THE PATTERN OF FINANCIAL ASSET

HOLDINGS IN AUSTRALIA

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2. A SYSTEM-WIDE PORTFOLIO MODEL

In this section we set out a simple portfolio allocation model which explains asset holdings in terms of wealth and interest rates. The model is applied to quarterly Australian data over the 1970s and then used for a number of applications. The results of this section are an extension of Clements and Taylor (1981) and Taylor and Clements (1981).

The Model

We write A_i for the nominal value of (financial) asset i ($i=1, \dots, n$), $a_i = A_i/P$ for the real value of asset i ($P = \text{CPI}$), $W = \sum_{i=1}^n a_i$ for total real wealth, $s_i = a_i/W$ for the i th portfolio share and r_i for the interest rate on asset i . We postulate that the demand for asset i takes the following form:

$$s_i = \alpha_i + \beta_i \log W + \sum_{j=1}^n \pi_{ij} r_j. \quad (1)$$

The first part of (1), $s_i = \alpha_i + \beta_i \log W$, is the Working (1943) - Leser (1963) Engel curve model from consumption theory. This model is very flexible and robust in consumer demand applications² and there is a presumption that it would probably also be satisfactory for asset demand. The coefficient β_i represents 100 times the effect on s_i of a 1 percent increase in wealth, all interest rates remaining constant. Equation (1) is linear in the levels of interest rate variables, which is a specification corresponding closely to the widely used Cagan (1956) form of the demand for money equation. The coefficient π_{ij} gives the effect on s_i of a 1 percentage point increase in r_j (other things equal). Finally, (1) is very similar to Deaton and Muellbauer's (1980) consumer demand model AIDS (the almost ideal demand system), which possesses a number of desirable aggregation and flexibility properties.

The Data

We use quarterly Australian data from March 1970-June 1979 and distinguish $n=4$ assets, (i) fixed deposits, (ii) total savings bank deposits, (iii) permanent building society deposits and (iv) special bonds plus Australian savings bonds (hereafter, bonds). Financial wealth is defined as the sum of the holdings of these assets. The data are given in Tables 1 and 2; see Taylor and Clements (1981) for full definitions and sources.

Results

Initial results suggest that savings bonds are qualitatively different to special bonds. We model this quality change by allowing the wealth coefficients to be different before and after the introduction of savings bonds. This allows the wealth elasticities to change with the introduction of the new asset. Our estimating equation becomes

$$s_i = \alpha_i + \beta_i D \cdot \log W + \beta'_i (1-D) \cdot \log W + \sum_{j=1}^n \pi_{ij} r_j,$$

where $D = 1$ before the introduction of savings bonds in 1976, 0 afterwards.

The homogeneity- and symmetry-constrained FIML results are given in Table 3.³ As can be seen, all the wealth coefficients are significant except for fixed deposits, implying that these have an elasticity insignificantly different from unity. All own-interest rate coefficients are significantly positive, as they should be. With the exception of π_{24} , which is not significantly different from zero, all the off-diagonal π_{ij} 's are negative and significant. This indicates that the assets are pairwise substitutes, which is reasonable as they all play a similar store-of-value role. Building society deposits are a strong luxury (wealth elasticity greater than unity), while the other three assets are necessities.

In Taylor and Clements (1981) the results of Table 3 are compared with the unrestricted estimates. The homogeneity and symmetry restrictions

TABLE 2
INTEREST RATES: AUSTRALIA,
MARCH 1970 - JUNE 1979

(Seasonally unadjusted; percent p.a.)

Year and quarter		Fixed deposits with trading banks	Savings bank deposits	Permanent building society deposits	Special/Australian savings bonds
1970	M	5.15	3.78	6.00	5.20
	J	5.30	3.79	6.00	5.80
	S	5.30	3.80	6.00	6.40
	D	5.30	3.80	6.30	6.40
1971	M	5.30	3.80	6.60	6.40
	J	5.30	3.80	6.60	6.40
	S	5.30	3.81	6.60	6.40
	D	5.30	3.82	6.60	6.10
1972	M	4.90	3.82	6.60	5.60
	J	4.50	3.82	6.55	5.40
	S	4.50	3.83	6.50	5.40
	D	4.50	3.85	6.50	5.40
1973	M	4.50	3.87	6.50	5.40
	J	4.50	3.88	6.35	5.60
	S	4.50	3.89	6.40	6.00
	D	5.75	4.06	7.20	7.10
1974	M	7.25	4.26	7.85	8.00
	J	7.50	4.32	8.90	8.00
	S	8.50	4.71	9.90	8.60
	D	9.50	5.15	9.90	9.20
1975	M	9.50	5.14	9.95	9.10
	J	9.50	5.08	9.70	9.00
	S	9.50	5.12	9.35	9.10
	D	9.25	5.14	9.10	9.20
1976	M	8.75	5.18	9.00	9.85
	J	8.63	5.24	9.10	9.85
	S	8.75	5.27	9.10	9.20
	D	8.75	5.27	9.15	9.60
1977	M	8.88	5.29	9.20	10.00
	J	9.00	5.33	9.20	10.00
	S	9.00	5.34	9.15	10.00
	D	9.00	5.33	9.10	9.63
1978	M	9.00	5.26	8.95	9.13
	J	9.00	5.19	8.80	9.00
	S	9.00	5.18	8.80	9.00
	D	8.75	5.11	8.50	8.88
1979	M	8.50	5.07	8.25	8.75
	J	8.50	5.11	8.30	9.00
Mean		7.21	4.57	7.96	7.82

Interest rates are arithmetic averages of the current and previous quarter's rates; they are thus lined up with the asset holdings given in Table 1.

are formally rejected by a likelihood ratio test; the observed value of the test statistic is $-2 \log \lambda = 19.1$, which is greater than the critical value of $\chi^2(6)$ at the 5 percent level of 12.6. However, there is a marked improvement in the signs and significance of the interest rate coefficients when the restrictions are imposed, so that there are good economic grounds for accepting the restricted model.

A Decomposition of the Change in the Portfolio Share

In this sub-section we use the model to decompose the change in each portfolio share into its components. In Tables 4-7 we give the wealth, interest rate and residual components of the change in the share Δs_i .

Over the 1970s one of the most dramatic changes in the Australian financial sector has been the growth of building societies at the expense of the savings banks (see Tables 5 and 6). The share of wealth held in the form of savings bank deposits fell by 18 percentage points over this period (from 72 to 54 percent; see Table 1), while building societies grew by 14 percentage points (from 9 to 23 percent). Looking at the second last row of Table 6 which gives the total change, the major reason for the growth of building societies is the increase in wealth; this accounts for 13.8 percentage points of the 14.2 total. This is consistent with the high wealth elasticity for this asset given in Table 3. The effect of increases in the interest rate on building society deposits is to increase the share by 3.4 points, but this is almost totally offset by increases in the other three rates; the effect of all interest rate changes is to decrease the share by .2 points. For a further analysis, see Clements and Taylor (1981).

