THE LONG-TERM BEHAVIOUR OF EXCHANGE RATES, PART VI: EQUILIBRIUM EXCHANGE RATES; SUMMARY AND CONCLUSIONS

by

Yihui Lan

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DISCUSSION PAPER 03.10
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* This is the Chapters 6 and 7 of my thesis The Long-Term Behaviour of Exchange Rates, UWA, 2003. The full thesis is available as Discussion Papers 03.05 to 03.10.
6.1. Introduction

Chapter 5 analysed the time-series properties of real exchange rates and the results generally favoured the idea that they are stationary. Hence, the real exchange rate is mean reverting, so that there exists a long-run steady-state value for the rate which acts as an attractor to the actual value. This stable rate is hereafter termed the “equilibrium exchange rate” (EER).

Recall from Chapter 2 that we considered several cases whereby shocks to the economy change the real exchange rate. This occurred in the short run when following a monetary expansion, the exchange rate overshoots its long-run value due to sticky prices; when long-term productivity differences between countries are amplified in the traded goods sector, which causes relative prices to differ systematically as we go across countries; and when there is an export boom. On the other hand, we showed that when prices are flexible, as they are likely to be over the longer term, the real exchange rate is unaffected by monetary shocks. Thus in theory real exchange rates can be either variable or constant in the long run, depending on the nature of shocks to the economy. However, the evidence seems to be unambiguous. The finding from Chapter 5 that real exchange rates are stationary means that they have a long-run tendency to converge to constant values. This finding is also supported by the results in Section 2.3 of Chapter 2 that over the last 30 years, for 68 countries, on average the changes in nominal exchange rates are more or less completely offset by changes in prices, so that the real exchange rate is approximately constant. Furthermore, in the same section of Chapter 2, we showed that over the longer term, the variability of nominal exchange rates is about the same as that
of relative prices. This is not inconsistent with the real rate being a constant in the long run. Finally, as discussed in Chapter 3, in the literature on PPP, there is now an emerging consensus that relative parity holds in the long run, which also amounts to the real exchange rate being constant.

Recall from Chapter 5 that we tested the stationarity of exchange rates based on the fixed-effects AR(1) model:

\[
q_{it} = \alpha_c + \beta q_{c,t-1} + u_{ct},
\]

where \( \alpha_c \) is a country-specific intercept, \( \beta \) the speed-of-adjustment parameter (common to all countries), and \( u_{ct} \) a stationary error term. This chapter uses the estimates of \( \alpha_c \) and \( \beta \) obtained in Chapter 5 to derive equilibrium exchange rates for the 16 countries. For other approaches to modelling equilibrium exchange rates, see Driver and Westaway (2001), MacDonald (2000), and Montiel and Hinkle (1999).

The organisation of the chapter is as follows: Section 6.2 introduces the concept of equilibrium exchange rates, presents estimates of these exchange rates and investigates their sampling variability based on asymptotic theory. In Section 6.3, the exact sampling variability of equilibrium exchange rates is derived using Monte Carlo methods, while Section 6.4 examines the whole distribution of EERs. Section 6.5 illustrates the practical usefulness of our equilibrium exchange rates by analysing the future time paths of the actual exchange rates as they adjust to their long-run equilibrium values. An out-of-sample evaluation of our exchange rate forecasts, based on EERs, is provided in Section 6.6. Our estimates are compared with those from other studies in Section 6.7. The final section summarises and concludes.
6.2. Estimating Equilibrium Exchange Rates

Recall from Section 2.2 that stochastic deviations from relative PPP were described in equation (2.4) as \( s_t = p_t - p^*_t - k + e_t \), where \( k \) is the equilibrium exchange rate and \( e_t \) is a zero-mean random term. This section is devoted to estimating the equilibrium real exchange rate \( k \) for each of the 16 Big Mac countries.

We proceed by taking expectations of both sides of model (1.1),
\[
E(q_{ct}) = \alpha_c + \beta E(q_{c,t-1}) + E(u_{ct}).
\]
As \( E(u_{ct}) = 0 \) and noting that in the long run \( E(q_{ct}) = E(q_{c,t-1}) = q^E_c \), the EER for country \( c \), it follows from (1.1) that
\[
(2.1) \quad q^E_c = \frac{\alpha_c}{1-\beta}, \quad c = 1, \ldots, 16.
\]

To evaluate the EERs, we use the bias-adjusted GLS estimates presented in Chapter 5. For convenience, we reproduce in Table 6.1 these estimates. As discussed in Section 5.6, the standard errors based on block-sectional independence (BSI) are generally larger, suggesting that they are not as efficient as those based on the common factor model (CFM). In addition, the point estimates based on BSI and the trade-weighted CFM are quite close to each other, but those based on the GDP-weighted CFM are quite different. Therefore, we use the estimates derived from the CFM using trade weights (column 3 of Table 6.1) to estimate equilibrium exchange rates.

Column 3 of Table 6.2 contains the estimates of EER calculated from equation (2.1). To illustrate their interpretation, take as an example the first entry of this column, -25.33 percent for Australia. This implies the equilibrium value of the Australian dollar is about 25 percent below the ratio of domestic to foreign prices; i.e., in the long run, the Australian dollar is estimated to be undervalued by about 25 percent, in comparison with its absolute PPP value. Accordingly, this is the value toward which the $A tends in the
TABLE 6.1
ADJUSTED GLS ESTIMATES FROM CHAPTER 5

\[ q_{ct} = \alpha_c + \beta q_{c,t-1} + u_{ct} \]

