ECONOMICS

Thinking Outside the Box: A New History of Edgeworth’s and Pareto’s Development of the Box Diagram

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DISCUSSION PAPER 17.12
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Abstract: Polemical views are held among many historians as to whether the ‘original version’ of the Edgeworth box was drawn by Edgeworth, in his 1881 Mathematical Psychics, or Pareto, in his 1906 Manual of Political Economy. This study demonstrates that that polemic is largely attributable to a failure to consider the relationship between ‘trade’ flow oriented indifference curves drawn in Edgeworth’s Figures 1 and 5 and the implications of the relationship between those two diagrams for the definition of the origin of Edgeworth’s Figure 1. A new history of Edgeworth’s and Pareto’s contributions to the development of the Edgeworth box diagram is presented that highlights the intermediate role played by Pareto’s graph, from his 1902 article ‘On a New Error in the Interpretation of the Theories of Mathematical Economics’, which derives from Edgeworth’s Figure 5 except Pareto maps the indifference curves in the standard ‘allocation’ orientation.

Key Words: Box diagram, Creedy, Edgeworth, Jaffé, Pareto, Tarascio
“A picture paints a thousand words”

1) Introduction

Views on the origin of the ‘Edgeworth box’ diagram are polarised. Some economists and historians of economics – such as Luigi Amoroso (1942), John Creedy (1980, 1986 and 2010), Thomas Humphrey (1996), Mary Morgan (2004, 2012) and Alberto Zanni (2014) – recognise F. Y. Edgeworth’s Figure 1 from *Mathematical Psychics* (Edgeworth 1881, p 28.) as the ‘original version’ of the Edgeworth box. Others working in the 1970s, however – such as Vincent Tarascio (1972, 1980), William Jaffé (1972, 1974), Maurice Allais (1975), James L. Weatherby Jr (1976) – explicitly state that Vilfredo Pareto’s Figure 16, or its companion diagram Figure 50, from the *Manual of Political Economy* (Pareto 1906/1909 [2014, p. 95 and p.180]) is the ‘original version’ of the Edgeworth box.

The primary goal of this paper is to consider the history of the Edgeworth box as a diagram in the context of the relationship between Edgeworth’s Figure 1 and his Figure 5, which is presented in Appendix V of *Mathematical Psychics* entitled “On Professor Jevons’s Formulæ of Exchange”. That contextual perspective establishes that Edgeworth mapped and discussed the indifference curves in both figures in a ‘trade’ perspective to the origin on an open axis graph where the origin indicates no trade, when considering the indifference curves, and nil allocation of goods to both individuals, when considering allocation in relation to the contract curve.

This paper establishes that those who suggest that Edgeworth’s Figure 1 is the original version of the Edgeworth box diagram have, consciously or unconsciously:
viewed $y$ as the quantity of time; interpreted Edgeworth as having mapped indifference curves in ‘allocation’ orientation and with reference to ‘implicit’ diagonal origins; and redefined the explicit origin of the diagram, point $O$, as the point where one individual has a pre-trade or initial allocation of all $x$ and no $y$ whereas the other individual has all $y$ and no $x$. In relation to Figure 1, there is no logical prohibition against redefinitions of coordinates and curves along those lines. Indeed, doing so establishes exactly how Edgeworth’s Figure 1 can be transformed into a box diagram. But such a transformation cannot be represented as an historical account of Edgeworth’s Figure 1 because he did not define $y$ as time (he presented it as labour time as an objective measure of sacrifice); he did not present his indifference curves in the now standard ‘allocation’ orientation (he presented them in ‘trade’ orientation); and he did not present the origin point $O$ as the pre-trade initial allocation for which one individual has all $x$ and no $y$ and the other has all $y$ and no $x$ (his pre-trade initial allocation is given as two coordinates within the $XY$ plane of Edgeworth’s Figure 1, one for each individual, with each individual having some of $x$ and some of $y$).

The secondary goal of the paper is to briefly re-write the history of the Edgeworth box as it pertains to the contributions of Edgeworth and Pareto. The main contribution in relation to that goal is the introduction of Pareto’s 1902 graph, from his article ‘On a New Error in the Interpretation of the Theories of Mathematical Economics’, which is related to Edgeworth’s Figure 5, in that both map two indifference curves for two individuals, which are for standard consumer goods and are presented in an open axes single origin graph. But Pareto’s diagram presents the indifference curves in ‘allocation’ orientations that are convex to the
origin for both individuals; whereas Edgeworth’s Figure 5 presents ‘trade’ orientation indifference curves that are mapped as concave to each dealer’s point of initial endowment. Nevertheless, Pareto’s 1902 diagram is important as it took him a step closer to the box diagram, which is first presented in 1906 in Pareto’s Manual of Political Economy.

To achieve these goals the paper is structured in four sections. Section 2 presents a critical review of the historical literature on this history of the Edgeworth box diagram, dating from the 1970s. Section 3 introduces an interpretation of Edgeworth’s Figure 1 in the light of his Figure 5. Specifically, the relationship between Edgeworth’s Figure 1 and his Figure 5 is used to correct the interpretation of Edgeworth’s Figure 1 and point to reasons for the polemical literature on the history of the Edgeworth box. Section 4 presents a brief new history of Edgeworth’s and Pareto’s contributions to the creation of the original Edgeworth box diagram. It is concluded, in Section 5, that the box diagram is rightly named in honour of Edgeworth, for his development of indifference curves and the contract curve, but, nonetheless, his Figure 1 is not an open axes box diagram. The ‘original version’ of the box diagram was first presented by Pareto in his 1906 Manual, which itself was an enhanced development of Pareto’s 1902 diagram, which is basically Edgeworth’s Figure 5 once indifference curves have been presented in ‘allocation’ re-orientation (i.e. convex to the origin for both individuals).

2) Literature on Edgeworth, Pareto and the Original Box Diagram
Edgeworth’s Figure 1 (Diagram A below) is an open axes graph that shows a *contract curve* \((CC')\); a single indifference curve for one individual that appears convex to the ‘north-west’ of the graph (the curve from 0 to \(\eta_0\xi_0\)); and a single indifference curve for the other individual that appears convex to the ‘south-east’ (the curve from 0 to \(y_0\xi_0\) and beyond). Two *reciprocal demand curves*, \(\eta_0\) and \(\xi_0\), are also drawn on that diagram which intersect on the contract curve marked at \(\eta\xi\).