TABLE 5

DECOMPOSITION OF CHANGE IN PORTFOLIO SHARE OF SAVINGS BANK DEPOSITS: AUSTRALIA,

MARCH 1970 - JUNE 1979

CHANGE IN SHARE OF WEALTH IN ASSET 2			COMPONENTS OF Δs_2					RESIDUAL
WEALTH			INTEREST RATES					
$\beta_2 D \cdot \Delta \log W$			$\sum_{j=1}^4 \pi_{2j} \Delta r_j$					
Δs_2	$+ \beta_2' (1-D) \cdot \Delta \log W$	$\pi_{21} \Delta r_1$	$\pi_{22} \Delta r_2$	$\pi_{23} \Delta r_3$	$\pi_{24} \Delta r_4$			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
1970 J	-0.55	-0.18	-0.07	0.01	0.00	0.11	0.06	-0.43
S	-0.56	-0.30	0.00	0.01	0.00	0.11	0.12	-0.38
D	-0.38	-0.30	0.00	0.00	-0.22	0.00	-0.22	0.14
1971 M	-0.42	-0.29	0.00	0.00	-0.22	0.00	-0.22	0.08
J	-0.83	-0.39	0.00	0.00	0.00	0.00	0.00	-0.44
S	-0.96	-0.47	0.00	0.01	0.00	0.00	0.01	-0.50
D	-0.76	-0.34	0.00	0.01	0.00	-0.06	-0.05	-0.37
1972 M	-0.45	-0.28	0.18	0.00	0.00	-0.10	0.08	-0.25
J	-0.39	-0.54	0.18	0.00	0.04	-0.04	0.17	-0.02
S	-0.33	-0.81	0.00	0.01	0.04	0.00	0.05	0.43
D	-0.15	-0.98	0.00	0.02	0.00	0.00	0.02	0.81
1973 M	-0.04	-0.93	0.00	0.02	0.00	0.00	0.02	0.87
J	0.16	-0.57	0.00	0.01	0.11	0.04	0.16	0.57
S	-0.20	-0.32	0.00	0.01	-0.04	0.08	0.05	0.07
D	-0.69	0.01	-0.55	0.16	-0.58	0.21	-0.75	0.06
1974 M	-0.50	0.13	-0.66	0.19	-0.47	0.17	-0.76	0.13
J	-0.25	0.22	-0.11	0.06	-0.76	0.00	-0.81	0.33
S	-1.01	0.59	-0.44	0.38	-0.72	0.11	-0.67	-0.93
D	-0.85	0.21	-0.44	0.43	0.00	0.11	0.10	-1.16
1975 M	-0.30	-0.39	0.00	-0.01	-0.04	-0.02	-0.06	0.15
J	-0.50	-0.52	0.00	-0.06	0.18	-0.02	0.10	-0.09
S	-0.43	-0.75	0.00	0.04	0.25	0.02	0.31	0.01
D	-0.18	-0.32	0.11	0.02	0.18	0.02	0.33	-0.19
1976 M	-0.79	-0.22	0.22	0.04	0.07	0.12	0.45	-1.02
J	-0.76	-0.51	0.05	0.06	-0.07	0.00	0.04	-0.29
S	-0.24	-0.31	-0.05	0.03	0.00	-0.12	-0.15	0.21
D	-0.77	0.08	0.00	0.00	-0.04	0.08	0.04	-0.89
1977 M	-0.69	0.18	-0.06	0.02	-0.04	0.08	0.00	-0.87
J	-0.52	-0.31	-0.05	0.04	0.00	0.00	-0.01	-0.20
S	-0.65	-0.27	0.00	0.01	0.04	0.00	0.05	-0.42
D	-0.54	-0.13	0.00	-0.01	0.04	-0.07	-0.04	-0.37
1978 M	-0.32	-0.30	0.00	-0.07	0.11	-0.10	-0.05	0.04
J	-0.33	-0.56	0.00	-0.07	0.11	-0.02	0.02	0.22
S	-0.37	-0.26	0.00	-0.01	0.00	0.00	-0.01	-0.10
D	-0.60	-0.07	0.11	-0.07	0.22	-0.02	0.24	-0.77
1979 M	-0.65	-0.54	0.11	-0.04	0.18	-0.02	0.23	-0.34
J	-0.25	-0.46	0.00	0.04	-0.04	0.05	0.05	0.16
SUM	-18.05	-11.17	-1.47	1.29	-1.66	0.72	-1.12	-5.76
MEAN	-0.49	-0.30	-0.04	0.03	-0.04	0.02	-0.03	-0.16

COLUMN (1)=(2)+(3)+(4)+(5)+(6)+(8). THE PERIOD 1970 J REFERS TO THE TRANSITION FROM THE MARCH TO JUNE QUARTER 1970 AND SO ON. ALL ENTRIES ARE TO BE DIVIDED BY 100.