(Standard errors in parentheses)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Block-sectional independence (1)</th>
<th>Common factor model (2)</th>
<th>Trade (3)</th>
<th>GDP (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept ( \alpha_c )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>-10.13 (3.88)</td>
<td>-9.93 (2.86)</td>
<td>-11.50 (2.79)</td>
<td></td>
</tr>
<tr>
<td>Belgium</td>
<td>13.06 (5.16)</td>
<td>13.13 (4.13)</td>
<td>15.39 (4.35)</td>
<td></td>
</tr>
<tr>
<td>Britain</td>
<td>8.31 (3.32)</td>
<td>8.47 (2.95)</td>
<td>9.62 (3.02)</td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>-5.12 (3.13)</td>
<td>-5.04 (2.55)</td>
<td>-5.70 (2.54)</td>
<td></td>
</tr>
<tr>
<td>Denmark</td>
<td>21.97 (8.20)</td>
<td>22.23 (6.06)</td>
<td>26.41 (6.40)</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>12.63 (5.48)</td>
<td>12.80 (4.31)</td>
<td>15.29 (4.39)</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>8.22 (4.33)</td>
<td>8.14 (3.65)</td>
<td>9.81 (3.73)</td>
<td></td>
</tr>
<tr>
<td>Hong Kong</td>
<td>-25.20 (8.07)</td>
<td>-25.44 (4.85)</td>
<td>-30.08 (5.52)</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>7.66 (4.86)</td>
<td>7.81 (4.21)</td>
<td>9.51 (4.54)</td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>6.47 (7.43)</td>
<td>6.92 (7.36)</td>
<td>8.91 (7.62)</td>
<td></td>
</tr>
<tr>
<td>Netherlands</td>
<td>9.24 (4.52)</td>
<td>9.41 (3.88)</td>
<td>11.07 (3.81)</td>
<td></td>
</tr>
<tr>
<td>Singapore</td>
<td>-6.63 (7.34)</td>
<td>-6.67 (6.98)</td>
<td>-7.69 (6.74)</td>
<td></td>
</tr>
<tr>
<td>South Korea</td>
<td>1.80 (3.13)</td>
<td>1.81 (2.80)</td>
<td>3.51 (3.70)</td>
<td></td>
</tr>
<tr>
<td>Russia</td>
<td>-15.34 (37.04)</td>
<td>-13.60 (29.79)</td>
<td>-17.59 (38.15)</td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>6.81 (4.41)</td>
<td>6.91 (4.14)</td>
<td>8.45 (4.18)</td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>16.18 (7.19)</td>
<td>16.49 (5.42)</td>
<td>20.27 (6.27)</td>
<td></td>
</tr>
<tr>
<td>Speed-of-adjustment ( \beta )</td>
<td>61.66 (28.79)</td>
<td>60.80 (17.69)</td>
<td>53.27 (19.87)</td>
<td></td>
</tr>
</tbody>
</table>

Note: All entries are to be divided by 100.
Source: Tables 5.5 and 5.6.

long run. It is to be noted that this 25 percent undervaluation is not eliminated in the long run; what is eliminated over time is the difference between the current over/undervaluation and this 25 percent. It can be seen that the estimated EERs for Australia, Canada, Hong Kong, Singapore and Russia are negative, which means that their currencies are undervalued in the long run; the currencies of the remaining 11 countries
## Table 6.2

**EQUILIBRIUM REAL EXCHANGE RATES**

(Logarithmic differences of nominal exchange rates from price ratio)

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean of real exchange rates 1989-98 (Standard error of the mean in parentheses)</th>
<th>GLS</th>
<th>Results based on Monte Carlo simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean of real exchange rates 1989-98 (Standard error of the mean in parentheses)</td>
<td>Estimate</td>
<td>Asymptotic standard error</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Australia</td>
<td>-22.57 (2.21)</td>
<td>-25.33</td>
<td>2.16</td>
</tr>
<tr>
<td>Belgium</td>
<td>29.38 (3.74)</td>
<td>33.49</td>
<td>4.25</td>
</tr>
<tr>
<td>Britain</td>
<td>17.52 (2.69)</td>
<td>21.61</td>
<td>3.03</td>
</tr>
<tr>
<td>Canada</td>
<td>-11.42 (2.50)</td>
<td>-12.86</td>
<td>2.33</td>
</tr>
<tr>
<td>Denmark</td>
<td>56.87 (3.87)</td>
<td>56.71</td>
<td>5.87</td>
</tr>
<tr>
<td>France</td>
<td>33.68 (3.29)</td>
<td>32.65</td>
<td>4.01</td>
</tr>
<tr>
<td>Germany</td>
<td>19.67 (3.09)</td>
<td>20.77</td>
<td>3.70</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>-66.20 (.97)</td>
<td>-64.90</td>
<td>1.27</td>
</tr>
<tr>
<td>Italy</td>
<td>20.78 (3.84)</td>
<td>19.92</td>
<td>4.51</td>
</tr>
<tr>
<td>Japan</td>
<td>23.53 (7.95)</td>
<td>17.65</td>
<td>8.08</td>
</tr>
<tr>
<td>Netherlands</td>
<td>23.34 (3.13)</td>
<td>24.01</td>
<td>3.66</td>
</tr>
<tr>
<td>Singapore</td>
<td>-20.08 (6.20)</td>
<td>-17.02</td>
<td>6.04</td>
</tr>
<tr>
<td>South Korea</td>
<td>20.52 (7.20)</td>
<td>4.62</td>
<td>2.84</td>
</tr>
<tr>
<td>Russia</td>
<td>-4.20 (23.56)</td>
<td>-34.69</td>
<td>32.97</td>
</tr>
<tr>
<td>Spain</td>
<td>18.93 (4.08)</td>
<td>17.63</td>
<td>4.16</td>
</tr>
<tr>
<td>Sweden</td>
<td>45.36 (4.76)</td>
<td>42.07</td>
<td>5.92</td>
</tr>
<tr>
<td>Mean</td>
<td>11.57 (7.53)</td>
<td>8.52</td>
<td>8.86</td>
</tr>
</tbody>
</table>

Notes: 1. Except for columns 5, 8, 10 and 11, all entries are to be divided by 100.
2. Column 2 is based on the last column of Table 4.1.
are overvalued in the long run. Column 2 of Table 6.2 gives the arithmetic average of the real exchange rates over the ten years for each country, reproduced from the last column of Table 4.1 with two decimal places added. These “naive” estimates of EERs are fairly closed to their GLS counterparts in column 3, except for Japan, South Korea and Russia. This suggests that the GLS estimation procedure does have some merit in picking up information that is missed by the simple averaging method. The last entry in column 3 indicates that on average the currencies are overvalued vis-à-vis the US dollar by about 9 percent. This means that the US dollar is undervalued by about 9 percent.

