITIONAL ENERGY DEMAND EQUATIONS WITHOUT CONSTANT TERMS,

WITH DUMMY VARIABLE: W.A., 1966/67-1979/80

$$\bar{w}_{it} Dq_{it} = \beta_i Z_t + \frac{\theta_i}{\theta_g} \bar{w}_{gt} DQ_{gt} + \epsilon_{it}$$

(Standard errors in parentheses)

Energy category	Coefficient of dummy $\beta_i \times 100$	Conditional marginal share $\frac{\theta_i}{\theta_g}$	DW	Conditional income elasticity $\frac{\theta_i \bar{w}_g}{\theta_g \bar{w}_i}$
Motor spirit	-.236 (.111)	.551 (.082)	1.81	.94
Land transport ADO	-.037 (.128)	.170 (.094)	1.55	.98
Bunker fuel	.278 (.207)	.168 (.153)	1.79	1.06
Aviation fuel	-.004 (.056)	.110 (.042)	2.58	1.39

The dummy variable Z=1 for the first observation (1966/67-1967/68), 0 otherwise. See notes to Table 6.

last subsection of Section 3. We first use the composite equation with a constant term, row 1 of Table 5, and the conditional equations of Table 7. The results are given in Table 9. The (unconditional) income and own-price elasticities are:

	<u>Income</u>	<u>Own-Price</u>
Motor spirit	.2	-.10
Land transport ADO	.2	-.04
Bunker fuel	.5	-.07
Aviation fuel	.4	-.03

Thus bunker and aviation fuels have income elasticities about twice those of the other two categories. Motor spirit has the highest price elasticity, followed by bunker fuel, although both are quite low.

Next, we compute the unconditional elasticities with the Table 8 conditional equations (which have the dummy variable) and the same group equation. The results are given in Table 10. In comparison to Table 9, the income elasticities of motor spirit and land transport ADO have risen and that of bunker fuel fallen; the own-price elasticity of motor spirit has risen (in absolute value), while that of bunker fuel has fallen.

The Information Inaccuracy

To measure the fit of the models we use a concept from information theory, the information inaccuracy. This measure, which has been extensively applied by Theil (1967, 1972, 1975/76, 1980), is well-suited to analysing the fit of allocation models such as ours.

Let $w_{1t}, \dots, w_{nt}$ be the observed budget shares of n goods in period t and $\hat{w}_{1t}, \dots, \hat{w}_{nt}$ the predicted shares. The information inaccuracy of these predictions is

TABLE 9

UNCONDITIONAL DEMAND ELASTICITIES FOR ENERGY:

W.A., 1966/67-1979/80

Energy category	Income elasticity $\theta_i/\bar{w}_i$	Price elasticities				
		$\pi_{i1}/\bar{w}_i$	$\pi_{i2}/\bar{w}_i$	$\pi_{i3}/\bar{w}_i$	$\pi_{i4}/\bar{w}_i$	$\pi_{i0}/\bar{w}_i$
Motor spirit	.20	-.10	-.03	-.03	-.01	.18
Land transport ADO	.23	-.12	-.04	-.03	-.02	.21
Bunker fuel	.50	-.27	-.08	-.07	-.04	.45
Aviation fuel	.36	-.19	-.06	-.05	-.03	.33

Based on the estimates given in row 1 of Table 5 and in Table 7 and the means of the arithmetic averages of the budget shares and the group budget share, $\bar{w}_i$ and $\bar{w}_g$. The means of $\bar{w}_i$ are .0463, .0136, .0124, .0062 (in the same order as above).