TABLE 7

DECOMPOSITION OF CHANGE IN PORTFOLIO SHARE OF SPECIAL/AUSTRALIAN SAVINGS BONDS: AUSTRALIA,

MARCH 1970 - JUNE 1979

CHANGE IN SHARE OF WEALTH IN ASSET 4			COMPONENTS OF Δs_4					RESIDUAL	
WEALTH			INTEREST RATES						
Δs_4			$\beta_4 D \cdot \Delta \log W$ $+ \beta_4' (1-D) \cdot \Delta \log W$	$\pi_{41} \Delta r_1$	$\pi_{42} \Delta r_2$	$\pi_{43} \Delta r_3$	$\pi_{44} \Delta r_4$	$\sum_{j=1}^4 \pi_{4j} \Delta r_j$	
YEAR & QUARTER	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
970 J	-0.13	-0.03	-0.07	0.00	0.00	0.37	0.30	-0.40	
S	0.19	-0.05	0.00	0.00	0.00	0.37	0.37	-0.13	
D	0.20	-0.05	0.00	0.00	-0.10	0.00	-0.10	0.35	
971 M	-0.06	-0.05	0.00	0.00	-0.10	0.00	-0.10	0.09	
J	-0.10	-0.07	0.00	0.00	0.00	0.00	0.00	-0.03	
S	0.08	-0.08	0.00	0.00	0.00	0.00	0.00	0.16	
D	0.12	-0.06	0.00	0.00	0.00	-0.19	-0.18	0.36	
972 M	-0.24	-0.05	0.20	0.00	0.00	-0.31	-0.11	-0.08	
J	-0.34	-0.09	0.20	0.00	0.02	-0.12	0.09	-0.33	
S	-0.13	-0.14	0.00	0.00	0.02	0.00	0.02	-0.01	
D	-0.20	-0.17	0.00	0.00	0.00	0.00	0.00	-0.03	
973 M	-0.38	-0.16	0.00	0.00	0.00	0.00	0.00	-0.22	
J	-0.31	-0.10	0.00	0.00	0.05	0.12	0.17	-0.39	
S	-0.15	-0.06	0.00	0.00	-0.02	0.25	0.23	-0.33	
D	0.01	0.00	-0.61	0.03	-0.26	0.68	-0.15	0.16	
974 M	0.01	0.02	-0.74	0.04	-0.21	0.56	-0.35	0.33	
J	-0.07	0.04	-0.12	0.01	-0.34	0.00	-0.45	0.34	
S	-0.05	0.10	-0.49	0.07	-0.32	0.37	-0.36	0.21	
D	-0.25	0.04	-0.49	0.08	0.00	0.37	-0.03	-0.25	
975 M	-0.38	-0.07	0.00	-0.00	-0.02	-0.06	-0.08	-0.23	
J	-0.30	-0.09	0.00	-0.01	0.08	-0.06	0.01	-0.22	
S	-0.12	-0.13	0.00	0.01	0.11	0.06	0.18	-0.17	
D	-0.03	-0.05	0.12	0.00	0.08	0.06	0.27	-0.24	
976 M	1.85	-0.04	0.25	0.01	0.03	0.40	0.69	1.20	
J	1.90	-0.08	0.06	0.01	-0.03	0.00	0.04	1.94	
S	-0.01	-0.05	-0.06	0.01	0.00	-0.40	-0.46	0.49	
D	0.02	0.01	0.00	0.00	-0.02	0.25	0.23	-0.22	
977 M	0.03	0.03	-0.06	0.00	-0.02	0.25	0.17	-0.17	
J	0.07	-0.05	-0.06	0.01	0.00	0.00	-0.05	0.17	
S	0.47	-0.04	0.00	0.00	0.02	0.00	0.02	0.50	
D	0.51	-0.02	0.00	-0.00	0.02	-0.23	-0.22	0.75	
978 M	-0.12	-0.05	0.00	-0.01	0.05	-0.31	-0.28	0.20	
J	-0.39	-0.09	0.00	-0.01	0.05	-0.08	-0.05	-0.26	
S	-0.04	-0.04	0.00	-0.00	0.00	0.00	-0.00	0.00	
D	0.05	-0.01	0.12	-0.01	0.10	-0.07	0.13	-0.07	
979 M	-0.33	-0.08	0.12	-0.01	0.08	-0.08	0.11	-0.36	
J	-0.18	-0.07	0.00	0.01	-0.02	0.16	0.15	-0.25	
SUM	1.20	-1.88	-1.64	0.25	-0.74	2.36	0.23	2.85	
MEAN	0.03	-0.05	-0.04	0.01	-0.02	0.06	0.01	0.08	

COLUMN (1)=(2)+(3)+(4)+(5)+(6)+(8). THE PERIOD 1970 J REFERS TO THE TRANSITION FROM THE MARCH TO JUNE QUARTER 1970 AND SO ON. ALL ENTRIES ARE TO BE DIVIDED BY 100.

TABLE 8

WELFARE COST OF INTEREST RATE CHANGES AND THE RATE OF INFLATION: AUSTRALIA,
MARCH 1970 - JUNE 1979

Year and quarter	Welfare cost as a percentage of wealth	Welfare cost in millions of 1966/67 dollars	Welfare cost as a percentage of real GDP	Rate of inflation (percent p.a.)
1970 M	.68	61	.90	3.20
J	.65	58	.82	3.73
S	.87	79	1.11	3.80
D	.62	57	.80	4.88
1971 M	.50	46	.63	4.92
J	.50	47	.65	5.40
S	.50	49	.65	6.70
D	.44	42	.56	7.19
1972 M	.78	76	1.01	7.12
J	1.28	128	1.68	6.23
S	1.27	132	1.72	5.70
D	1.29	139	1.77	4.50
1973 M	1.31	147	1.83	5.67
J	1.36	155	1.94	8.19
S	1.55	179	2.18	10.62
D	.46	53	.64	13.23
1974 M	.04	5	.06	13.57
J	.73	84	1.02	14.40
S	1.78	198	2.41	16.05
D	1.52	168	2.00	16.25
1975 M	1.67	187	2.25	17.56
J	1.61	184	2.16	16.94
S	1.29	152	1.79	12.10
D	.81	97	1.16	14.04
1976 M	.33	40	.46	13.38
J	.26	32	.37	12.32
S	.23	28	.32	13.93
D	.23	28	.32	14.40
1977 M	.34	41	.47	13.63
J	.37	46	.52	13.44
S	.37	47	.53	13.15
D	.34	43	.49	9.26
1978 M	.47	60	.67	8.20
J	.59	77	.86	7.88
S	.59	78	.85	7.86
D	.51	67	.72	7.76
1979 M	.45	60	.64	8.20
J	.41	56	.60	8.84
Mean	.76	85	1.04	9.85

Real GDP is in terms of 1966/67 prices and is seasonally adjusted.

Source: Australian Bureau of Statistics, *Historical Series of Quarterly Estimates of National Income and Expenditure, Australia, September Quarter 1959 to June Quarter 1980*, cat. no. 5207.0. The rate of inflation is the quarter-on-quarter percentage change of the Consumer Price Index (weighted average of the six State capital cities).

Source: ABS, *Consumer Price Index, Australia*, cat. no. 6401.0, various issues.

An Out-of-Sample Test

We now use the model to predict asset holdings for the ten quarters following the estimation period. The data are given in Tables 9 and 10, while the forecasts are presented in Table 11. As can be seen, there is a tendency for the forecast errors to grow over time and, with the exception of fixed deposits and bonds, most are significant.⁴ However, it should be stressed that this represents a stringent test of the model for the following reasons. (i) In contrast to standard practice, these forecasts are entirely "hands off" in that no *ad hoc* adjustments are made. (ii) There is more variability of relative interest rates over this period than over the estimation period. In part this is due to the deregulation of bank interest rates in December 1980.

3. OTHER APPROACHES

We now use basically the same data as before to explore the applicability of two alternative approaches to modeling asset holdings. The first is the Markov chain model, which is based on and extends Anderson and Clements (1982). The second approach is an application of the logistic growth model to one of the assets; this is based on Broadbent, Clements and Johnson (1981).