We now examine the sampling variability of the EERs using the delta method. Write the parameters of equation (1.1) for \( c = 1, \ldots, 16 \) in vector form as \( \theta = [\alpha, \beta]' \). Let the vector \( \theta_c = [\alpha_c, \beta]' \) be the subset of parameters for country c. It follows from (2.1) that \( q_c^E \) is a nonlinear function of \( \theta_c \), which we write as \( q_c^E = f_c(\theta_c) \), with \( \partial f_c / \partial \theta_c = [1 / (1 - \beta), -\alpha_c / (1 - \beta)^2] \). Let \( \hat{q}_c^E = \hat{f}_c(\hat{\theta}_c) \) be an estimate of the EER, where \( \hat{\theta}_c \) is an estimate of \( \theta_c \). The asymptotic variance of \( \hat{q}_c^E \) is

\[
\text{var} \hat{q}_c^E = \left| \frac{\partial \hat{f}_c}{\partial \theta_c} \right| \Sigma_c \left| \frac{\partial \hat{f}_c}{\partial \theta_c} \right|,
\]

where \( \Sigma_c = \text{var} \theta_c \) is a \( 2 \times 2 \) matrix which is obtained from the intersections of the \( c^{th} \) and \( 17^{th} \) rows and columns of the \( 17 \times 17 \) matrix \( \text{var} \theta \). We use equation (2.2) to compute the asymptotic standard errors (ASEs) of the 16 EERs and the results are given in column 4 of Table 6.2. The ratio of the point estimate of \( q_c^E \) to its ASE provides a test of whether in the long run country c’s currency is over- or under-valued. As can be seen from column 5, the absolute values of the t-ratios for South Korea and Russia are less than 1.9, indicating that the \( \hat{q}_c^E \) for these two countries are not significantly different.
from zero, so that absolute PPP holds in these two instances. In addition, the ASEs are lowest for Hong Kong and largest for Russia, as expected.\footnote{The ASEs are generally larger than the corresponding standard errors of the means of the exchange rates given in column 2 of Table 6.2.} Columns 6 to 11 of Table 6.2 will be discussed in subsequent sections. In the next section, we will investigate the reliability of the ASEs contained in column 4.

6.3. How Reliable are the Estimated Equilibrium Rates?

In Section 6.2, we estimated the EERs for the 16 countries and examined the sampling variability of these estimates using the delta method, which has an asymptotic justification. However, as there are only nine observations for each country, it is a fair question to ask, how reliable are the asymptotic standard errors? In addition to a possible small-sample problem, there is also a further issue. The EER is defined by equation \( (2.1) \) and its estimate is obtained by replacing the unknown parameters on the right-hand side with estimates, \( \hat{\alpha}_c = \hat{\alpha}_c / (1 - \hat{\beta}) \). Accordingly, the estimated EER involves a ratio of estimated parameters. Under normality, these ratios are typically not normally distributed and do not possess finite moments; see, e.g., Bewley and Fiebig (1990), Chen (1999) and Zellner (1978). Under such circumstances, the application of asymptotic theory can be risky and misleading. We address these problems by means of Monte Carlo simulations.\footnote{Other approaches to estimating a ratio of parameters include Zellner’s minimum expected loss (Zellner, 1978, Zellner and Park, 1979), generalised maximum entropy (Golan et al., 1996) and the Bayesian method of moments (Zellner, 1996, 1997). See also Shen and Perloff (2001).}

We first generate multivariate error terms from \( \mathcal{N}(0, S) \) corresponding to the disturbance in equation (1.1), where \( S \) is the estimated disturbance covariance matrix based on either block-sectional independence or the common factor model. Then we use these generated errors, the value of simulated exchange rates in the previous period, and
the data-based (bias-adjusted GLS) estimates to form the simulated value of the current-period exchange rates. This yields nine artificial observations on $q_{ct}$ for each currency. Equation (1.1) is then re-estimated by bias-adjusted GLS to yield the trial's estimates, $\alpha^{(s)}_c$ and $\beta^{(s)}$, and the covariance matrices, $\text{var}\theta^{(s)}$ and $\Sigma^{(s)}_c$. These estimates are then used in equation (2.1) to yield the estimated EER for trial $s$, $q^{E(s)}_c = \alpha^{(s)}_c / (1 - \beta^{(s)})$, and its asymptotic variance is obtained from (2.2) as $\text{var} q^{E(s)}_c = (\partial f^{(s)}_c / \partial \theta^{(s)}_c)' \Sigma^{(s)}_c (\partial f^{(s)}_c / \partial \theta^{(s)}_c)$. We perform 1,000 trials and obtain the mean $\bar{q}^{E}_c = (1/1000) \sum_{s=1}^{1000} q^{E(s)}_c$. In addition, we compute the root-mean-squared error (RMSE) and the root-mean-squared asymptotic standard error (RMSASE) for the EER of country $c$ as

$$
\text{RMSE} = \sqrt{\frac{1}{1000} \sum_{s=1}^{1000} (q^{E(s)}_c - \bar{q}^{E}_c)^2}, \quad \text{RMSASE} = \sqrt{\frac{1}{1000} \sum_{s=1}^{1000} \text{var} q^{E(s)}_c}.
$$

If the asymptotic procedure is working satisfactorily, the mean $\bar{q}^{E}_c$ should be close to the corresponding data-based estimate in column 3 of Table 6.2, indicating unbiasness, and the ratio RMSE / RMSASE should be centered around unity. For more details of the simulation procedure, see Appendix 6.1.