![Diagram A: Edgeworth’s Figure 1 (1881, p. 28)](image)

In contrast, Pareto’s Figure 16 (Diagram B below) is a closed box diagram that has two origins, one given by the ‘south-west’ corner \(O\) of the box and the other given in the ‘north-east’ corner \(\omega\) of the box, with each origin associated with a different trader. Ordinal systems of indifference curves are mapped for two individuals, each convex to the respective origin for each individual. Relative to the

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1 Edgeworth refers to these curves as “demand curves”, as they are locus of points that equate (i) the ratio of the marginal utility of the good given up, say good X, to the marginal utility of the good acquired, say Good Y, to (ii) the quantity of a good that an individual demands, \(y\), relative to the quantity of the other good, \(x\), that the individual is willing to offer in exchange for \(y\). The curve is plotted by rotating the price ray from the origin by 90° in the XY plane. Following Bowley (1924), these curves have subsequently been labelled as “offer curves”, although Edgeworth did not like that name and many economists now refer to them as *reciprocal demand curves* (see Gram 1987).
modern stylised Edgeworth box diagram, Edgeworth’s contract curve is a notable omission.

Diagram B: Pareto’s Figure 16 (1906 [2014, p.95])

The suggestion that the origin of the ‘Edgeworth box’ goes back to Edgeworth’s Figure 1 was implicit by at least as early as 1942, when the Italian Paretian Luigoi Amoroso (1942) outlined Pareto’s system and then discussed the “Reppresentazione di Edgeworth” (a slight variation of Diagram A) in his *Meccanica Economica* (Amoroso 1942), which was recently discussed by Alberto Zanni (2014). Notably, Amoroso used an open axes diagram in the plane to illustrate a contract curve and two indifference curves and explain the “Reppresentazione di Edgeworth”. The implicit origin for one indifference curve, for one dealer, appears in the ‘north-west’ of the diagram; and the implicit origin for the other indifference curve, for the other dealer, appears in the ‘south-east’. Only a modest transformation is required to transpose the diagram that Amoroso developed, as a representation of Edgeworth’s Figure, into the ‘Box diagram’. On
that reckoning, labelling of the box diagram as the Edgeworth box would be entirely reasonable.

A few economists, such as Erich Schneider (1952), G. L. S. Shackle (1967) and Maurice Dobb (1969), subsequently reported on Pareto’s diagram and its relation to the Edgeworth box diagram, as William Jaffe (1974) had noted. It was not until the 1970s, however, following the translation of the French edition of the *Manual* by A. S. Schweir and A. N. Page (Pareto 1909 [1971]), that the relationship between Edgeworth’s Figure 1 and the box diagram was seriously brought into question, starting with Jaffé’s (1972, p. 1190) dramatic announced that the Edgeworth box is not to be found in Edgeworth’s *Mathematical Psychics*. In the same year, Vincent Tarascio reviewed the origins of the Edgeworth box (or the Edgeworth-Bowley box as it has sometimes be called) and reached a similar conclusion, although he observed that two possible diagrams in *Mathematical Psychics* that could be considered as possible origins for the box diagram: Edgeworth’s Figure 1 (our Diagram A) and Figure 5 (our Diagram C). But he concluded that these diagrams cannot be converted into a box diagram by “simple geometric manipulation” (Tarascio 1972, p. 193).

In regard to Edgeworth’s Figure 1, Tarascio reminded us that Edgeworth drew his diagram to illustrate the particular economic relationship between Robinson Crusoe and Friday:

“It might be pointed out that Edgeworth had a box diagram in this sense: the ordinate of Figure 1 (p.28) is the amount of labour (time) given up by Friday; the abcissa is the remuneration received by Friday.
Then Y is received by Crusoe, and X is given up. Hence quantities are conserved in the diagram; this mode inherently limits the quadrant to tenable points.” (Tarascio 1972, p. 193).

Any geometrical manipulation of Edgeworth’s Figure 1 would result, in Tarascio’s assessment, in a ‘nonsensical’ box diagram in the sense that “the indifference curve for one trader is parallel to the indifference curve of the other trader.” (Tarascio 1972, p. 193).

For his Figure 5, Edgeworth’s specific objective was to explain how each individual’s semi-circular reciprocal demand curve may be derived as the “the locus of the point where lines from the origin touch the curves of indifference” (Edgeworth 1881, p. 113). Importantly, unlike his Figure 1, Edgeworth’s Figure 5 is not a representation of the exchange of labour for remuneration in terms of goods produced by labour. Rather, goods X and Y are both consumer goods and the axes on the graph are such that x indicates the quantity of corn and y indicates the quantity of beef. The initial endowment of goods for the two dealers are given by point C, which represents dealer A’s initial allocation (no corn and all the beef), and point C’, which represents dealer B’s initial allocation (all the beef and no corn). The origin, point O, indicates no trade, with the semicircle OQTC representing person A’s reciprocal demand curve and; the semicircle OTC’ representing person B’s reciprocal demand curve. Person A’s indifference curve is tangential to the market price ray at point T and is concave to the Y axis; and person B’s indifference curve is concave to the X axis and passes through point Q.
Tarascio rejected any suggestion that Diagram C is an early version of the Edgeworth box because the “results are inconsistent with the theory underlying the analysis” (1972, p. 194). He attempts to clarify his reasoning, noting the following:

“Again, the result is parallel indifference curves. If, on the other hand, one begins with individual diagrams and then superimposes them by reversing the origins and axes, the result is indifference curves along a contract curve, but the direction of the contract curve is from northwest to the south east and the indifference curves are not concave to/from the origins.” (Tarascio 1972, p. 194)

Tarascio writes nothing more on the issue, so his rationale for the abovementioned conclusions is not explained. In regard to his first observation, he appears to be suggesting that parallel indifference curves will ‘result’ when the indifference curve are redrawn with reference to point O; and this would be inconsistent with theory because Edgeworth has drawn the indifference curves as ‘concave’ to points C and C’, which implies that the concave (not convex)
orientation would be maintained when the curves are redrawn with respect to point O. His ‘on the other hand’ alternative appears to involve a 180º rotation of person B’s $XY$ plane, which is then superimposed over Person A’s $XY$ plane. The implied outcome is suggested below in Diagram D.

![Diagram D](image)

**Diagram D: Tarascio’s Implied ‘Box’ Reorientation of Edgeworth’s Figure 5**

However, as demonstrated in Section 3, Tarascio’s assessment reflects a fundamental misinterpretation of Edgeworth’s Figure 5. Edgeworth’s diagram is not, as Tarascio suggests, inconsistent with theory. Nevertheless, his article does serve as an important reminder that Edgeworth’s Figure 5 should be indeed be considered in the history of Edgeworth’s contribution to the box diagram.