TABLE 10

UNCONDITIONAL DEMAND ELASTICITIES FOR ENERGY:

W.A., 1966/67-1979/80

Energy category	Income elasticity $\theta_i/\bar{w}_i$	Price elasticities				
		$\pi_{i1}/\bar{w}_i$	$\pi_{i2}/\bar{w}_i$	$\pi_{i3}/\bar{w}_i$	$\pi_{i4}/\bar{w}_i$	$\pi_{i0}/\bar{w}_i$
Motor spirit	.25	-.13	-.04	-.04	-.02	.22
Land transport ADO	.26	-.14	-.04	-.04	-.02	.23
Bunker fuel	.28	-.15	-.04	-.04	-.02	.25
Aviation fuel	.37	-.20	-.06	-.05	-.03	.33

Based on estimates given in row 1 of Table 5 and in Table 8. See notes to Table 9.

$$(5.1) \quad I_t = \sum_{i=1}^n w_{it} \log \frac{w_{it}}{\hat{w}_{it}}.$$

When the predictions are perfect ($\hat{w}_{it} = w_{it}$ for $i=1, \dots, n$) $I_t = 0$; otherwise $I_t > 0$.

To apply the information inaccuracy to the energy group, we simply divide the n goods into two groups, energy and all other, so that (5.1) becomes

$$(5.2) \quad I_{gt} = W_{gt} \log \frac{W_{gt}}{\hat{W}_{gt}} + (1 - W_{gt}) \log \frac{1 - W_{gt}}{1 - \hat{W}_{gt}}.$$

We can also apply it conditionally to the within group allocation by replacing the budget shares in (5.1) with the conditional budget shares, so that the conditional information inaccuracy is

$$(5.3) \quad I_t = \sum_{i=1}^4 \frac{w_{it}}{W_{gt}} \log \frac{w_{it}/W_{gt}}{\hat{w}_{it}/\hat{W}_{gt}}.$$

Finally, we can consider the prediction for the i^{th} category separately by dividing the group into i and all other within the group, so that the observed and predicted conditional budget shares are

$$\frac{w_{it}}{W_{gt}}, 1 - \frac{w_{it}}{W_{gt}}$$

$$\frac{\hat{w}_{it}}{\hat{W}_{gt}}, 1 - \frac{\hat{w}_{it}}{\hat{W}_{gt}}.$$

The information inaccuracy for the i^{th} good and its complement within the group is thus

$$(5.4) \quad I_{it} = \frac{w_{it}}{W_{gt}} \log \frac{w_{it}/W_{gt}}{\hat{w}_{it}/\hat{W}_{gt}} + \left(1 - \frac{w_{it}}{W_{gt}}\right) \log \frac{1 - w_{it}/W_{gt}}{1 - \hat{w}_{it}/\hat{W}_{gt}}.$$

We shall evaluate (5.2), (5.3) and (5.4) with the demand models providing the predicted budget shares. Consider first the prediction of the group budget share $\hat{W}_{gt}$. As the model determines the change in the share we write this prediction as

$$(5.5) \quad \hat{W}_{gt} = W_{g,t-1} + \text{prediction of } \Delta W_{gt}.$$

The finite-change version of (3.2) is

$$(5.6) \quad \Delta W_{gt} = \bar{W}_{gt} DP_{gt} + \bar{W}_{gt} DQ_{gt} - \bar{W}_{gt} DM_t$$

and the expectation of (4.1) provides the prediction of the quantity component $\bar{W}_{gt} DQ_{gt}$. As this expectation is $\bar{W}_{gt} DQ_{gt}$ minus the residual $\hat{E}_{gt}$, it follows that the prediction of ΔW_{gt} is $\Delta W_{gt} - \hat{E}_{gt}$ and (5.5) becomes

$$(5.7) \quad \hat{W}_{gt} = W_{gt} - \hat{E}_{gt}.$$

The actual and predicted budget shares of the energy group are given in Table 11. In addition to using the composite demand equation to give predictions of W_g , we also use no-change extrapolation, $\hat{W}_{gt} = W_{g,t-1}$. The performance of the demand model can be judged by a comparison with no-change extrapolation. The results of applying the information inaccuracy (5.2) to the two sets of predictions are given in the last two columns of Table 11. As can be seen, in most years the demand model yields a lower information inaccuracy than does no-change extrapolation. In the latter half of the 1970s, when W_g is quite volatile, the demand model wins handsomely. On average, the information inaccuracy for the demand model is almost 90 percent lower than that for the no-change extrapolation.