The results of the simulation are contained in columns 6 to 9 of Table 6.2. It can be seen that the means of the simulated equilibrium exchange rates (column 6) are fairly close to the GLS estimates in column 3 of Table 6.2, except for Russia. This suggests that the estimate of EER for this country could be somewhat biased. It is to be noted from column 10 that the RMSEs are at least twice as large as the corresponding

---

3 These two covariance matrices are defined the same as in Section 6.2 -- $\text{var} \theta^{(s)}$ is the $17 \times 17$ covariance of estimated parameters from the simulated data, with $\theta^{(s)} = [\alpha^{(s)}_1, \ldots, \alpha^{(s)}_{16}, \beta^{(s)}]'$; and $\Sigma^{(s)}_c = \text{var} \theta^{(s)}_c$ is a $2 \times 2$ matrix which is obtained from the intersections of the $c^{th}$ and $17^{th}$ rows and columns of the $17 \times 17$ matrix $\text{var} \theta^{(s)}$, as before.
RMSASEs, which indicates that the asymptotic standard errors obtained from equation (2.2) give an overly optimistic picture of the sampling variability of the estimated EERs. We conclude that the estimated EERs are more or less unbiased, but the asymptotic procedure leads to an overstatement of the precision of the estimates. Recall from Section 6.2 that we used the t-ratios of the EER estimates (based on the asymptotic standard errors) to test whether the estimates are significantly different from zero. As the ASEs are shown to be too small, we re-compute the t-ratios based on empirical sampling variability of estimates, i.e., using the RMSEs instead. The new t-ratios are given in column 8 of Table 6.2. It can be seen that the absolute values of the t-ratios for Japan, Singapore, South Korea and Russia are less than 1.9, suggesting that the equilibrium values of their currencies are not significantly different from zero. This is in contrast with the results from column 5 whereby the EERs for only two countries (South Korea and Russia) are not significantly different from zero.

6.4. The Distribution of Equilibrium Exchange Rates

This section explores further the properties of the simulated EERs by examining the distributions of the 1,000 simulated equilibrium rates.

Figure 6.1 gives the histograms of the 1,000 simulated values for each of the 16 EERs. It can be seen that the distributions are unimodal and some display a certain degree of asymmetry (especially for Australia, France and Hong Kong). The top-right corner of the graph for each country presents the same histogram, but now with the scale (of both axes) the same for each country. Therefore, the location and dispersion of the smaller histograms are directly comparable across countries. As the mid-point of the horizontal axis is zero, if a histogram is located towards the right (left) in the smaller graph, then according to the centre of gravity of the simulation, the country’s currency is over (under)-valued in the long-run. If the middle range of a histogram includes zero,
FIGURE 6.1
SIMULATED EQUILIBRIUM EXCHANGE RATES

(100 × logarithmic differences of nominal exchange rates from price ratios)
FIGURE 6.1 (continued)
SIMULATED EQUILIBRIUM EXCHANGE RATES

Spain
Mean = 18
SD = 10

Japan
Mean = 19
SD = 18

South Korea
Mean = 5
SD = 7

Netherlands
Mean = 24
SD = 9

Singapore
Mean = -17
SD = 18

Italy
Mean = 21
SD = 10

Russia
Mean = -52
SD = 93

Sweden
Mean = 44
SD = 12

Mean = 19
SD = 18
the EER is likely to be insignificantly different from zero. This is the case for Japan, Singapore, South Korea and Russia, which agrees with the t-tests discussed at the end of the previous section. Figure 6.1 also shows that Russia has the biggest variance, while Australia, Canada, Hong Kong and South Korea have relatively small variances.

Figure 6.2 summarises the simulation results by plotting the means of 1,000 simulated values for each country and their 95 percent confidence intervals. These confidence intervals are obtained by ranking the 1,000 simulated EERs and then recording the 25th and the 975th values. It can be seen that the 95 percent confidence intervals for four countries -- Japan, Singapore, South Korea and Russia -- include zero. This is again consistent with the t-tests of the previous section. For country c, we have 1,000 simulated values of its EER, \( q_c^{E(1)}, \ldots, q_c^{E(1,000)} \). Suppose that the mean over these 1,000 values happens to be positive; that is, \( \bar{q}_c^E > 0 \). If the number of trials for which the value of \( q_c^{E(s)} \) is negative is \( X \), then \( X/1,000 \) can be considered as an estimate of the p-value for the null hypothesis that absolute PPP holds; that is, that \( q_c^E = 0 \). The procedure is modified in an obvious way for \( q_c^E < 0 \). Column 11 of Table 6.2 presents the results. It can be seen that for twelve countries we can reject the null of long-run absolute parity at the 2 percent level. Only for Japan, Singapore, South Korea and Russia we cannot reject the null; the p-values for these four countries range from 14 to 28 percent. This is consistent with Figures 6.1 and 6.2, where the equilibrium exchange rates for these four countries are not significantly different from zero.