A Markov Chain Analysis

Consider \$1 of financial wealth which can be held in any of n assets. The Markov chain model is a stochastic process describing where this \$1 will be held, with the probability of it being held in a certain

TABLE 10

INTEREST RATES: AUSTRALIA,

SEPTEMBER 1979 - DECEMBER 1981

(Seasonally unadjusted; percent p.a.)

Year and quarter		Fixed deposits with trading banks	Savings bank deposits	Permanent building society deposits	Special/Australian savings bonds
1979	S	8.50	5.11	8.35	9.25
	D	8.75	5.08	8.40	9.25
1980	M	9.00	5.14	8.40	9.50
	J	9.50	5.30	8.45	9.75
	S	10.00	5.61	8.55	10.00
	D	10.75	5.83	8.60	10.88
1981	M	11.75	6.13	8.55	11.88
	J	12.13	6.42	8.60	12.25
	S	12.63	6.75	9.10	12.25
	D	13.00	7.09	9.60	12.25
Mean		10.60	5.85	8.66	10.73

See notes to Table 2

asset depending only on where it was last held. Thus the Markov model specifies that where the wealth was held before the previous period is irrelevant for the current placement of the \$1.

Let $p_{ij} \geq 0$ be the probability that the \$1 will be held in the form of asset j given that it was previously in asset i ; we shall refer to p_{ij} as the $(i,j)^{\text{th}}$ transition probability. We assume that the \$1 will continue to be held in the form of one of the assets, implying $\sum_{j=1}^n p_{ij} = 1$ ($i=1, \dots, n$). Writing s_{i0} for the initial proportion of wealth held in i , $s_{i0}p_{ij}$ of this goes to asset j next period. The total proportion of wealth in j next period is the sum of this over i , $s_{j1} = \sum_i s_{i0}p_{ij}$, which we write in vector form as

$$s'_1 = s'_0 P, \quad (2)$$

where $s_t = [s_{it}]$ is the t^{th} asset distribution ($t=0,1$) and $P = [p_{ij}]$ is the transition matrix. For the subsequent period we have $s'_2 = s'_1 P = s'_0 P^2$ [from (2)]. Repeated application of this procedure yields $s'_t = s'_0 P^t$. The $(i,j)^{\text{th}}$ element of P^t is thus the probability of moving \$1 from asset i to j in t steps (periods).

If P^t for some t consists exclusively of nonzero elements (implying that any transition can be made in at most t steps), then the Markov chain is said to be regular. In this case, it can be shown (Theil, 1972, Chapter 5) that no matter what initial distribution s_0 , there is always convergence to the same steady-state distribution $\pi = [\pi_i]$ as $t \rightarrow \infty$,

$$\lim_{t \rightarrow \infty} s'_0 P^t = \pi'.$$

Furthermore, the steady-state proportions π_i sum to one and are all positive. The vector π is the steady-state distribution in the sense that once it is attained, the system will stay there: $\pi' = \pi' P$. This does not mean that there is no movement of wealth between assets in the steady state, but that movements cancel each other out to keep the asset distribution constant.

We apply the Markov model to the Australian asset data with savings bank deposits disaggregated into (i) investment accounts and

TABLE 12

MAXIMUM LIKELIHOOD ESTIMATES OF TRANSITION PROBABILITIES: AUSTRALIA,

SEPTEMBER 1973 - JUNE 1979

Previous asset (i of p_{ij})	Current asset (j of p_{ij})				
	1	2	3	4	5
1. Fixed deposits	.733	0	.166	.102	0
2. Savings bank investment accounts	.022	.913	.004	.003	.058
3. Savings bank ordinary accounts	.020	.053	.908	.019	0
4. Permanent building society deposits	.113	0	0	.837	.050
5. Special/Australian savings bonds	0	0	.073	.200	.727

FIGURE 1 FIGURE 2

FIGURE 5 KEY

—————	Asset 1: Fixed Deposits
.....	Asset 2: Savings bank investment accounts
-----	Asset 3: Savings bank ordinary accounts
-----	Asset 4: Permanent building society deposits
-----	Asset 5: Special/Australian savings bonds

FIGURE 6

FOOTNOTES

1. See Evans (1979) for a recent survey in which this conclusion emerges very clearly.
2. See, e.g., Clements and Theil (1979), Izan and Clements (1979) and Leser (1963).
3. We use Wymer's (1978) RESIMUL program.
4. We use Salkever's (1976) dummy variable technique to obtain the forecast error variances.
5. We are grateful to John Anderson for providing us with these graphs which were produced with his program MATGRAF (Anderson, 1981).
6. The variable $\bar{r}$ is defined as the rate of return on SIAs less a weighted average of the rates on the five assets listed in Table 12, where the weights are the current period shares of the assets in the total.

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1. INTRODUCTION

An understanding of the determinants of holdings of financial assets is of crucial importance for a number of macroeconomic issues. The growth of the non-bank financial intermediaries, the substitutability between money and other assets, the determination of interest rates, and so on are all related to the pattern of financial asset holdings. The most popular approach to modeling holdings of financial assets is the multivariate stock adjustment model due to Brainard and Tobin (1968). This has the distinct advantage over single equation approaches that the balance sheet constraints are built in, so that the asset demands sum to total wealth. The Brainard-Tobin model does, however, suffer from the problem that there are a large number of unconstrained parameters to be estimated, with the result that many applications are characterized by statistical imprecision of a substantial proportion of the estimates.¹ The source of the problem is that interest rates are highly collinear, so that the data cannot determine such a large number of free parameters.

In this paper we review and extend our recent work on alternative approaches to modeling holdings of financial assets by households. In Section 2 we present a new portfolio model in the Brainard-Tobin tradition, but which has fewer free parameters. The model is estimated and then used (i) to decompose changes in asset holdings into effects due to wealth and interest rate movements; (ii) to measure the welfare cost of interest rate movements; and (iii) in an out-of-sample forecasting exercise. Section 3 contains a Markov chain analysis of the evolution of asset holdings, together with an application of a logistic model.

Wealth and interest rate elasticities for asset i are

$$\partial \log a_i / \partial \log W = 1 + \beta_i / s_i \quad \partial \log a_i / \partial \log r_j = r_j \pi_{ij} / s_i.$$

Thus an asset with positive (negative) β_i has a wealth elasticity larger (smaller) than unity.

The balance sheet restrictions can be derived by summing equation (1) over $i=1, \dots, n$. Since the asset shares sum to unity this gives

$$1 = \sum_i \alpha_i + \sum_i \beta_i \log W + \sum_i \sum_j \pi_{ij} r_j.$$

For this equality to hold for all values of the variables on the right, the coefficients must satisfy the following constraints:

$$\sum_i \alpha_i = 1 \quad \sum_i \beta_i = 0 \quad \sum_i \pi_{ij} = 0.$$

With these restrictions, equation (1) satisfies the balance sheet constraint ($\sum_i a_i = W$ or $\sum_i s_i = 1$).