We now turn to conditional budget shares of the four energy categories. Applying a similar argument that led to (5.7), the prediction of the i^{th} conditional budget share from the demand model (4.2) is

$$(5.8) \quad \frac{\hat{w}_{it}}{\hat{W}_{gt}} = \frac{w_{it}}{W_{gt}} - \frac{\hat{e}_{it}}{\bar{W}_{gt}},$$

where $\hat{e}_{it}$ is the residual of the i^{th} conditional demand equation.

Table 12 gives the actual and predicted conditional budget shares, with the predictions based on the demand equations of Table 7.

Table 13 gives the predictions based on the demand equations with the

TABLE 11

ACTUAL AND PREDICTED BUDGET SHARES OF ENERGY AND INFORMATION INACCURACIES:

W.A., 1966/67-1979/80

Year	Actual energy budget share $\times 100$	Predicted energy budget shares $\times 100$		Information inaccuracies $\times 10^6$	
		No-change $\hat{W}_{gt} = W_{g,t-1}$	Demand model $\hat{W}_{gt} = W_{gt} - \hat{E}_{gt}$	No-change	Demand model
1967/68	8.065	7.809	7.562	44.9	177.0
1968/69	7.643	8.065	7.741	121.9	6.8
1969/70	7.780	7.643	7.546	13.2	38.8
1970/71	7.662	7.780	7.872	9.7	30.7
1971/72	7.425	7.662	7.450	40.1	.5
1972/73	7.549	7.425	7.690	11.0	14.1
1973/74	7.402	7.549	7.367	15.4	.9
1974/75	7.246	7.402	7.597	18.0	89.2
1975/76	6.836	7.246	7.132	127.5	67.0
1976/77	7.203	6.836	6.973	104.3	40.2
1977/78	8.073	7.203	8.020	545.9	1.9
1978/79	9.820	8.073	9.574	1934.0	34.7
1979/80	11.227	9.820	11.404	1074.3	15.6
Mean	7.995	7.732	7.994	312.3	39.8

The demand model is given in row 1 of Table 5.

TABLE 12

ACTUAL AND PREDICTED CONDITIONAL BUDGET SHARES OF ENERGY:

W.A., 1966/67-1979/80

Year	Energy category							
	Motor spirit		Land transport ADO		Bunker fuel		Aviation fuel	
	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted
1967/68	59.56	61.27	13.79	14.06	20.40	18.38	6.26	6.28
1968/69	62.35	60.14	14.41	13.53	16.37	19.87	6.87	6.47
1969/70	61.84	61.42	15.16	14.62	16.08	16.60	6.91	7.36
1970/71	59.16	60.04	16.14	15.41	16.22	17.20	8.48	7.34
1971/72	56.97	57.52	16.60	16.37	17.78	17.19	8.65	8.92
1972/73	60.99	59.78	17.38	14.94	14.12	16.90	7.51	8.38
1973/74	59.43	58.67	17.04	16.56	15.93	17.91	7.60	6.86
1974/75	59.14	57.74	16.65	16.71	16.36	18.33	7.86	7.21
1975/76	57.24	55.54	20.45	18.88	14.05	17.27	8.26	8.31
1976/77	55.91	55.65	18.91	20.38	16.39	15.31	8.79	8.66
1977/78	58.42	57.79	18.20	18.30	14.45	15.12	8.94	8.79
1978/79	58.53	57.49	20.21	18.50	12.13	14.82	9.13	9.19
1979/80	49.73	50.52	26.28	25.22	14.83	15.19	9.16	9.07
Mean	58.40	57.97	17.79	17.19	15.78	16.93	8.03	7.91

The predictions are based on the demand equations of Table 7. All entries are to be divided by 100.