Tarascio also observes that Edgeworth’s Figure’s 1 and 5 both show a single indifference curve, rather than a system of indifference curves, that is mapped for each individual. He interprets this as evidence for the proposition that Edgeworth was not thinking in terms of systems of indifference curves. In support of that, he argues that neither the text nor the equations in *Mathematical Psychics* suggest a
basis for doing so. Jaffé (1974) a few years later similarly noted the difference between Edgeworth and Pareto over the number of indifference curves shown for each person.

Jaffé’s assessment of Edgeworth’s ‘pristine’ diagram (our Diagram A) failed to account for relationship between the Friday-Crusoe and the associated economic scenario that the diagram was drawn to illustrate. Instead, he read Edgeworth Figure 1 as if it were a representation of two dealers, a Mr A and a Mr B, exchanging consumer goods that, implicitly, have positive marginal utility to both individuals. On that basis, he pointed out that the dealers’ initial endowment poses serious problems for the box interpretation of Edgeworth’s Figure 1.

“Edgeworth’s Contract Curve diagram could only be converted into a box diagram of specific dimensions if the initial endowments, which Edgeworth deliberately chose to leave implicit, were explicitly specified. (Jaffé 1974, pp. 343-344).

To make explicit that which had been left implicit, Jaffé (1974, p. 347) devised Diagram E (below), which treats exchange as incremental variations in quantities of goods from that given by the initial endowment marked by the origin O.
The fundamental and path breaking feature of Jaffé’s work and diagram E is that it clarifies the distinction between an indifference mapped in an ‘allocation’ plane, as is commonly the case today, and a ‘trade’ plane, as Edgeworth did. In diagram E, the ‘northwest’ and the ‘southeast’ quadrants map indifference for ‘Mr A’ and ‘Mr B’ respectively in the ‘allocation’ plane. That is, the ‘northwest’ quadrant maps the allocations of quantities $x$, of $X$, and $y$, of $Y$, to which “Mr A” is indifferent and the ‘southeast’ quadrant maps the allocations that ‘Mr B’ is indifferent to. They are united by a non-zero initial endowment given at point O. The ‘northeast’ quadrant, however, maps both dealers’ indifference curves in a ‘trade’ plane, such that indifference curve 1 for dealer A in the ‘allocation’ plane is $I_A$ in the ‘northwest’ quadrant and his indifference curve in the ‘trade’ plane is the $I_A$ in the ‘northeast’ quadrant in bold font. Conversely, the indifference curve 1 for dealer B in the ‘allocation’ plane is $I_B$ in the southeast’ quadrant and his
indifference curve in the ‘trade’ plane is $I_B$ in the ‘northeast’ quadrant. But, in the light of Tarascio (1972), the problem associated with Jaffé’s interpretation of Edgeworth’s Figure 1 is that he interprets $x$ and $y$ as quantities of consumer goods, and not labour and remuneration.

Notwithstanding the difficulties associated with both Tarascio’s and Jaffé’s explanation of Edgeworth’s Figure 1, they each contributed a precious gem to the historical assessment of Edgeworth’s diagrams. Tarascio for the labour-remuneration aspect of the diagram; and Jaffé for his recognition that Edgeworth drew his indifference curves in a ‘trade’ oriented XY plane. But neither Tarascio’s point about the Friday-Crusoe scenario; nor Jaffé’s point about orientation of the indifference curves proved influential. In the case of Tarascio (1972), this is partly also because he confined his critical contributions to footnotes 4 and 5 of his paper, without articulating the full ramifications of his point. In the case of Jaffé, the lack of influence was partly a result of his overly complicated way or relating ‘trade’ and ‘allocation’ indifference curves to each other and partly related to a slight tendency to exaggerate the significance of points.²

But the lack of influence of Tarascio (1972) and Jaffé (1974) is also linked to errors or misinterpretations in their respective papers, which Creedy (1980) succeeded in highlighting. Specifically, Creedy (1980) insisted that it is hardly legitimate for Tarascio or Jaffé to suggest that Edgeworth did not appreciate that

² For example, Jaffé (1974, p. 343) states that the ‘earliest adumbration’ of the box diagram is Figure 5.3 from the fifth instalment of Pareto’s series of article titled “Considerations on the Fundamental Principles of Pure Political Economy (1893 [2007, p.112]). That diagram is a box, yes, but it is not an Edgeworth Box diagram in any sense that we are all familiar with today. It simply maps a system of indifference curves (as well as attaching vertical lines of different length to each indifference curve to indicate total utility) for a single individual. There is no mapping of indifference for two individuals from diagonally opposite corners of the box.
systems of indifference curves may be modelled for each individual because the very definition of a contract curve “implies a ‘system’ of indifference curves” (Creedy 1980, p. 274). He also pointed to many instances in the text of *Mathematical Psychics* which envisage more than one indifference curve per dealer.

The context to Tarascio’s point on ‘systems’ of indifference curves is given by his important observation that Pareto (1902) had provided the mathematical formalism for ordinal systems of indifference curves, in which utility is explicitly a function of the index numbers used to label indifference curves. But the lack of such an explicit equation is hardly a weakness of *Mathematical Psychics* when one recalls that Edgeworth’s utility functions are valued in cardinal terms. As a result, the implicit index number for each individual’s indifference curve is simply the total utility that is associated with a curve; and a separate index number function would not need to be specified because an individual’s potential total utilities constitute perfectly sound index numbers for his, or her, system of indifference curves.

Creedy also notes that Jaffé was critical of Edgeworth for ignoring the initial endowment, but rejects that criticism because Edgeworth (1881, p. 105) had made it clear that he fully appreciated the role of endowment. It is just that his main equations and his diagram reflected a greater interest in exchange rather than allocations. That is, Creedy sees Edgeworth as simply focused on the pre-trade case in which one dealer has all of one good and the other dealer has all of the

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3 Pareto actually introduced the mathematical formalism for ordinal theory for systems of indifference curves a few years earlier, in Pareto (1900).
other, with that particular pre-trade allocation given at the origin of the graph, as a deliberate simplification. Creedy then looked to Bowley for insight into generalising the treatment on the initial endowment to the point where both dealers could have some of both goods (see Figure F below).