The interest rate coefficients of (1) are subject to homogeneity and Slutsky symmetry restrictions. Homogeneity means that it is only changes in relative rates of return which matter. With specification (1), the homogeneity condition is formulated in terms of equal percentage point changes, not equi-proportionate changes. Homogeneity implies

$$\sum_j \pi_{ij} = 0 \quad i=1, \dots, n.$$

Slutsky symmetry means that interest rate substitution effects are symmetric,

$$\pi_{ij} = \pi_{ji} \quad i, j=1, \dots, n.$$

These restrictions are linear in the coefficients and can thus be easily tested and then imposed.

TABLE 1
FINANCIAL ASSET HOLDINGS: AUSTRALIA,
MARCH 1970 - JUNE 1979
(Seasonally adjusted)

Year and quarter		Total financial wealth (real \$M)	Fixed deposits with trading banks (% of total)	Savings bank deposits (% of total)	Permanent building society deposits (% of total)	Special/Australian savings bonds (% of total)
1970	M	8911	11.68	72.15	8.99	7.18
	J	8974	11.82	71.60	9.54	7.05
	S	9080	11.72	71.04	10.00	7.24
	D	9187	11.37	70.66	10.53	7.44
1971	M	9290	11.26	70.24	11.12	7.38
	J	9432	11.54	69.41	11.77	7.28
	S	9605	11.82	68.45	12.36	7.36
	D	9732	11.87	67.69	12.96	7.48
1972	M	9837	11.75	67.24	13.78	7.24
	J	10046	11.46	66.85	14.80	6.90
	S	10366	10.96	66.52	15.76	6.77
	D	10768	10.35	66.37	16.72	6.57
1973	M	11165	9.87	66.33	17.61	6.19
	J	11414	9.79	66.49	17.84	5.88
	S	11556	10.12	66.29	17.86	5.73
	D	11553	10.48	65.60	18.18	5.74
1974	M	11494	10.38	65.10	18.77	5.75
	J	11394	10.30	64.85	19.17	5.68
	S	11137	11.39	63.84	19.15	5.63
	D	11045	13.12	62.99	18.51	5.38
1975	M	11212	14.09	62.69	18.22	5.00
	J	11440	14.19	62.19	18.92	4.70
	S	11777	13.96	61.76	19.70	4.58
	D	11923	13.64	61.58	20.24	4.55
1976	M	12025	13.12	60.79	19.69	6.40
	J	12260	12.71	60.03	18.96	8.30
	S	12404	12.56	59.79	19.36	8.29
	D	12367	12.46	59.02	20.22	8.31
1977	M	12283	12.48	58.33	20.84	8.34
	J	12430	12.54	57.81	21.25	8.41
	S	12561	12.46	57.16	21.51	8.88
	D	12622	12.39	56.62	21.60	9.39
1978	M	12769	12.70	56.30	21.73	9.27
	J	13047	13.12	55.97	22.03	8.88
	S	13178	13.10	55.60	22.46	8.84
	D	13213	13.02	55.00	23.09	8.89
1979	M	13487	13.82	54.35	23.26	8.56
	J	13727	14.34	54.10	23.18	8.38
Mean		11334	12.10	63.13	17.68	7.10

Stocks are arithmetic averages of holdings in the current and previous quarters; they are thus interpreted as at mid-quarter. Total financial wealth is the sum of holdings of the four assets and the CPI (= 1 in 1966/67) is used as the deflator for real wealth. Deposits with savings banks are ordinary and investment accounts plus deposit stock.

TABLE 3
ASSET DEMAND EQUATIONS: AUSTRALIA,
MARCH 1970 - JUNE 1979

Equation	Parameters							Elasticities						R ²
	α_i	β_i	β'_i	π_{i1}	π_{i2}	π_{i3}	π_{i4}	W	W'	r ₁	r ₂	r ₃	r ₄	
1. Fixed Deposits	.2720 (2.0)	-.0169 (1.2)	-.0165 (1.2)	.0135 (8.2)	-.0044 (3.2)	-.0042 (2.8)	-.0049 (3.3)	.86	.86	.81	-.17	-.28	-.32	.73
2. Savings Bank Deposits	3.0722 (15.0)	-.2572 (11.5)	-.2612 (11.8)		.0097 (4.6)	-.0072 (5.3)	.0019 (1.4)	.59	.59	-.05	.07	-.09	.02	.96
3. Permanent Building Society Deposits	-2.8247 (27.6)	.3187 (28.3)	.3189 (28.8)			.0146 (6.3)	-.0032 (2.0)	2.80	2.80	-.17	-.18	.66	-.14	.99
4. Special/Austral- ian Savings Bonds	.4805 (4.1)	-.0446 (3.4)	-.0412 (3.2)				.0062 (3.0)	.37	.42	-.50	.12	-.35	.68	.83

Asymptotic t-values are given in parentheses. R² is the squared correlation coefficient between the actual and predicted value of the dependent variable. Elasticities are evaluated at sample means. Wealth elasticities before the introduction of savings bonds in 1976 are given in the column headed W, and those after the introduction in the column headed W'.