In total there are three sets of results for testing \( q_c^E = 0 \): The new t-ratios from column 8 of Table 6.2, the 95 confidence intervals in Figure 6.2 and the p-values given in the last column of Table 6.2. These tests are not independent, of course. Table 6.3 summarises these results. The results all indicate that the equilibrium exchange rates for Japan, Singapore, South Korea and Russia are not significantly different from zero.
FIGURE 6.2
MEANS AND CONFIDENCE INTERVALS OF SIMULATED EQUILIBRIUM RATES

\( q^E_c \times 100 \)

- Orange: Upper bound
- Magenta: Mean
- Blue: Lower bound

Countries: Australia, Belgium, Britain, Canada, Denmark, France, Germany, Hong Kong, Italy, Japan, Netherlands, Singapore, South Korea, Russia, Spain, Sweden
TABLE 6.3
DOES ABSOLUTE PPP HOLD? A SUMMARY

<table>
<thead>
<tr>
<th>Country</th>
<th>t-value for $H_0: \hat{\sigma}_c = 0$</th>
<th>Is zero included in the 95 confidence interval?</th>
<th>p-value for $H_0: \hat{\sigma}_c = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Australia</td>
<td>-3.56 ×</td>
<td>x</td>
<td>.00</td>
</tr>
<tr>
<td>Belgium</td>
<td>3.34 ×</td>
<td>x</td>
<td>.00</td>
</tr>
<tr>
<td>Britain</td>
<td>2.85 ×</td>
<td>x</td>
<td>.00</td>
</tr>
<tr>
<td>Canada</td>
<td>-2.07 ×</td>
<td>x</td>
<td>.01</td>
</tr>
<tr>
<td>Denmark</td>
<td>4.63 ×</td>
<td>x</td>
<td>.00</td>
</tr>
<tr>
<td>France</td>
<td>3.51 ×</td>
<td>x</td>
<td>.00</td>
</tr>
<tr>
<td>Germany</td>
<td>2.41 ×</td>
<td>x</td>
<td>.01</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>-7.58 ×</td>
<td>x</td>
<td>.00</td>
</tr>
<tr>
<td>Italy</td>
<td>2.03 ×</td>
<td>x</td>
<td>.02</td>
</tr>
<tr>
<td>Japan</td>
<td>1.06 √</td>
<td>x</td>
<td>.14</td>
</tr>
<tr>
<td>Netherlands</td>
<td>2.67 ×</td>
<td>x</td>
<td>.00</td>
</tr>
<tr>
<td>Singapore</td>
<td>-.94 √</td>
<td>x</td>
<td>.17</td>
</tr>
<tr>
<td>South Korea</td>
<td>.71 √</td>
<td>x</td>
<td>.24</td>
</tr>
<tr>
<td>Russia</td>
<td>-.56 √</td>
<td>x</td>
<td>.28</td>
</tr>
<tr>
<td>Spain</td>
<td>1.92 ×</td>
<td>x</td>
<td>.02</td>
</tr>
<tr>
<td>Sweden</td>
<td>3.62 ×</td>
<td>x</td>
<td>.00</td>
</tr>
</tbody>
</table>

Note: Columns 2 and 4 are from columns 8 and 11 of Table 6.2 respectively; column 3 is based on Figure 6.2.

6.5. Convergence to Equilibrium

Suppose it is the end of 1998, the last year of the sample period, and we wish to know the likely future course of the 16 exchange rates. In this section, we illustrate the practical usefulness of the equilibrium exchange rate concept by showing how it can be applied in “real time” to analyse the future time path of the actual rate as it adjusts to its long-run equilibrium value. We proceed by presenting the material in three subsections: (i) The simulation procedure; (ii) converting from real to nominal exchange rates; and (iii) the results.
The Simulation Procedure

The data-generating process of the real exchange rate of country \( c \) is

\[
q_{ct} = \alpha_c + \beta q_{c,t-1} + u_{ct},
\]

where \( \beta < 1 \).\(^4\) By successive substitution, this can be written as

\[
q_{ct} = \alpha_c + \frac{1 - \beta^{t-1}}{1 - \beta} \beta^{t-1} q_{cl} + \sum_{t=1}^{t} \beta^{t-1} u_{ct}.
\]

Note that \( \alpha_c / (1 - \beta) \) is the EER for country \( c \), \( q_c^E \). Denoting the number of years ahead of 1998 (the last year in the sample) by \( j \), we can write the real-time version of the above process from the perspective of 1998 as

\[
q_{c,1998+j} = q_c^E + (q_{c,1998} - q_c^E) \beta^j + \sum_{t=1}^{q} \beta^{j+t} u_{c,1998-t} + \sum_{t=1}^{j} \beta^{j-t} u_{c,1998+t}.
\]

This equation defines the time path of the exchange rate into the future. There are four components of equation (5.1):

- The equilibrium exchange rate \( q_c^E \).

- The initial deviation from equilibrium \( q_{c,1998} - q_c^E \). Given the speed of adjustment \( 0 < \beta < 1 \), by the year 1998 + \( j \), this deviation will have declined to \( (q_{c,1998} - q_c^E) \beta^j \), a fraction of its initial value. When \( j \to \infty \), \( \beta^j \to 0 \) and thus \( (q_{c,1998} - q_c^E) \beta^j \to 0 \).

\(^4\) This equation also has another interpretation. Write it as a generic AR(1) process \( x_t = a + bx_{t-1} + \epsilon_t \), where \( a \) and \( b \) are constants and \( \epsilon_t \) is an error term. This is the discrete-time version of the Ornstein-Uhlenbeck (OU) model. In continuous time, the OU process is \( dx = \eta(x - \bar{x}) dt + \sigma dz \), where \( \bar{x} \) is the mean of \( x \), \( dz \) follows a Wiener process, \( \eta \) is the speed-of-adjustment parameter, and \( \sigma \) is a variance parameter. Given an initial value \( x_0 \), the expected value of \( x \) at some future date, \( t \), is

\[
E(x_t) = \bar{x} + (x_0 - \bar{x}) e^{-\eta t}.
\]

Interpreting \( x_0 \) as \( x_{t-1} \), we have \( t = 1 \) in the exponential of (a). Write \( x_t \) as the sum of its expected value and an error term \( (\epsilon_t) \), \( x_t = E(x_t) + \epsilon_t \). Substituting the RHS of equation (a) with \( t = 1 \) for \( E(x_t) \) yields the AR(1) model \( x_t = a + bx_{t-1} + \epsilon_t \), with \( a = (1 - e^{-\eta}) \bar{x} \) and \( b = e^{-\eta} \). For details, see Dixit and Pindyck (1994, p. 76).