TABLE 13

ACTUAL AND PREDICTED CONDITIONAL BUDGET SHARES OF ENERGY:

W.A., 1966/67-1979/80

Year	Energy category							
	Motor spirit		Land transport ADO		Bunker fuel		Aviation fuel	
	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted
1967/68	59.56	59.56	13.79	13.79	20.40	20.40	6.26	6.26
1968/69	62.35	60.59	14.41	13.60	16.37	19.33	6.87	6.48
1969/70	61.84	62.26	15.16	14.75	16.08	15.62	6.91	7.37
1970/71	59.16	60.13	16.14	15.42	16.22	17.11	8.48	7.34
1971/72	56.97	58.05	16.60	16.46	17.78	16.57	8.65	8.92
1972/73	60.99	59.87	17.38	14.96	14.12	16.79	7.51	8.38
1973/74	59.43	59.29	17.04	16.65	15.93	17.18	7.60	6.87
1974/75	59.14	57.55	16.65	16.68	16.36	18.56	7.86	7.21
1975/76	57.24	55.63	20.45	18.89	14.05	17.16	8.26	8.31
1976/77	55.91	56.43	18.91	20.50	16.39	14.40	8.79	8.67
1977/78	58.42	57.87	18.20	18.31	14.45	15.03	8.94	8.79
1978/79	58.53	57.60	20.21	18.51	12.13	14.69	9.13	9.19
1979/80	49.73	50.20	26.28	25.17	14.83	15.56	9.16	9.06
Mean	58.40	58.08	17.79	17.21	15.78	16.80	8.03	7.91

The predictions are based on the demand equations of Table 8. All entries are to be divided by 100.

TABLE 14

CONDITIONAL INFORMATION INACCURACIES FOR ENERGY BASED ON NO-CHANGE EXTRAPOLATION:

W.A., 1966/67-1979/80

Year	Energy category				Total
	Motor spirit	Land transport ADO	Bunker fuel	Aviation fuel	
1967/68	3114.6	26.7	5231.5	90.2	5589.8
1968/69	1632.7	159.1	5263.4	311.4	5335.4
1969/70	55.0	229.3	31.3	1.5	244.1
1970/71	1513.0	366.9	7.2	1781.9	2556.6
1971/72	987.7	77.3	873.0	18.8	1219.4
1972/73	3326.2	216.8	4860.7	856.7	6410.3
1973/74	507.7	40.7	1308.6	5.4	1355.4
1974/75	17.7	55.7	66.6	49.1	154.7
1975/76	741.6	4937.3	2026.9	108.2	6173.1
1976/77	363.2	742.3	2177.7	182.0	2773.0
1977/78	1284.5	167.9	1426.0	13.5	1906.1
1978/79	2.7	1323.5	2287.3	23.5	3071.3
1979/80	15719.8	10698.7	3224.7	.5	18109.8
Mean	2251.2	1464.8	2214.2	264.8	4223.0

All entries are to be divided by 10^6 .

TABLE 15

CONDITIONAL INFORMATION INACCURACIES FOR ENERGY BASED ON DEMAND MODEL:

W.A., 1966/67-1979/80

Year	Energy category				Total
	Motor spirit	Land transport ADO	Bunker fuel	Aviation fuel	
1967/68	617.1	30.8	1319.7	.7	1338.0
1968/69	1029.2	323.3	4015.6	129.8	4068.2
1969/70	36.8	117.8	97.5	148.2	333.5
1970/71	163.2	204.1	344.9	905.9	1360.5
1971/72	61.3	19.1	120.0	44.7	182.0
1972/73	305.1	2244.9	2878.1	511.1	4901.9
1973/74	119.0	84.6	1372.1	408.8	1634.6
1974/75	402.9	1.7	1342.8	302.6	1555.6
1975/76	587.2	793.6	3833.2	1.8	4112.5
1976/77	13.1	674.3	444.6	10.3	930.4
1977/78	79.7	3.1	180.0	13.6	201.6
1978/79	220.3	954.9	3039.5	1.9	3484.5
1979/80	123.7	296.5	53.0	5.2	331.9
Mean	289.1	442.2	1464.7	191.1	1879.6

The demand model is given in Table 7. All entries are to be divided by 10^6 .