Diagram F: Bowley’s Figure 1 (1924, p. 5)

On that basis, Creedy viewed Edgeworth’s Figure 1 as the basic Edgeworth box diagram. Quite simply, “Edgeworth’s diagram [our Diagram A] is rotated by 90° when compared with the conventional textbook figure” (1980, p. 273). But this characterisation logically depends on a unique allocation of $x$ and $y$ between the two individuals being given at every point in the $XY$ plane, with the origin the origin of the graph, point O, representing a unique initial allocation of goods between two dealers.

In his reply, Tarascio (1980, p. 279) pointed out again that Edgeworth’s Figure 1 is an illustration of the particular Friday-Crusoe scenario and that Creedy’s
paper was silent on that issue. But that point is ignored by Creedy and all other historians in the subsequent literature except Morgan (2012), although, she did not attribute any significance to that when interpreting Edgeworth’s Figure 1 and its relationship to the Edgeworth box.

In his entry on “The Edgeworth box” (Creedy 2010) in *Famous Diagrams and Figures in Economics*, Creedy completely writes Pareto out of the history of the Edgeworth box diagram. He does so partly because of his earlier stated view that “the use of a box has of course no special analytical value, though it is perhaps a useful pedagogical addition” (Creedy 1986, p.59), but mainly because he had characterised the origin of Edgeworth’s Figure 1 as a unique allocation between two individuals on the presumption that the origin of Edgeworth’s graph indicates the initial endowment under which one dealer has all of \( x \) and the other has all of \( y \). The end result is that his account of the Edgeworth box is confined to the contributions of Edgeworth (1881) and Bowley (1924).

“Edgeworth’s diagram [our Diagram A] was drawn with the origin as the endowment point... This meant that he did not actually draw two of the edges of the ‘box’. The way in which the two sets of indifference curves and their related axes are related was later clarified by Bowley (1924)” (Creedy 2010 pp. 233-234.)

The subsequent, and more extensive, literature on the history of the Edgeworth box is primarily written by Thomas Humphrey, in his “The Early History of the Box Diagram” (1996), and Mary Morgan, provisionally in “Imagination and Imaging in Model Building” (2004) and more fully in “The
History of the Edgeworth Box Diagram – as Told by Itself” from her *The World in the Model: How Economists Work and Think* (2012). Both Humphrey and Morgan are far more appreciative and nuanced in their treatment of Pareto than Creedy, in that the close resemblance between Pareto’s box diagram and the modern stylized Edgeworth box is fully recognised. But neither Humphrey nor Morgan cite or consider Tarascio (1972) and both treated Edgeworth’s Figure 1 as the ‘original version’ of the Edgeworth box diagram. And they do so on a similar basis as Creedy by treating goods $X$ and $Y$ as allocations of stocks that have positive marginal utilities to both dealers under which: (i) Edgeworth’s Figure 1 implicitly, but not explicitly, incorporates a system of indifference curves for each individual because of the inclusion of the contract curve; (ii) the initial endowments can be treated more generally by moving from the origin point O to a point inside the box; and (iii) by creating the box diagram in modern (Paretian) orientation by rotating Edgeworth’s diagram by $90^\circ$.

Morgan reflects the above point (i) when she discusses movement toward the point $\eta \xi$ on Edgeworth’s diagram (i.e. the point where the *reciprocal demand curves* and the contract curve intersect) in the following terms:

“he [Edgeworth] notes that this process arrives at the point $\eta \xi$ on his figure 1 …, where the price ray from the origin will be at a tangent to both indifference curves (not shown on his diagram ....)” (Morgan 2012, p. 113)

Thomas Humphrey reflects a similar view of Edgeworth’s contributions to indifference curves.
“From the indifference curves radiating out from their respective origins, Edgeworth selects one particular curve for each trader, namely, the curves passing through the endowment point.” (Humphrey, 1996, p. 41)

In regard to point (ii), Humphrey notes that Edgeworth’s diagram is not an allocation box diagram, as the stylised Edgeworth box diagram is, but an ‘exchange’ box diagram. That is: “Edgeworth invented the exchange box.” (Humphrey, 1996, p. 37). This is not to suggest that Humphrey saw indifference curves as ‘trade’ indifference curves, as already noted by Jaffé, just that the initial endowment gives all of one good to one dealer and all of the other to the other dealer. Like Creedy, he notes that it was Bowley who modified the assumption of that each trader having all of one good “to each trader holding some of both goods. The result was to fix the endowment point in the interior of the box, rather than at one of its corners.” (Humphrey 1996, p. 51) Morgan makes a similar point when discussing the area of the box diagram, but she makes the important additional observation that Marcel Lenoir, not Bowley, was the first to bring the initial endowment point inside the box:

“He [Pareto] represents a fixed quantity of goods, but, by extending the axes beyond the rectangle, invites the possibility of extension. Marcel Lenoir (1913), who was familiar with Edgeworth’s and Pareto’s work, picked up the latter’s formulation of the box and seems to have anticipated Bowley’s innovation by moving the starting point for trade inside the Box” (Morgan 2012, p. 121)
In regard to point (iii), Creedy’s suggestion that the Edgeworth box is created, in all its substantive features, by rotating it by 90° is also repeated, although both Humphrey and Morgan are careful enough to explicitly attribute this to Pareto:

“He [Pareto] located the box in its now conventional form. That is, he located the origin of the indifference maps in the south east and northwest corners, rather than the other corners as Edgeworth had done.” (Humphrey 1996, p. )

“[Pareto] creates a box from Edgeworth’s open axes, and reorients it (by ninety degrees), with the two individuals at opposite corners, each with a whole set of indifference curves (rather than the single one for each individual drawn by Edgeworth).” (Morgan 2012 p. 115)

In regard to point (i) above, Creedy, Humphrey and Morgan appear to be on solid ground. But that is not the case for points (ii) and (iii), on which the thesis that “Edgeworth’s Figure 1 is the original Edgeworth Box diagram” ultimately rests. It is argued in Section 3 that Edgeworth’s ‘trade’ oriented account of Figure 1 and the origin of that diagram, as evident from a comparison of Edgeworth’s Figures 1 and 5, are inconsistent with both the presumption that the origin of Figure 1 indicates the initial endowment; and the argument that each and every point in the $XY$ plane of Edgeworth’s Figure 1 constitutes a unique allocation of $x$ and $y$ between Friday and Crusoe.