TABLE 4

DECOMPOSITION OF CHANGE IN PORTFOLIO SHARE OF FIXED DEPOSITS: AUSTRALIA,

MARCH 1970 - JUNE 1979

YEAR & QUARTER		CHANGE IN SHARE OF WEALTH IN ASSET 1	COMPONENTS OF Δs_1					RESIDUAL	
		WEALTH	INTEREST RATES						
		Δs_1	$\beta_1 D \cdot \Delta \log W$ $+ \beta_1' (1-D) \cdot \Delta \log W$	$\pi_{11} \Delta r_1$	$\pi_{12} \Delta r_2$	$\pi_{13} \Delta r_3$	$\pi_{14} \Delta r_4$	$\sum_{j=1}^4 \pi_{1j} \Delta r_j$	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1970	J	0.14	-0.01	0.20	-0.00	0.00	-0.29	-0.10	0.25
	S	-0.10	-0.02	0.00	-0.00	0.00	-0.29	-0.30	0.22
	D	-0.35	-0.02	0.00	0.00	-0.13	0.00	-0.13	-0.20
1971	M	-0.11	-0.02	0.00	0.00	-0.13	0.00	-0.13	0.03
	J	0.28	-0.03	0.00	0.00	0.00	0.00	0.00	0.31
	S	0.28	-0.03	0.00	-0.00	0.00	0.00	-0.00	0.32
	D	0.05	-0.02	0.00	-0.00	0.00	0.15	0.14	-0.07
1972	M	-0.12	-0.02	-0.54	0.00	0.00	0.25	-0.29	0.19
	J	-0.29	-0.04	-0.54	0.00	0.02	0.10	-0.42	0.17
	S	-0.50	-0.05	0.00	-0.00	0.02	0.00	0.02	-0.46
	D	-0.61	-0.06	0.00	-0.01	0.00	0.00	-0.01	-0.54
1973	M	-0.48	-0.06	0.00	-0.01	0.00	0.00	-0.01	-0.41
	J	-0.08	-0.04	0.00	-0.00	0.06	-0.10	-0.04	-0.00
	S	0.33	-0.02	0.00	-0.00	-0.02	-0.20	-0.22	0.57
	D	0.36	0.00	1.69	-0.07	-0.34	-0.54	0.74	-0.38
1974	M	-0.10	0.01	2.03	-0.09	-0.27	-0.44	1.22	-1.33
	J	-0.08	0.01	0.34	-0.03	-0.44	0.00	-0.13	0.04
	S	1.09	0.04	1.35	-0.17	-0.42	-0.29	0.46	0.59
	D	1.73	0.01	1.35	-0.19	0.00	-0.29	0.86	0.85
1975	M	0.97	-0.03	0.00	0.00	-0.02	0.05	0.03	0.96
	J	0.10	-0.03	0.00	0.03	0.11	0.05	0.18	-0.05
	S	-0.23	-0.05	0.00	-0.02	0.15	-0.05	0.08	-0.26
	D	-0.32	-0.02	-0.34	-0.01	0.11	-0.05	-0.29	-0.01
1976	M	-0.52	-0.01	-0.68	-0.02	0.04	-0.32	-0.97	0.46
	J	-0.41	-0.03	-0.16	-0.03	-0.04	0.00	-0.23	-0.15
	S	-0.15	-0.02	0.16	-0.01	0.00	0.32	0.47	-0.60
	D	-0.10	0.00	0.00	0.00	-0.02	-0.20	-0.22	0.11
1977	M	0.02	0.01	0.18	-0.01	-0.02	-0.20	-0.05	0.06
	J	0.06	-0.02	0.16	-0.02	0.00	0.00	0.14	-0.06
	S	-0.08	-0.02	0.00	-0.00	0.02	0.00	0.02	-0.08
	D	-0.07	-0.01	0.00	0.00	0.02	0.18	0.21	-0.27
1978	M	0.31	-0.02	0.00	0.03	0.06	0.25	0.34	-0.01
	J	0.42	-0.04	0.00	0.03	0.06	0.06	0.16	0.30
	S	-0.02	-0.02	0.00	0.00	0.00	0.00	0.00	-0.01
	D	-0.08	-0.00	-0.34	0.03	0.13	0.06	-0.12	0.05
1979	M	0.80	-0.03	-0.34	0.02	0.11	0.06	-0.15	0.99
	J	0.52	-0.03	0.00	-0.02	-0.02	-0.12	-0.16	0.71
SUM		2.66	-0.72	4.52	-0.59	-0.97	-1.86	1.11	2.28
MEAN		0.07	-0.02	0.12	-0.02	-0.03	-0.05	0.03	0.06

COLUMN (1)=(2)+(3)+(4)+(5)+(6)+(8). THE PERIOD 1970 J REFERS TO THE TRANSITION FROM THE MARCH TO JUNE QUARTER 1970 AND SO ON. ALL ENTRIES ARE TO BE DIVIDED BY 100.

TABLE 6

DECOMPOSITION OF CHANGE IN PORTFOLIO SHARE OF BUILDING SOCIETY DEPOSITS: AUSTRALIA,

MARCH 1970 - JUNE 1979

CHANGE IN SHARE OF WEALTH IN ASSET 3			COMPONENTS OF Δs_3					RESIDUAL
WEALTH			INTEREST RATES					
Δs_3	$\beta_3 D \cdot \Delta \log W$ $+ \beta_3' (1-D) \cdot \Delta \log W$	$\pi_{31} \Delta r_1$	$\pi_{32} \Delta r_2$	$\pi_{33} \Delta r_3$	$\pi_{34} \Delta r_4$	$\sum_{j=1}^4 \pi_{3j} \Delta r_j$		
YEAR & QUARTER	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1970 J	0.55	0.22	-0.06	-0.01	0.00	-0.19	-0.26	0.59
S	0.46	0.37	0.00	-0.01	0.00	-0.19	-0.20	0.28
D	0.53	0.37	0.00	0.00	0.44	0.00	0.44	-0.28
1971 M	0.59	0.36	0.00	0.00	0.44	0.00	0.44	-0.20
J	0.65	0.48	0.00	0.00	0.00	0.00	0.00	0.17
S	0.59	0.58	0.00	-0.01	0.00	0.00	-0.01	0.02
D	0.60	0.42	0.00	-0.01	0.00	0.10	0.09	0.09
1972 M	0.82	0.34	0.17	0.00	0.00	0.16	0.33	0.15
J	1.02	0.67	0.17	0.00	-0.07	0.06	0.16	0.19
S	0.96	1.00	0.00	-0.01	-0.07	0.00	-0.08	0.04
D	0.96	1.21	0.00	-0.01	0.00	0.00	-0.01	-0.24
1973 M	0.89	1.15	0.30	-0.01	0.00	0.00	-0.01	-0.25
J	0.23	0.70	0.00	-0.01	-0.22	-0.06	-0.29	-0.18
S	0.02	0.39	0.00	-0.01	0.07	-0.13	-0.06	-0.31
D	0.32	-0.01	-0.53	-0.12	1.17	-0.35	0.17	0.16
1974 M	0.59	-0.16	-0.63	-0.14	0.95	-0.29	-0.11	0.87
J	0.40	-0.28	-0.11	-0.04	1.53	0.00	1.38	-0.71
S	-0.02	-0.73	-0.42	-0.28	1.46	-0.19	0.57	0.14
D	-0.64	-0.26	-0.42	-0.32	0.00	-0.19	-0.93	0.55
1975 M	-0.29	0.48	0.00	0.01	0.07	0.03	0.11	-0.88
J	0.70	0.64	0.00	0.04	-0.37	0.03	-0.29	0.35
S	0.78	0.93	0.00	-0.03	-0.51	-0.03	-0.57	0.43
D	0.54	0.39	0.11	-0.01	-0.36	-0.03	-0.31	0.45
1976 M	-0.55	0.27	0.21	-0.03	-0.15	-0.21	-0.17	-0.65
J	-0.73	0.62	0.05	-0.04	0.15	0.00	0.15	-1.50
S	0.40	0.37	-0.05	-0.02	0.00	0.21	0.14	-0.11
D	0.86	-0.10	0.00	0.00	0.07	-0.13	-0.06	1.01
1977 M	0.62	-0.22	-0.05	-0.01	0.07	-0.13	-0.12	0.95
J	0.41	0.38	-0.05	-0.03	0.00	0.00	-0.08	0.11
S	0.26	0.33	0.00	-0.01	-0.07	0.00	-0.08	0.01
D	0.09	0.15	0.00	0.01	-0.07	0.12	0.05	-0.12
1978 M	0.13	0.37	0.00	0.05	-0.22	0.16	-0.01	-0.23
J	0.30	0.69	0.00	0.05	-0.22	0.04	-0.13	-0.26
S	0.43	0.32	0.00	0.01	0.00	0.00	0.01	0.10
D	0.63	0.08	0.11	0.05	-0.44	0.04	-0.24	0.79
1979 M	0.17	0.65	0.11	0.03	-0.36	0.04	-0.19	-0.29
J	-0.08	0.56	0.00	-0.03	0.07	-0.08	-0.04	-0.61
SUM	14.19	13.77	-1.41	-0.96	3.36	-1.22	-0.22	0.64
MEAN	0.38	0.37	-0.04	-0.03	0.09	-0.03	-0.01	0.02