TABLE 16

CONDITIONAL INFORMATION INACCURACIES FOR ENERGY BASED ON DEMAND MODEL:

W.A., 1966/67-1979/80

Year	Energy category				Total
	Motor spirit	Land transport ADO	Bunker fuel	Aviation fuel	
1967/68	0.0	0.0	0.0	0.0	0.0
1968/69	653.4	271.8	2927.4	124.8	2995.2
1969/70	36.9	67.3	80.8	157.6	285.5
1970/71	194.3	196.9	282.4	903.6	1313.1
1971/72	238.4	7.6	520.8	47.7	583.2
1972/73	259.5	2216.3	2662.9	513.0	4682.1
1973/74	3.9	53.5	557.5	396.7	878.6
1974/75	520.8	.5	1656.8	305.8	1863.5
1975/76	524.6	778.4	3589.3	2.0	3870.7
1976/77	54.9	789.5	1554.6	8.2	1984.0
1977/78	61.0	3.9	134.7	13.4	155.7
1978/79	176.7	935.4	2772.5	2.0	3222.6
1979/80	45.1	324.9	208.1	5.8	446.0
Mean	213.0	434.3	1303.7	190.8	1713.9

The demand model is given in Table 8. All entries are to be divided by 10^6 .

dummy variable, those given in Table 8. The conditional information inaccuracies are given in Tables 14-16. Table 14 is based on no-change extrapolation, $\hat{w}_{it}/\hat{w}_{gt} = w_{i,t-1}/w_{g,t-1}$; Table 15 on the demand equations of Table 7; and Table 16 on the demand equations of Table 8. As can be seen from the mean information inaccuracies, both demand models perform substantially better than does no-change extrapolation. To compare the two demand models, we need to allow for the fact that the model of Table 8 has more parameters due to the dummy variable. As the inclusion of the dummy is equivalent to omitting the first observation, it is natural to compare the two models on the basis of the mean information inaccuracies over all observations except the first. This gives:

<u>Model</u>	<u>Motor spirit</u>	<u>Land transport ADO</u>	<u>Bunker fuel</u>	<u>Aviation fuel</u>	<u>Total</u>
Table 7	261.8	476.5	1476.8	207.0	1924.7
Table 8	230.8	470.5	1412.3	206.7	1856.7

Although there are no substantial differences, the model of Table 8 yields a lower inaccuracy for the total and all four categories. This is our preferred model.

We conclude this subsection with a few words about no-change extrapolation. This is quite a popular method of prediction and it does have an economic interpretation. Adding and subtracting from the right hand side of (5.6) the CPI log-change DP^* , we obtain

$$\Delta W_{gt} = \bar{W}_{gt}[DP_{gt} - DP_t^*] + \bar{W}_{gt}DQ_{gt} - \bar{W}_{gt}DQ_t$$

as $DM_t - DP_t^* = DQ_t$. As $\Delta W_{gt} = 0$ under no-change extrapolation it follows that

$$\bar{W}_{gt}DQ_{gt} = \bar{W}_{gt}DQ_t - \bar{W}_{gt}[DP_{gt} - DP_t^*].$$

This is a composite demand equation of the same form as (4.1) with

$\theta_g = \bar{W}_{gt}$ and $\phi = -1$; i.e., with an income and price elasticity of 1 and -1

respectively. By a similar argument, no-change extrapolation of the conditional budget shares implies unitary conditional income elasticities and conditional own-price elasticities of minus 1.