3) Reinterpretation of Edgeworth’s Figure 1 and its Relationship to Figure 5
Following Tarascio (1972), it should now be evident that Edgeworth’s Figure 1 is a graphical representation of a very particular scenario: Robinson Crusoe (X) pays the quantity \( x \) to Friday (Y) as remuneration for labour; and Friday supplies the quantity \( y \), being the contracted hours of work, to Crusoe. Jevons (1871 [1888]), of course, had already developed his theory of labour in which market equilibrium emerges when the decrement in the degree of utility from the ‘painful exertion of body and mind’ endured by a worker is exactly offset by the increment in the pleasure that the worker gains from the remuneration received. The contention of this paper is that the indifference curves are intended to illustrate the utility/disutility relationship of Jevon’s theory of labour; whereas Edgeworth’s discussion of ‘contract’ and the ‘contract curve’ focuses on exchange complement Jevon’s characterisation of market exchange by considering the case of bilateral exchange of labour and remuneration.

In the context of Edgeworth’s particular example, Jevons’s theory of labour means that the negative marginal utility that Friday incurs from the ‘toilsome sensations’ associated with his labour would, in equilibrium, be matched by the positive marginal utility he derives from the in-kind remuneration he receives from Crusoe. A corollary of that position is that the positive marginal utility that Crusoe derives from Friday’s labour (i.e. an ‘indirect’ measure of utility for the ‘direct’ utility gained from retaining the product of Friday’s labour) is matched by the negative marginal utility he endures from paying the remuneration good, presumably food, to Friday. That, in essence, is the subject of Edgeworth’s indifference analysis in Figure 1. And there is no doubt that Edgeworth’s Figure 1 is built upon Jevons’s market theory of exchange. That can be readily inferred from
discussions on pages 34 to 36 of *Mathematical Psychics* and in the appendix on Jevons’s theory of exchange. If $\Pi$ indicates $Y$’s (Friday’s) utility, his utility function would be given by $\Pi = \Phi(x, y)$, with utility continuously increasing with respect to quantity $x$ (units of remuneration) and continuously decreasing with respect to quantity $y$ (labour hours). Similarly, if Crusoe’s utility is given by $P = F(x, y)$, utility increases continuously with respect to the quantity of labour supplied by Friday, $y$, and decreases continuously respect to the quantity of remuneration paid to Friday as $x$.

An important consequence of these utility relations is that they impose particular shapes on the indifference curves. As Friday’s marginal utility from labour (on the $Y$ axis) is negative and his marginal utility from the remuneration good (on the $X$ axis) is positive, Friday’s indifference curves are convex to the $Y$ axis. That is, the curve from the origin O to $\eta_0x_0$ is Friday’s indifference curve relative to the no-trade point. As Crusoe’s indirect marginal utility from Friday’s labour (on the $Y$ axis) is positive, and his marginal utility from paying remuneration to Friday (on the $X$ axis) is negative, it follows that his indifference curves would be mapped as convex to the $X$ axis. On that basis, the curve from O to $y_0\xi_0$, and beyond, is Crusoe’s indifference curve.

On that basis, the $XY$ plane is in ‘trade’ orientation for the purpose of indifference curve mapping. But Figure 1 also includes a contract curve and the initial allocation for both individuals, which are typically shown in ‘allocation’ orientation and not trade orientation. As such, the relationship between the $XY$ plane of Figure 1 and indifference curves, on the one hand, and the initial allocations and
the contract curve, on the other, must be made explicit. In that regard, the relationship between Edgeworth’s Figure’s 1 and 5 is instructive.

The initial endowment is not given explicitly by Edgeworth for his Figure 1. So one must ask, from Edgeworth’s discussion, where could Friday’s and Crusoe’s initial endowment be indicated? The answer to that question is both straightforward and challenging. As both indifference curves meet at the origin which, given the trade orientation of the indifference curves, indicates no trade, it follows that Friday’s initial allocation must be at some point on his indifference curve; and Crusoe’s initial allocation must be at some point on his indifference curve. If each and every coordinate reflects a unique allocation between Friday and Crusoe, the only possible location of the initial allocation is given by the origin. That could indicate nil allocation of labour and remuneration to both Friday and Crusoe; or, as Creedy and others argue, Friday has all of y and Crusoe has all of x. But the second of these two options effectively impose the condition that the initial allocation between Friday and Crusoe must be given by a single point. It is akin to assuming that Edgeworth’s Figure 1 is an open axes Edgeworth box diagram and then going on to demonstrate that it is an open axes Edgeworth box diagram.

Having established that the initial allocation for Friday lies on his indifference curve and for Crusoe, on his indifference curve, the next question that must be asked is: does Edgeworth envisage one initial allocation coordinate, which indicates allocation of x and y between Friday and Crusoe, or does he envisage two allocation coordinates, one for Friday and another one for Crusoe? If we look to Edgeworth’s Figure 5 for insight, it is evident that he drew his contract curve as a linear approximation between two points of initial endowment, from point C, the
initial for dealer A, to point C’, the initial endowment for dealer B (see diagram C). Assuming that a similar arrangement also applies to Figure 1, it follows that Friday’s initial endowment is given at point $\eta_0x_0$, where the northwest end of the contract curve intersects Friday’s indifference curve (i.e. near to point C but not exactly at point C); and Crusoe’s is given at point $y_0\xi_0$, where the southeast end of the contract curve intersects Crusoe’s indifference curve (i.e. near to point C’ but not exactly at point C’).  

On that basis, there would two initial allocations represented in Edgeworth’s Figure 1. Does that make sense? It does when regard is given to the character of the specific Friday-Crusoe scenario being illustrated by that diagram. Friday attempts to survive through personal foraging, and, without entering into contract with Crusoe to undertake labour, he can do so at the initial allocation of $x$ and $y$ given at $\eta_0x_0$. Similarly, Crusoe’s past abstinence to obtain a stock of wage good $X$ and the benefit he gains from Friday’s uncontracted labour, including observing Friday’s uncontracted labour, appears to permit Crusoe to survive without trade to contract Friday’s labour at the initial allocation of $x$ and $y$ given at $y_0\xi_0$. If correct, the above assessment suggests that Edgeworth’s Figure 1 has three coordinates of significance: first, the origin, point O, which indicates no trade (for indifference curve analysis) and nil allocation of both ‘contracted labour’ and remuneration; second, Friday’s initial allocation, given at point $\eta_0x_0$; and third, Crusoe’s initial allocation at $y_0\xi_0$.