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The term $\sum_{t=1}^{9} \beta^{j+t} u_{c,1998-t}$. As $u_{c,1997}, \ldots, u_{c,1989}$ are the disturbances within the sample period, this term is a weighted sum of these disturbances. The weight $\beta^{j+\tau}$ is accorded to the disturbance $(j+\tau)$ years before the year 1998 + $j$ and serves to dampen the impact of this disturbance on the value of the exchange rate in the year 1998 + $j$.

The term $\sum_{t=1}^{9} \beta^{j-\tau} u_{c,1998+\tau}$. This is the analogous weighted sum of disturbances in the forecasting period. The weight $\beta^{j-\tau}$ is given to the disturbance $(j-\tau)$ years before the year 1998 + $j$. In particular, when $\tau = j$, the weight is $\beta^{0} = 1$, so that the “contemporaneous” shock is given full weight in determining the forecast of that year’s exchange rate.

It is to be noted that a smaller value of $\beta$ means that the convergence of $q_{c,1998+j}$ to $q_{c}^{E}$ is faster. Equation (5.1) implies that if the deviation is positive (negative) at the end of 1998, i.e., $q_{c,1998}$ lies above (below) $q_{c}^{E}$, the time path of $q_{c,1998+j}$ will always be negatively (positively) sloped as $q_{c,1998+j}$ approaches $q_{c}^{E}$. Such behaviour is implied by the first-order model.

We derive the adjustment path using a Monte Carlo approach similar to that employed earlier in this chapter. As shown in Table 6.1, the bias-adjusted estimate of the speed-of-adjustment coefficient lies in the range of .53 – .62, which implies that the half-life of the real exchange rate is between 1.1 – 1.4 years. This means that exchange rates converge to their long-run equilibrium values fairly quickly, so we only look at six years ahead of 1998 (the last year in the sample), from 1999 to 2004 (i.e., $j = 1, \ldots, 6$). In trials of the simulation experiment, we draw $u_{c,1998-\tau}^{(s)}$ ($\tau = 1, \ldots, 9$) and $u_{c,1998+\tau}^{(s)}$ ($\tau = 1, \ldots, j$) for the 16 countries to generate exchange rates. We then obtain bias-adjusted GLS estimates of equation (1.1), $\alpha_{c}^{(s)}$ and $\beta^{(s)}$, and compute $q_{c}^{E(s)}$ via equation (2.1). Finally, $q_{c,1998+j}^{(s)}$ is calculated via equation (5.1). This procedure is replicated 1,000 times. We finally obtain the mean $\bar{q}_{c,1998+j}$ and the $2\frac{1}{2}$ and $97\frac{1}{2}$.
percentiles of the 1,000 simulated values, denoted by \(q_{c, 1998+j}^L\) and \(q_{c, 1998+j}^U\). For details of the simulation procedure, see Appendix 6.1.

**Converting from Real to Nominal Exchange Rates**

Recall from Section 4.3 that the real exchange rate is defined in terms of natural logarithms, i.e.,

\[
q_{ct} = \log \left( \frac{P_{ct}}{S_{ct} P_t^*} \right).
\]

As it is more convenient to examine the path of future exchange rates in terms of currency units, we convert the logarithmic value back to the domestic currency cost of one US dollar. As the real exchange rate \(q_{ct}\) in equation (5.2) consists of two parts, the logarithm of the relative price, \(p_{ct} = \log (P_{ct} / P_t^*)\) and the log of the nominal exchange rate \(s_{ct} = \log S_{ct}\), we write it as

\[
q_t = p_t - s_t,
\]

where, for convenience, we have dropped the country subscripts. Given an estimate of \(q_t\) for some year in the future, \(q_{1998+j}\), the problem is to then infer the value of \(s_{1998+j}\). As the future value of relative price \(p_{1998+j}\) is unknown, we treat this as a signal extraction problem. That is, we form the optimal forecast of \(p_{1998+j}\) conditional on \(q_{1998+j}\) and the past history of these variables.

We assume that (1) \(p_t\) has mean \(\bar{p}\) and variance \(\sigma_p^2\); (2) \(s_t\) has mean \(\bar{s}\) and variance \(\sigma_s^2\); and that (3) \(p_t\) and \(s_t\) are orthogonal, i.e., \(\text{cov} (p_t, s_t) = 0.5\)

---

5 The justification for the orthogonality of the exchange rate and the relative price is the empirical regularity that over the short term, exchange rates seem to be more or less unrelated to relative prices.
We wish to find an optimal forecast of $p_{1998+j}$, $\hat{p}_{1998+j}$, based on the available information $I_{1998+j}$, which consists of $q_{1998+j}$, $\bar{p}$ and $\bar{s}$. That is, we seek to obtain

$$\hat{p}_{1998+j} = E(p_{1998+j} | I_{1998+j}) = E(p_{1998+j} | q_{1998+j}, \bar{p}, \bar{s}),$$

$\text{to infer the future values of the nominal exchange rate } s_{1998+j}$. In Appendix 6.2, we show that minimising the mean squared forecast error yields

$$(5.4) \quad \hat{p}_{1998+j} = (1 - \lambda) \bar{p} + \lambda (\bar{s} + q_{1998+j}),$$

where $\lambda = \sigma_p^2 / (\sigma_p^2 + \sigma_s^2)$. As is well known, relative prices are much less variable than exchange rates (see, e.g., Mussa, 1986); i.e., the ratio $\sigma_p / \sigma_s$ is likely to be very small. Thus, it is natural to consider the situation in which this ratio tends to zero. Write $\sqrt{\theta} = \sigma_p / \sigma_s$, so that $\lambda = \theta / (1 + \theta)$. Then, $\lim_{\theta \to 0} \lambda = 0$. In words, the limiting value of the coefficient $\lambda$ is zero when the variance of relative prices becomes very small (relative to the variance of the exchange rate), i.e., when $\theta \to 0$. Equation (5.4) is thus simplified to $\hat{p}_t = \bar{p}$, and equation (5.3) for future values of $t$ becomes

$$(5.5) \quad q_t = \bar{p} - s_t.$$  

When the relative price is taken to be a constant, there is a one-for-one co-movement in real and nominal exchange rates. Such a result is consistent with the findings on the behaviour of exchange rates in the short run (see, e.g., Obstfeld, 1995).