The result that the demand models dominate no-change extrapolation can thus be interpreted as saying that the income and price elasticities are not unitary.

6. CONCLUDING COMMENTS

In this paper we have extensively analysed the recent consumption in Western Australia of four categories of energy used in transport, (i) motor spirit, (ii) automotive diesel oil used in land transport, (iii) bunker fuel and (iv) aviation fuel. We developed Divisia indexes of the volume and price of energy which provide convenient summary measures.

We introduced an energy demand model which relates consumption to income and relative prices. This is a system-wide model, firmly based in microeconomic theory and which has the attractive feature of analysing the consumption of the four categories simultaneously. The model draws a distinction between the total consumption of energy and the allocation of that total to the four categories.

We estimated two versions of the model and found that income and relative prices were significant determinants of consumption.

On the basis of the information inaccuracy, our preferred income and own-price elasticities are (from Table 10):

	<u>Income</u>	<u>Own-Price</u>
Motor spirit	.25	-.13
Land transport ADO	.26	-.04
Bunker fuel	.28	-.04
Aviation fuel	.37	-.03

We note that these elasticities are quite low (but not zero!). They are to be interpreted as short-run elasticities. Over the longer term as the capital stock of the transport sector is adjusted in response to changes in real income and relative prices, all elasticities could be expected to be higher in absolute value. We also found that our demand model performs substantially better than no-change extrapolation, which is based on unitary elasticities.

APPENDIX

In this appendix we use consumer demand theory to justify the demand equations used in the text. We use consumption theory notwithstanding the fact that some components of the demand for energy used in transport derive from their use as factors of production. This is because the same form of demand equations emerge when the differential approach is applied to factor demand theory (Theil, 1980).

The Economic Theory of the Consumer

Letting $p = [p_1, \dots, p_n]'$ and $q = [q_1, \dots, q_n]'$, the consumer chooses q to maximize the utility function $u(q)$ subject to the budget constraint $p'q = M$. The first-order conditions are the budget constraint and $\partial u / \partial q = \lambda p$, where λ is the marginal utility of income. Differentiating these conditions with respect to p and M gives Barten's (1964) fundamental matrix equation,

$$(A1) \quad \begin{bmatrix} U & p' \\ p' & 0 \end{bmatrix} \begin{bmatrix} \partial q / \partial M & \partial q / \partial p' \\ -\partial \lambda / \partial M & -\partial \lambda / \partial p' \end{bmatrix} = \begin{bmatrix} 0 & \lambda I \\ 1 & -q' \end{bmatrix},$$

where $U = \partial^2 u / \partial q \partial q'$ and $\partial q / \partial M$ and $\partial q / \partial p'$ are the income and price derivatives of the demand functions. Solving (A1) gives

$$(A2) \quad \frac{\partial q}{\partial p'} = \lambda U^{-1} - \frac{\lambda}{\partial \lambda / \partial M} \frac{\partial q}{\partial M} \frac{\partial q'}{\partial M} - \frac{\partial q}{\partial M} q',$$

for the price derivatives and

$$(A3) \quad \frac{\partial q}{\partial M} = \frac{\partial \lambda}{\partial M} U^{-1} p$$

for the income derivatives.

Defining $P = \text{diag } [p]$, we premultiply (A2) by P and post-multiply by P/M to give

$$(A4) \quad P \frac{\partial q}{\partial p}, P/M = v - \phi \theta \theta' - \theta w',$$

where $v = \lambda P U^{-1} P/M = [v_{ij}]$, $\phi = \frac{\lambda/M}{\partial \lambda / \partial M} = \left(\frac{\partial \log \lambda}{\partial \log M} \right)^{-1}$, $\theta = P \partial q / \partial M = [\theta_i]$ and $w = P q / M = [w_i]$. Dividing (A3) by $\partial \lambda / \partial M$ and premultiplying by $\lambda P / M$ gives