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4 In Figure 1, point C cannot be Friday’s initial endowment, as it indicates negative total utility (i.e. less utility than the no trade state), not zero total utility (same utility as the no trade state). Similarly, point C’ cannot be Crusoe’s initial allocation, as it indicates negative total utility (i.e. less utility than the no trade state), not zero total utility (i.e. same utility as the no trade state).

5 For example, Crusoe can obtain information about natural resources and their location, and survival generally, by watching Friday work even in a no-trade environment.
allocation, given at point $y_0x_0$. This account is consistent with the following comments by Edgeworth, when he notes that the:

“available portion of the contract curve lies between two points, say $\eta_0x_0$ to the north-west, and $y_0\xi_0$ south-east; which are respectively the intersections with the contract-curve of the curves of indifference for each part drawn though the origin. Thus the utility of the contract represented at $\eta_0x_0$ is for Friday zero, or rather, the same as if there was no contract. At that point he would as soon be off with the bargain – work by himself perhaps. (Edgeworth 1881, p. 29).

Importantly, when Edgeworth is writing about Friday’s indifference at point $\eta_0x_0$, or to any point on the curve $O\eta_0x_0$, he is writing about indifference to ‘trade’, not indifference to allocation. Diagram G below derives ‘allocation’ indifference curves that pass through the initial endowment points $O\eta_0x_0$, for Friday and $y_0\xi_0$, for Crusoe.
Diagram G: Edgeworth’s Figure 1 with ‘Allocation’ Indifference Curves Added While Maintaining a Single Origin Graph

From Friday’s allocation at point \( \eta_0x_0 \), he would be indifferent to an increase in trade by moving to point \( A_T \) (i.e. point A on the ‘trade’ flow indifference curve), in which case his ‘allocation’ would be shown by point \( A_A \) (i.e. point A on the stock ‘allocation’ indifference curve). The allocation of uncontracted labour (which is the stock of labour with the potential to be used in contracted employment) decreases as the allocation of contracted labour increases; and the quantity of remuneration also increases. Similarly, Friday would be indifferent to less trade by moving from the initial allocation to point \( B_T \) (i.e. point B on the ‘trade’ flow indifference curve), in which case his stock ‘allocation’ would be shown by point \( B_A \) (i.e. point B on the ‘allocation’ indifference curve). That is associated with an increase in leisure, which, like uncontracted labour, also adds to the stock of potential labour hours available for paid employment; and a corresponding reduction in remuneration.

The resulting ‘allocation’ indifference curve emerges when a curve is fitted to points \( A_A, \eta_0x_0 \) and \( B_A \). That curve is convex to the origin, at point O, reflecting the fact that an available ‘stock’ of hours available to work (as either ‘contracted’ labour with Crusoe or ‘uncontracted’ self employment) is a good to Friday that has positive marginal utility, as distinct from a flow of toil and effort which has negative marginal utility, while remuneration remains a good with a positive marginal utility. When the process is repeated for Crusoe, his ‘allocation’ indifference curve for the stock of labour and wage goods is \( A_A' - y_0z_0 - B_A' \), which is also convex to the origin. In his case, the ‘stock’ of the remuneration good is a
capital good with positive marginal utility to Crusoe, as is Friday’s labour (because of the product that it produces for Crusoe).

But Edgeworth did not map ‘allocation’ indifference curves in his Figure 1. He did not do so in reference to the origin, as illustrated above in Figure G. And neither did he do so with implicit reference to the northwest, for Friday, and southeast, for Crusoe, even though the shape of Friday’s and Crusoe’s indifference curves in Edgeworth’s Figure 1 are consistent with that interpretation.

When Friday’s indifference curve $O_{\eta_0\xi_0}$ is interpreted as a stock or ‘allocation’ indifference curve, it must be viewed with an implicit origin in the northwest. In that case, Friday’s indifference curve does not illustrate the indifference trade-off between the sacrifice associated with the exertion of labour service (negative marginal utility) and remuneration, as is the case when it is considered with reference to point O. Rather, it illustrates the indifference trade-off between the allocation of time not working (24 hours per day less the hours of contract labour) and remuneration, both of which have positive marginal utility to Friday. Similarly, when Crusoe’s indifference curve $O_{\eta_0\xi_0}$ is perceived as an ‘allocation’ indifference curve it is considered as being drawn to an implicit origin in the southeast.

The legitimacy or otherwise of that interpretation, in an historical sense, turns on the implications for the origin of the diagram, point O, and whether Edgeworth had those implications in mind. Interpreting the ‘trade’ indifference curves, which are mapped to a ‘single’ origin for both Friday and Crusoe, as if they are ‘allocation’ indifference curves that are mapped to two diagonal ‘implicit’
origins in the northwest and southeast, transforms point O into a point of initial endowment under which Friday has all y and Crusoe has all x.

But, notwithstanding the consistency of Edgeworth’s Figure 1 as an image with both the ‘trade’ flow interpretation and the stock ‘allocation’ interpretation, the relationship between Edgeworth’s Figures 1 and 5 makes it absolutely clear that Edgeworth had drawn his indifference curves in ‘trade’ orientation. In that regard, Edgeworth’s Figure 5 is drawn for two standard consumer goods, x (corn) and y (beef), and two individuals (‘dealer A’ and ‘dealer B’). That is, it deals with the exchange of two goods which both have positive marginal utility to both ‘dealers’, which is exactly how Creedy and Humphrey characterise Figure 1. To confirm the ‘trade’ orientation of dealer A’s and dealer B’s indifference curves from Figure 5, and to illustrate their relationship to ‘trade’ flow and ‘allocation’ orientation, it is useful to present that figure as grid referenced diagram, as shown in diagram G.

![Diagram G: Edgeworth’s Figure 5 in Grid Format](image)

Edgeworth does not give the ‘allocation’ implications of trade in this diagram directly, but they can be readily inferred. In this ‘trade’ oriented diagram,
dealer A gives up \(Oy_2\) and dealer B receives \(Oy_2\); and dealer A receives \(Ox_1\) and dealer B gives up \(Ox_1\). As the origin of the trade diagram is given by point O, the post-trade ‘allocation’ for dealer A will not be given at point T; rather, it will correspond to \(C_1\) at coordinates \(x_1\) and \(y_1\) as point O indicates both a nil allocation and nil trade. For the same reasons, the post trade ‘allocation’ for dealer B will not be point T, it will be point \(C’_1\) at coordinates \(x_2\) and \(y_2\). There is, therefore, two explicit ‘allocation’ coordinates in Figure 5, one for each dealer, in the pre-trade setting; and two implicit ‘allocation’ coordinates in the post-trade context (unless, by chance, both dealers have identical endowments), one for each dealer.