COLUMN (1)=(2)+(3)+(4)+(5)+(6)+(8). THE PERIOD 1970 J REFERS TO THE TRANSITION FROM THE MARCH TO JUNE QUARTER 1970 AND SO ON. ALL ENTRIES ARE TO BE DIVIDED BY 100.

The Welfare Cost of Interest Rate Movements

In this sub-section we give a brief analysis of the effects of inflation on financial markets. The effect we focus on is that in an inflationary environment, not all interest rates are free to instantaneously adjust upwards in a uniform manner. Some rates adjust fully while others adjust only partially, so that there is a change in the structure of relative rates of interest. In this case, inflation can be said to distort relative interest rates, which can be expected to have the usual welfare cost as funds leave those assets whose relative rates have fallen and go to those yielding higher rates.

We obtain an upper estimate of the welfare cost of this distortion by attributing all observed changes in relative interest rates to inflation. The observed rates need to be compared with a base; for this base we use sample means, which amounts to assuming that on average over the 1970s the returns on these assets were not distorted by inflation. Writing r'_i for the deviation of r_i from its sample mean (given in Table 2), the welfare cost can be expressed as (Harberger, 1964)

$$L = \frac{1}{2} \sum_{i=1}^4 \sum_{j=1}^4 r'_i S_{ij} r'_j ,$$

where $S_{ij} = \partial a_i / \partial r_j$ is the effect on holdings of asset i of a change in r_j , real wealth remaining constant. From (1), $\partial a_i / \partial r_j = W \pi_{ij}$, so that the cost can be formulated as a fraction of wealth as

$$\frac{L}{W} = \frac{1}{2} \sum_{i=1}^4 \sum_{j=1}^4 r'_i \pi_{ij} r'_j .$$

It should be noted that demand homogeneity ($\sum_j \pi_{ij} = 0$) implies that the loss is zero when all rates increase by the same percentage point amount. We evaluate this expression with the data given in Table 2 and the estimates of π_{ij} given in Table 3.

The welfare cost and the rate of inflation are given in Table 8. As can be seen the losses are not trivial; the average loss is 1 percent of

GDP, which is quite substantial by the usual standards. Furthermore, there is a clear tendency for the loss to rise with inflation (in the mid-1970s in particular).

It should, however, be repeated that these welfare costs represent upper bounds. In further work it may be worthwhile to extend this analysis to decompose changes in relative interest rates into effects due to (i) inflation and (ii) other factors. It would then be interesting to see by how much the welfare cost falls when the analysis is applied solely to the inflation component of changes in relative rates.

TABLE 9
 FINANCIAL ASSET HOLDINGS: AUSTRALIA,
 SEPTEMBER 1979 - DECEMBER 1981
 (Seasonally adjusted)

Year and quarter		Total financial wealth	Fixed deposits with trading banks	Savings bank deposits	Permanent building society deposits	Special/ Australian savings bonds
		(real \$M)	(% of total)	(% of total)	(% of total)	(% of total)
1979	S	13723	14.29	53.65	23.38	8.68
	D	13682	14.31	52.87	23.88	8.94
1980	M	13699	14.07	52.60	24.55	8.78
	J	13725	14.00	52.59	25.07	8.35
	S	13688	14.35	52.28	25.36	8.01
	D	13825	14.91	51.92	25.49	7.68
1981	M	13980	15.83	51.80	25.31	7.06
	J	14064	17.00	51.46	24.94	6.60
	S	14019	17.81	50.79	24.97	6.44
	D	13803	18.05	50.60	25.23	6.12
Mean		13821	15.46	52.06	24.82	7.67

See notes to Table 1

TABLE 11

OUT-OF-SAMPLE FORECASTS OF FINANCIAL ASSET HOLDINGS: AUSTRALIA,

SEPTEMBER 1979 - DECEMBER 1981

(Seasonally adjusted)

Year and quarter	Fixed deposits with trading banks (% of total)		Savings bank deposits (% of total)		Permanent building society deposits (% of total)		Special/Australian savings bonds (% of total)	
	Forecast	Forecast error	Forecast	Forecast error	Forecast	Forecast error	Forecast	Forecast error
1979 S	12.67	1.62 (2.6)	55.31	-1.66 (1.8)	23.35	.03 (.1)	8.67	.00 (.0)
D	12.99	1.32 (2.2)	55.22	-2.35 (2.5)	23.24	.64 (1.4)	8.55	.39 (.7)
1980 M	13.19	.88 (1.4)	55.17	-2.57 (2.8)	23.05	1.50 (3.1)	8.59	.20 (.4)
J	13.64	.36 (.6)	55.08	-2.49 (2.7)	22.78	2.29 (4.6)	8.51	-.16 (.3)
S	14.03	.32 (.5)	55.19	-2.91 (3.1)	22.33	3.03 (5.9)	8.45	-.44 (.8)
D	14.48	.43 (.7)	54.94	-3.02 (3.2)	21.97	3.52 (6.1)	8.61	-.93 (1.6)
1981 M	15.21	.62 (.9)	54.73	-2.93 (3.0)	21.30	4.01 (5.8)	8.76	-1.70 (2.6)
J	15.38	1.62 (2.2)	54.73	-3.27 (3.3)	21.06	3.88 (5.3)	8.83	-2.22 (3.3)
S	15.71	2.10 (2.9)	54.56	-3.77 (3.8)	21.24	3.73 (5.3)	8.50	-2.06 (3.1)
D	15.88	2.17 (3.0)	54.76	-4.16 (4.2)	21.08	4.15 (6.1)	8.28	-2.16 (3.4)
Mean	14.32	1.14	54.97	-2.91	22.14	2.68	8.58	-.91

The forecast error is actual minus forecast. Asymptotic t-values are given in parentheses.
See notes to Table 1.