$$\frac{\lambda/M}{\partial \lambda / \partial M} P \frac{\partial q}{\partial M} = \lambda P U^{-1} P/M = (\lambda P U^{-1} P/M) \mathbf{1},$$

where $\mathbf{1} = [1, \dots, 1]'$, or

$$(A5) \quad \phi \theta = v \mathbf{1}.$$

We write the demand equations in differential form as

$$dq = \frac{\partial q}{\partial M} dM + \frac{\partial q}{\partial p}, dp.$$

Premultiplying by P/M gives

$$W d(\log q) = \theta d(\log M) + \left[P \frac{\partial q}{\partial p}, P/M \right] d(\log p),$$

where $W = \text{diag } [w]$. Substituting the right side of (A4) for $P \frac{\partial q}{\partial p}, P/M$, we obtain

$$(A6) \quad \begin{aligned} W d(\log q) &= \theta d(\log M) + (v - \phi \theta \theta' - \theta w') d(\log p) \\ &= \theta [d(\log M) - d(\log P)] + v(I - \mathbf{1} \theta') d(\log p), \end{aligned}$$

where $d(\log P) = w' d(\log p)$ and where we have used (A5) to substitute $v \mathbf{1}$ for $\phi \theta$. The term $d(\log M) - d(\log P)$ is the change in real income, which we write as $d(\log Q)$. Writing $d(\log P') = \theta' d(\log p)$ for the Frisch price index, (A6) can be expressed as

$$W d(\log q) = \theta d(\log Q) + v[d(\log p) - \mathbf{1} d(\log P')].$$

This is a system of n demand equations, the i^{th} being

$$(A7) \quad w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^n v_{ij} [d(\log p_j) - d(\log P')].$$

Block-Independent Preferences

Let the n goods now be divided into G groups, $S_1, \dots, S_G$, such that each good belongs to only one group. Further, let the consumer's preferences be such that the utility function is the sum of G sub-utility functions, each involving the quantities of only one group,

$$u(q) = \sum_{g=1}^G u_g(q_g),$$

where q_g is the vector of the q_i 's that fall under S_g . Then, when the goods are numbered appropriately, the Hessian U and its inverse are block-diagonal. Accordingly, this form of the utility function is known as block-independent preferences (Theil, 1975/76), or strong separability.

Under block-independence, $v_{ij} = 0$ for i and j in different groups and (A5) becomes

$$(A8) \quad \sum_{j \in S_g} v_{ij} = \phi \theta_i \quad i \in S_g, \quad g=1, \dots, G.$$

The demand equation (A7) for $i \in S_g$ becomes

$$(A9) \quad w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j \in S_g} v_{ij} d\left(\log \frac{p_j}{p_i}\right),$$

so that the only deflated prices which appear are those of goods belonging to the same group as i .

Composite Demand Equations

We write $W_g = \sum_{i \in S_g} w_i$ and $\Theta_g = \sum_{i \in S_g} \theta_i$ for the average and marginal shares of group g , and define the group volume and Frisch price indexes as $d(\log Q_g) = \sum_{i \in S_g} (w_i/W_g) d(\log q_i)$, $d(\log P'_g) = \sum_{i \in S_g} (\theta_i/\Theta_g) d(\log p_i)$. If we then add (A9) over $i \in S_g$ and use (A8) and the symmetry of $[v_{ij}]$, we obtain the composite demand equation for S_g as a group (see below),

$$(A10) \quad W_g d(\log Q_g) = \theta_g d(\log Q) + \phi \theta_g d\left(\log \frac{P'_g}{P'}\right).$$

Thus only the deflated price of the group $d\left(\log \frac{P'_g}{P'}\right) = d(\log P'_g) - d(\log P')$ and income determine the demand for the group as a whole. Equation (3.1) is (A10) with the Frisch relative price replaced by $d(\log P_g) - d(\log P)$.