The indifference curve for dealer A is shaped as ‘concave’ to his/her initial endowment. Tarascio (1972) appears to note that these indifference curves are mapped as concave, but he fails to recognise that the indifference curve is in ‘trade orientation’ or that, within its own terms, Edgeworth’s Figure 5 is entirely consistent with theory. As it is trade that is represented by point T, not allocation, it follows that dealer A’s allocation has changed from \(C_0\) to \(C_1\) as a result of trade at point T; and the shape of the ‘allocation’ indifference curve at point \(C_1\) will not correspond to the shape of the ‘trade’ indifference curve at trade point T. That is, although the ‘trade’ indifference curves for both dealers are concave to their initial allocation. But when the indifference curves are rotated into their ‘allocation’ orientation using Jaffé’s (1994) procedure, the indifference curves are both convex to the origin of the graph through point \(C_1\), for dealer A, and through point \(C’_1\) for dealer B (see Diagram H).
But because the Friday-Crusoe scenario involves two categories of labour, uncontracted labour (individual fossicking), which is the only form of labour in the ‘pre-trade’ state, and contracted labour, the allocation implications in Figure 1 are not identical to those of Figure 5 (as indicated in Figures G and H). Under the scenario that Edgeworth presents when illustrating his Figure 1, trade involves substitution of ‘contracted labour’ for ‘uncontracted labour’. As the extent of trade increases, so too does the extent of the substitution.

In a pre-trade state, Figure I illustrates that Friday devotes the segment $Oy_0^F$ as his time working in uncontracted labour to attain his initial allocation given by point $\eta_0\chi_0$. At that point he is indifferent to devoting $Oy_0^F$ as uncontracted labour\(^6\) and the following combination of labour categories: $y_0^Fy_0^F$ in uncontracted labour and $Oy_0^{c}\!$ in contracted labour.

\(^6\) Although, even in the no trade context the part $Oy_0^{c}\!$ of Friday’s uncontracted labour is of value to Crusoe (observation of such labour reveals information about nature and survival).
If trade were introduced in a manner that increases Friday’s total utility and takes him from the initial endowment point $\eta_o$ to point $\alpha$ on the contract curve, a likely allocation scenario is shown in Figure I, for which the post trade allocation involves two coordinates. Specifically, when Friday trades with Crusoe to attain point $\alpha$, his ‘contracted’ labour increases from $y_{oF}$ to $y_{F}$, with that labour exchanged with Crusoe for the quantity of $X$ given by $x_{F} - x_{oF}$. But that would only bring Crusoe to a post trade allocation given at point $\beta$, with his stock of remuneration good $X$ falling by the amount $x_{oC} - x_{C}$ and his access to labour increasing by the amount $y_{oF} - y_{F}$. 

Not only does Edgeworth’s Figure 1 include two pre-trade initial allocations, one for Friday and one for Crusoe, but two coordinates for the post trade allocations are also possible when the outcome is not Pareto optimal. In the
above example, both individuals could gain from a further increase in trade (i.e. an optimal allocation could be achieved somewhere between points \( \alpha \) and \( \beta \)) and, under bilateral trade between Friday and Crusoe, there is no determinate point of efficiency.

The most fundamental historical point to draw from this discussion is that, as Edgeworth’s Figure 1 deals with factor exchange and his Figure 5 deals with exchange of consumer goods, historians are wrong, and they mislead others, when they present Edgeworth’s Figure 1 as if it were Figure 5. That is, as if it were illustrating exchange of consumer goods - like corn and beef – and contend that Figure 1 is an Edgeworth box in all but its framing. For example:

“Edgeworth’s diagram depicts two isolated individuals, A and B, trading fixed stocks of two goods, x and y, whose quantities determine the dimensions of the box ... Individual A initially holds the entire stock of good x and individual B the entire stock of good y. Superimposing indifference maps on the box, Edgeworth sites the origin of A’s map in the lower right corner and the origin of B’s map in the upper left corner”. (Humphrey , 1996 p 40)

The box-esque features of Figure 1 are a consequence of the labour-remuneration trade flows that Edgeworth was illustrating, and that feature cannot be ignored. Furthermore, the focus on the relationship between the Edgeworth Figure 1 and the Edgeworth box has come at the cost of losing Edgeworth’s significant insights into the relationship between the exchange of labour services and the production undertaken by such services (see McLure and Montesano 2017).
4) Moving Towards the ‘Original Version’ of the Edgeworth Box

The contention that Edgeworth’s Figure 1 is the ‘original version’ Edgeworth box, to use Morgan’s phrase (2012, p. 109) depends on redefining the quantity $y$ from labour hours, to Friday’s time, and then imposing the requirement that the origin of the graph indicates an initial allocation in which Friday supplies all labour and Crusoe all of the remuneration good. Neither of those requirements accord with Edgeworth’s account once the relationship between Edgeworth’s Figure 1 and his Figure 5 is considered.

Nevertheless, Edgeworth’s Figures 1 and 5 must both feature prominently in the history of the creation of the Edgeworth box as a diagram; and there can be no serious suggestion that it is incorrect to name the diagram in honour of Edgeworth. These two diagrams are the first diagrams to map a contract curve and two indifference curves, both of which feature in the Edgeworth box diagram. But, just as unequivocally, neither of Edgeworth’s two diagrams can be considered as the ‘original version’ of the Edgeworth box.

As Pareto was an admirer of Edgeworth’s *Mathematical Psychics*\(^7\), it is certain that he was influenced by Edgeworth in some contextual way when the box diagram was being devised. That contextual influence is evident from Pareto’s

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\(^7\) Pareto’s entry on ‘Economie Mathématique’ for the *Encyclopédie des Sciences Mathématiques* follows the characterisation of endowment and exchange set out in Edgeworth’s Figure 5, and in the discussion around that figure in the appendix, when he introduces the case of two commodities, $X$ and $Y$, of which “the first is being offered and its initial quantity is $x_0$; the second is demanded and its initial quantity is zero. The price of $Y$ in terms of $X$ is $p$” (1911 [1955, p61). The agrees with Edgeworth’s Figure 5 and the discussion around that diagram. In the footnote attached to this sentence, Pareto makes Edgeworth’s influence explicit: “Some problems of this type are treated by F. Y. Edgeworth, *Mathematical Psychics*, London 1881. This work is important and remarkable in many respects, especially for the period when it was written.” Pareto (1911 [1955, p61)
polemic with Gaetano Scorza over welfare economics, which has been discussed by John Chipman (1976) and McLure (2000). Specifically, in ‘On a New Error in the Interpretation of the Theories of Mathematical Economics’, Pareto (1902 [2008]) presented the first proof of the first order conditions for that theorem in a world with exchange and production. He then introduced a simple numeric example to illustrate the concepts and, in the process, he provided, for the first time, an illustrative image that explicitly shows the association between efficiency and the point of tangency (m on ray AB) between two individual’s indifference curves (see Figure H).