Based on equation (5.5), we have $s_{1998+j} = \bar{p} - q_{1998+j}$, or in terms of domestic currency units,

$$(5.6) \quad S_{1998+j} = e^{\bar{p} - q_{1998+j}}.$$  

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The Results

We convert real exchange rates, $q_c$, to their nominal counterparts, $S_c$, by using equation (5.6) as follows:\(^6\)

(i) The 1,000 values of real equilibrium rates $q_{c}^{E}$ are used to compute 1,000 values of the nominal EER, $S_{c}^{E}$, by using $S_{c}^{E} = e^{\bar{p}-q_{c}^{E}}$. The means and RMSEs of the 1,000 values of $S_{c}^{E}$ are contained in columns 3 and 4 of Table 6.4. The actual exchange rates at 1998 are presented in column 5. Column 6 gives the under/over-valuations of the 1998 actual rates compared to $S_{c}^{E}$. As can be seen, most nominal rates were undervalued at 1998: the currency of South Korea was most undervalued, and those of Britain and Japan were just slightly below their respective equilibrium; on the other hand, Hong Kong’s currency was more or less at equilibrium and Russia’s moderately overvalued. Recall from Section 4.3 that most of real rates in 1998 were undervalued relative to the US dollar, undervaluations that can be attributed to a misalignment of the nominal rates.\(^7\)

(ii) For a given year in the “future” 1998 $+$ $j$, we compute the mean over the 1,000 simulated values of the future nominal exchange rate for country $c$, $S_{c,1998+j}$, from $q_{c,1998+j}$. For $j = 1, ..., 6$, these values define the future time path and are indicated in Figure 6.3 by the middle curve in each panel.

(iii) The lower and upper bounds of the 95 percent confidence interval are obtained from the nominal versions of the $2\frac{1}{2}$ and $97\frac{1}{2}$ percentiles $q_{c,1998+j}^{L}$ and $q_{c,1998+j}^{U}$. These are shown as the lower and upper curves in each panel of Figure 6.3. Note that the three curves in each panel start with the same point -- the actual nominal exchange rate in 1998. The following comments can be made:

---

\(^6\) The relative price for Russia requires a special treatment. First, as the price level in that country for the first three years of the sample period seems to be unrealistically low, we compute its $\bar{p}$ over the remaining seven years. Second, as the post-January 1998 rouble is equal to 1,000 old roubles, we divide the average relative price of Russia by 1,000 when using equation (5.6).

\(^7\) It is appropriate to clarify the valuation of the US dollar. Here the undervaluations of nominal and real rates relative to the US dollar pertain to the year 1998. By contrast, in Sections 4.3 and 6.2 we noted that the US dollar was undervalued by about 10 percent; this refers to the value of the US dollar over the ten-year sample period. Over the ten-year period, the dollar was undervalued in the first nine years and overvalued in the last; see Figure 4.5 of Chapter 4.
### TABLE 6.4

**EQUILIBRIUM NOMINAL EXCHANGE RATES**

<table>
<thead>
<tr>
<th>Country</th>
<th>Currency unit</th>
<th>Equilibrium nominal rates</th>
<th>Actual exchange rate as of 1998</th>
<th>Under(-)/over(+) valuation relative to (3) (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean (3)</td>
<td>RMSE (4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>A$</td>
<td>1.39</td>
<td>.11</td>
<td>1.51</td>
</tr>
<tr>
<td>Belgium</td>
<td>Bfr</td>
<td>32.32</td>
<td>3.34</td>
<td>38.00</td>
</tr>
<tr>
<td>Britain</td>
<td>£</td>
<td>.59</td>
<td>.04</td>
<td>.60</td>
</tr>
<tr>
<td>Canada</td>
<td>C$</td>
<td>1.31</td>
<td>.09</td>
<td>1.42</td>
</tr>
<tr>
<td>Denmark</td>
<td>Dkr</td>
<td>6.30</td>
<td>.78</td>
<td>7.02</td>
</tr>
<tr>
<td>France</td>
<td>Frf</td>
<td>5.63</td>
<td>.53</td>
<td>6.17</td>
</tr>
<tr>
<td>Germany</td>
<td>DM</td>
<td>1.62</td>
<td>.15</td>
<td>1.84</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>HK$</td>
<td>7.82</td>
<td>.71</td>
<td>7.75</td>
</tr>
<tr>
<td>Italy</td>
<td>Lire</td>
<td>1,488.36</td>
<td>152.24</td>
<td>1,818</td>
</tr>
<tr>
<td>Japan</td>
<td>£</td>
<td>128.77</td>
<td>23.47</td>
<td>135.00</td>
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<td>Netherlands</td>
<td>Fl</td>
<td>1.84</td>
<td>.17</td>
<td>2.07</td>
</tr>
<tr>
<td>Singapore</td>
<td>S$</td>
<td>1.45</td>
<td>.26</td>
<td>1.62</td>
</tr>
<tr>
<td>South Korea</td>
<td>Won</td>
<td>958.65</td>
<td>66.15</td>
<td>1474</td>
</tr>
<tr>
<td>Russia</td>
<td>Rouble</td>
<td>7.92</td>
<td>12.87</td>
<td>6.00</td>
</tr>
<tr>
<td>Spain</td>
<td>Pta</td>
<td>122.69</td>
<td>11.66</td>
<td>156.00</td>
</tr>
<tr>
<td>Sweden</td>
<td>Skr</td>
<td>7.00</td>
<td>.84</td>
<td>8.00</td>
</tr>
</tbody>
</table>

- In 1998, the nominal exchange rate in most countries is undervalued compared to their long-run equilibrium, except for Hong Kong and Russia. Thus as the rates adjust to their long-run equilibrium values, they mostly appreciate, so that the time paths are negatively sloped.