Conditional Demand Equations

Combining (A9) and (A10) we obtain (see below),

$$(A11) \quad w_i d(\log q_i) = \frac{\theta_i}{\theta_g} W_g d(\log Q_g) + \sum_{j \in S_g} v_{ij} d\left(\log \frac{p_j}{p'_g}\right).$$

This is the demand equation for $i \in S_g$, given the demand for the group as a whole $W_g d(\log Q_g)$. As the variables on the right of this equation are exclusively concerned with the group S_g to which the i th commodity belongs, it is known as a conditional demand equation. Equation (3.3) is (A11) without the price substitution term.

Derivation of Equations (A10) and (A11)

Equation (A10) is obtained by summing both sides of (A9) over $i \in S_g$. The derivation of the variable on the left and the first term on the right of equation (A10) is straightforward. We obtain the substitution term as follows. The sum over $i \in S_g$ of the substitution term of equation (A9) is

$$(A12) \quad \sum_{i \in S_g} \sum_{j \in S_g} v_{ij} d\left(\log \frac{p_j}{p'_g}\right) = \phi \sum_{j \in S_g} \theta_j [d(\log p_j) - d(\log P')],$$

where we have used $\sum_{i \in S_g} v_{ij} = \phi \theta_j$, which follows from the symmetry of $[v_{ij}]$ and (A8). Since $\theta_g = \sum_{j \in S_g} \theta_j$, the second member of (A12) can be expressed as

$$\phi\theta_g \left[\sum_{j \in S_g} \frac{\theta_j}{\theta_g} d(\log p_j) - d(\log P') \right] = \phi\theta_g [d(\log P'_g) - d(\log P')],$$

which is the substitution term of equation (A10).

To derive equation (A11), we rearrange (A9) and (A10) to give

$$(A13) \quad d(\log Q) = \frac{w_i}{\theta_i} d(\log q_i) - \sum_{j \in S_g} \frac{v_{ij}}{\theta_i} d\left(\log \frac{p_j}{P'}\right),$$

$$(A14) \quad d(\log Q) = \frac{W_g}{\theta_g} d(\log Q_g) - \phi d\left(\log \frac{P'_g}{P'}\right).$$

Equating the right side of (A13) with that of (A14) and rearranging gives

$$\begin{aligned} w_i d(\log q_i) &= \frac{\theta_i}{\theta_g} W_g d(\log Q_g) + \sum_{j \in S_g} v_{ij} d(\log p_j) \\ &\quad - \sum_{j \in S_g} v_{ij} d(\log P') - \phi\theta_i d(\log P'_g) + \phi\theta_i d(\log P') \\ &= \frac{\theta_i}{\theta_g} W_g d(\log Q_g) + \sum_{j \in S_g} v_{ij} [d(\log p_j) - d(\log P')], \end{aligned}$$

which follows from (A8). This is equation (A11).

FOOTNOTES

1. As (4.2) contains identical regressors OLS is equivalent to GLS.
2. See, e.g., Clements (1981), Clements and Nguyen (1980), Clements and Theil (1979), Theil (1975/76, 1980) and Theil and Suhm (1981).
3. To test for lagged adjustment, we added a lagged dependent variable to the two equations. This did not result in an improvement as this variable was highly insignificant.
4. We also estimated the conditional demand equations under the Working (1943) - Leser (1963) parametrization whereby the conditional marginal share is not constant, but exceeds the conditional budget share by a constant, $\theta_i/\theta_g = \bar{w}_{it}/\bar{w}_{gt} + \alpha_i$. The demand equation for i is then

$$\bar{w}_{it}(Dq_{it} - DQ_{gt}) = \alpha_i \bar{w}_{gt} DQ_{gt} + \varepsilon_{it},$$

which implies a conditional income elasticity of the form

$1 + \alpha_i/(\bar{w}_{it}/\bar{w}_{gt})$. The resulting elasticities were not substantially different.

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