(ii) ordinary accounts. No information is available about the micro transitions from one asset to another, so that we need to apply regression-type techniques to (2) to estimate the p_{ij} 's; see Lee *et al.* (1970). The ML estimates of the transition probabilities are given in Table 12. Each row of the table gives the probability of moving in one quarter \$1 previously held in the asset at the left of the row (asset i) into each of the five assets (asset j). According to the first row, for \$1 previously in a fixed deposit the probability of continuing to hold it in that form is .73. The probability of moving it into a savings bank investment account is zero; to a savings bank ordinary account .17; to a building society .10; and there is a zero probability of moving it into bonds. As the wealth must go somewhere, these probabilities sum to one.

For a given row, the diagonal probability is always much larger than any of the other elements. This means that for each asset, the most likely one quarter "move" is no move; i.e. wealth is most likely to stay where it was previously. Thus, there is a good deal of inertia in holdings of these assets. There are nonetheless a number of substantial off-diagonal elements which reflect the (one quarter) interrelationships between assets. There are large cross-probabilities for fixed deposits to savings bank ordinary accounts (p_{13}), fixed deposits to building societies (p_{14}), building societies to fixed deposits (p_{41}), and bonds to building societies (p_{54}).

It can be shown that $1/(1-p_{jj})$ is the average time that wealth stays in asset j . Thus, the average time wealth spends in each asset is (3.7, 11.5, 10.9, 6.1, 3.7) quarters (in the same order as Table 12). As can be seen, the turnover of the two types of savings bank deposits is half (or less) that of the other three assets. This corresponds with the notion that very little "smart money" is held in these low interest bearing forms; people who hold money in savings banks are not concerned with actively managing their wealth.

The steady-state asset distribution corresponding to the transition matrix is $\pi' = (.141, .203, .332, .235, .087)$. Thus in the steady-state, fixed deposits make up 14 percent of financial wealth, savings bank investment accounts 20 percent, savings bank ordinary accounts 33 percent, building society deposits 24 percent, with the remaining 9 percent in bonds. It is of interest that the system is very close to the steady state by the end of the sample period (June 1979) when the observed asset distribution is (.140, .209, .334, .231, .087). This is to be expected if the data are generated by a process which can be well-approximated by a Markov chain; i.e. as we move through time the observed asset distribution should get closer to the steady state.

As an illustrative application of the model, we analyse the convergence of the process to the steady state after an exogenous increase in wealth. Before the shock let the system be in the steady state, so that $s = \pi$. Then let wealth increase from W_0 to W_1 , with all the increase held initially in asset j . In this case, the initial distribution is

$$s'_0(j) = (1/W_1)[W_0\pi_1, \dots, W_0\pi_j + W_1 - W_0, \dots, W_0\pi_5],$$

and the evolution of the system can be simulated as

$$s'_t(j) = s'_0(j)P^t.$$

We assume that wealth increases by 2 percent from the actual value in June 1979 and that it is all held first in asset 1, then all in asset 2 and so on. In Figures 1-5 we plot the percentage deviation of the dollar

FIGURE 3 FIGURE 4

holdings of each asset from the old steady state,

$$z_{it}(j) = 100 \left(\frac{W_1 s_{it}(j) - W_0 \pi_i}{W_0 \pi_i} \right) \quad i, j=1, \dots, 5,$$

where $s_{it}(j)$ is the i^{th} element of $s_t(j)$.⁵ As the steady state is independent of where the additional wealth was initially held, we have

$$\lim_{t \rightarrow \infty} s_{it}(j) = \pi_i$$

and it follows that $z_{it}(j)$ converges to $100(W_1 - W_0)/W_0 = 2$ percent for all i and j . In words, holdings of each asset increase by 2 percent in the steady state; this follows from the constancy of the steady-state asset distribution. As can be seen from the graphs, the system approaches the steady state quite rapidly, except when the wealth is initially held in savings bank ordinary accounts.

A Logistic Growth Model

The rate of adoption of new products and technology has been extensively analysed with the logistic model (see, e.g., Fitzroy, 1976, Griliches, 1957). The objective is to estimate the rate of adoption, the saturation level and sometimes the responsiveness of the saturation level to economic variables. Savings bank investment accounts (SIAs) were first introduced in Australia in December 1969. In this sub-section we show that the growth of this new asset is similar to that of other new products in that the evolution of SIAs can be explained very well in terms of a logistic model. This type of growth of a new asset can be understood in terms of a learning process whereby people perceive SIAs as becoming less risky over time.

In Figure 6 we plot against time the percentage of total financial wealth (as defined in Section 2) held in SIAs. As can be seen, it does have a distinct S-shape, with the slope first increasing and then decreasing as the percentage tends to an upper asymptote. We use Oliver's (1969) generalized logistic function,

$$p_t = \gamma + \frac{\kappa_t}{1 + \exp(\alpha_1 - \alpha_2 t)} + \varepsilon_t,$$

where p_t is the percentage of wealth held in SIAs at time t ; $\kappa_t = \kappa_1 + \kappa_2 \bar{r}_t$ where $\bar{r}$ is the relative rate of return on SIAs;⁶ γ and $\gamma + \kappa_t$ are lower and upper asymptotes (for a fixed value of $\bar{r}$); and ε is an ARI disturbance, $\varepsilon_t = \rho\varepsilon_{t-1} + e_t$ with e_t white noise.

Using the quarterly Australian data from December 1969 to June 1979 we obtain the following ML estimates (asymptotic t-values errors are given in parentheses);

$$\begin{array}{lll} \hat{\gamma} = 2.65 \text{ (11.0)} & \hat{\alpha}_1 = 6.70 \text{ (12.9)} & R^2 = .998 \\ \hat{\kappa}_1 = 17.33 \text{ (32.7)} & \hat{\alpha}_2 = .363 \text{ (12.5)} & \text{S.E.} = .432 \\ \hat{\kappa}_2 = 1.62 \text{ (2.9)} & \hat{\rho} = .669 \text{ (5.0)} & \text{D.W.} = 1.72. \end{array}$$

As can be seen, all estimates are highly significant and the model fits the data well, with the coefficient of determination (R^2) equal to .998 and the standard deviation of the residuals (S.E.) being .43 percent. The interest elasticity of holdings is $\kappa_2 \bar{r} / p[1 + \exp(\alpha_1 - \alpha_2 t)] = .07$ at sample means. This agrees well with the system-wide elasticity given in Table 3 for all savings bank deposits.

The conclusion which emerges from this analysis is that the evolution of holdings of SIAs conforms well with the logistic process.

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