Diagram H: Pareto 1902 [2008, p. 532]

In this diagram, Pareto, like Edgeworth in his Figure 5, had shown two individuals, two consumer goods, and two indifference curves (one for each individual) in one graph with a common origin. The initial endowment also follows Edgeworth’s Figure 5, although the initial endowments are labelled as points A and B. The stylistic difference is that Pareto’s diagram is rotated clockwise by 90° relative to Edgeworth’s Figure 5. The substantive difference, however, is that both of Pareto’s indifference curves are mapped in conventional ‘allocation’ orientation
for both individuals, which means they are convex to the origin. His main purpose for introducing Diagram H was to illustrate that, when the price ray under free competition, $AB$, is replaced by a different price ray, $rt$, one individual would move to $s$ and the other to $t$, (such that $t$ and $s$ are equidistant from $m$). Consequently, at least one person is made worse off in welfare terms.

As such, Pareto’s 1902 diagram comes to represent an intermediate image in the history of the Edgeworth box. From diagram H it was possible for Pareto to notice that the indifference map for one individual can be rotated 180º and then superimposed over the map of the other individual to create a box diagram that shows, for a price ray given under free competition, the efficient allocation is the one given by the point of tangency between the two individuals’ indifference curves. That is what he did in the 1906 *Manual*.

On balance, then, the diagrammatic history of Edgeworth’s and Pareto’s contributions to the development of the Edgeworth box may be briefly characterised by the following sequence. It starts from Edgeworth’s Figures 1 and 5, for they illustrate the concepts of contract curve and ‘trade’ oriented indifference curve on an open XY diagram with single origin that indicates both no trade and no allocation. It progresses with Pareto’s 1902 Figure from ‘On a New Error in the Interpretation of the Theories of Mathematical Economics’, which builds on Edgeworth’s Figure 5. It highlights efficient allocation attained from trade between two individuals at a market price in a single origin open axes XY diagram, with a ‘no trade’ and a ‘no allocation’ origin, but with parallel indifference curves now presented in ‘allocation’ orientation i.e. convex to the origin. The story culminates in Pareto’s Figure 16 (as well as his figure 50) from his *Manual of Political
Economy, which builds on his 1902 graph. The 1906 figure is a box diagram with indifference curves presented in ‘allocation’ orientation, such that one individual’s curves are mapped with respect to the southwest corner and the other’s mapped with respect to the northeast corner, and every coordinate within the box indicating a unique allocation of two goods between two individuals.

While Mark Blaug did not contrast Edgeworth’s Figure 1 with Pareto’s Figure 16, he nevertheless provided a clear outline of how the Edgeworth box diagram is constructed in his textbook Economic Theory in Retrospect, which is consistent with the historical sequence presented in this paper.8 No comparable procedure is associated with either Edgeworth’s Figure 1 or Figure 5.9

7) Conclusion

Prior to this study, there has been no complete and sound account of the relationship between Edgeworth’s Figure 1, Edgeworth’s Figure 5 and the box diagram. But this does not mean that efforts related to earlier studies have been wasted because each is predicated on important and correct elements of the story. Tarascio failed to recognise the relationship between the trade and allocation

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8 “Following Pareto’s procedure in the Manual, we now combine the two indifference maps in a box diagram by rotating figure 13.2b [an open axes graph showing a system of indifference curves for 1 individual] and imposing it on figure 13.2a [another open axes graph showing a system of indifference curves for a different individual] 180° and imposing it on figure 13.2a until M [indifference curve 1 in Figure 13.2a] and N [indifference curve 1’ in Figure 13.2b] coincide.” (Blaug 1996, p. 571)

9 Perhaps it should be noted that Peter Newman (1965, p 54), a noted Edgeworth scholar, refrains from attributing the 180° rotation process for one agent when creating a box diagram to Edgeworth. He even used the term ‘the box diagram’ not the Edgeworth box diagram. When he does introduce his readers to the Edgeworth’s Figure 1 (Newman 1994, p. 103), he is also careful not to claim that that figure is an Edgeworth box. He simply explains the Friday-Crusoe scenario in a note at the base of the reproduced Figure, noting that the Figure may help readers follow Edgeworth’s arguments in relation to settlements under barter, but then states that “no explicit reference will be made to it” (Newman 1994, p. 102) in his paper.
orientation of indifference curve and he was wrong to suggest that Edgeworth’s Figure 1 cannot be readily transformed into a box diagram and wrong to criticise Edgeworth’s Figure 5. But his insistence on placing the labour-remuneration issue at the core of the diagram was invaluable and is essential to a legitimate historical interpretation of Edgeworth’s Figure 1. Jaffé failed to recognise the labour-remuneration dimension to Figure 1, instead, appearing to think that Figures 1 and 5 are different versions of the same graph, both of which deal with consumer goods. But his insistence on the importance on accounting for the ‘trade’ orientation was essential for the development of a sound account of Figure 1. Creedy’s, Humphrey’s and Morgan’s accounts of Figure 1 as an Edgeworth box with indifference curves in standard allocation, which ignore (or deemphasise in the case of Morgan) the labour-remuneration trade flows in relation to indifference curves; and they also ignore the important production aspect of that diagram. But their work serves the useful purpose of reminding historians that only modest change is required to transform Edgeworth’s Figure 1 from its ‘trade’ orientation into an ‘allocation’ orientation and, from there, into a box diagram.

Edgeworth’s Figures 1 and 5 remain essential elements in the diagrammatic history of the Edgeworth box as they contain the original illustrations of indifference curves and the contract curve. They are important elements of the history of the Edgeworth box; but they are not Edgeworth boxes. Pareto’s overlooked 1902 open axis graph from ‘On a New Error in the Interpretation of the Theories of Mathematical Economics’ also plays a significant role in the history of the Edgeworth box. It constitutes an intermediate diagram, between Edgeworth’s
diagrams and the box diagram in Figure 16 of Pareto’s *Manual*, from which Pareto’s procedure for creating the box diagram could emerge.

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