- This suggests that the US dollar was overvalued in 1998 and was expected to depreciate from 1998 onwards.

- The confidence intervals mostly have a tendency to get wider as time moves further into the future. This makes sense as the more distant future is usually more uncertain.
FIGURE 6.3
FUTURE TIME PATHS OF EXCHANGE RATES
(Domestic currency cost of US$1)

(continued on next page)
FIGURE 6.3 (continued)
FUTURE TIME PATHS OF EXCHANGE RATES
(Domestic currency cost of $US1)
• The confidence bands of most currencies are more or less symmetric, except for Russia.8

It is worthwhile noting two further issues: (i) Currencies in the euro area are fixed vis-à-vis each other after 1999, but not with respect to the US dollar. Another reason for these currencies to have different under/over-valuations is because Big Mac prices vary within Euroland. Accordingly, for these countries the EERs and adjustment paths are not the same. (ii) Hong Kong has (since 1983) operated under a currency-board system whereby its exchange rate is fixed at $HK7.80/$US1. While there is no compelling reason for its real exchange rate to be constant, this in part explains why its EER has a relatively small standard error (7 percent, from Figure 6.1), compared to other countries. As the deviation of $HK from its EER in 1998 is quite small, its adjustment path is almost horizontal, as indicated by Figure 6.3.

6.6. An Out-of-Sample Evaluation

As we now have realised exchange rates for 1999-2001, it is appropriate to discuss the ex post quality of the forecasts. Recall that the data-generating process is

\[
q_{ct} = \alpha_c + \beta q_{ct-1} + u_{ct},
\]

In Section 6.5, this model was used to forecast real exchange rates which were then converted to their nominal counterparts. For simplicity, we shall refer to these nominal

---

8 It is to be noted that the asymmetry of the confidence interval in Figure 6.3 pertaining to Russia results from the requirement that prices cannot be negative. As \( q = \log(P/SP^*) \), we have \( q \in (-\infty, +\infty) \). Due to the large standard deviation of its estimated EER (93 percent, from Figure 6.1), Russia has a wide range of 1,000 forecast real rates in each year. When we convert these real rates back to their nominal counterparts via the exponential function in equation (5.6), there is no limit to the upper bound (i.e., \( \lim_{q \to -\infty} e^{q} = \infty \)), while the lower bound is zero (\( \lim_{q \to -\infty} e^{-q} = 0 \)), reflecting the fact that value of the currency cannot be negative. This results in the asymmetric confidence band for Russia’s nominal rate.
forecasts as being derived from the fixed-effects (FE) model (6.1), although it is to be understood that something a bit more elaborate is involved. This section first compares our forecasts from the fixed-effects model with the actual values and then investigates whether this model beats the random walk model.

The First Set of Forecasts

Columns 2, 3, 6, 7, 10 and 11 of Table 6.5 give a comparison of actual and forecast exchange rates, with the forecasts from the FE model. It is fair to say that the point forecasts are not too close to the corresponding realisations. However, most are contained within the 95 percent confidence bands, especially for 1999 and 2000 when there is only one forecast outside the band each year (columns 4 and 8). But for the year 2001, 50 percent of the actual rates are outside (more precisely, above) their respective band (column 12). These forecasting errors serve to remind us of the difficulties in forecasting nominal exchange rates.

To further analyse the discrepancy between the actual and forecast exchange rates, we compute the logarithmic forecast error for country \( c \) as

\[
\delta_c = \log(\frac{\text{actual}_c}{\text{forecast}_c}).
\]

The results are given in columns 5, 9 and 13 of Table 6.5. It can be seen that the absolute values of the differences are generally under 35 percent, except for Russia. We summarise the results over all 16 countries by computing a weighted average of \( \delta_1, \ldots, \delta_{16} \) as

\[
\overline{\delta} = \sum w_c \delta_c,
\]

the weight \( w_c \) being country \( c \)'s share in the total trade for the group. Due to the great uncertainty regarding the estimate of Russia’s EER and hence the forecasts of its nominal rate, we compute the weighted average of \( \delta_c \) with Russia included and excluded.\(^9\) Rows 17 and 18 of Table 6.5 present \( \overline{\delta} \) for each year. It can be seen that (1) the inclusion of Russia increases the

\(^9\) It is to be noted that both the actual and forecast exchange rates for Russia in Table 6.5 are in terms of new roubles which came into effect in 1998; see footnote 6 of this chapter. That is, the forecasts in Table 6.5 for Russia reflect this change in the currency unit.
<table>
<thead>
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<tbody>
<tr>
<td></td>
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<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
<td>(10)</td>
<td>(11)</td>
<td>(12)</td>
</tr>
<tr>
<td>1. Australia A$</td>
<td>1.56</td>
<td>1.48</td>
<td>√</td>
<td>5.26</td>
<td>1.68</td>
<td>1.44</td>
<td>√</td>
<td>15.42</td>
<td>1.99</td>
<td>1.42</td>
<td>x</td>
<td>33.75</td>
</tr>
<tr>
<td>2. Belgium Bfr</td>
<td>37.71</td>
<td>37.45</td>
<td>√</td>
<td>0.69</td>
<td>42.69</td>
<td>35.40</td>
<td>√</td>
<td>18.73</td>
<td>45.18</td>
<td>34.23</td>
<td>x</td>
<td>27.76</td>
</tr>
<tr>
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<td>0.61</td>
<td>√</td>
<td>1.63</td>
<td>0.63</td>
<td>0.60</td>
<td>√</td>
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<td>0.70</td>
<td>0.60</td>
<td>√</td>
<td>15.42</td>
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<td>4. Canada C$</td>
<td>1.49</td>
<td>1.42</td>
<td>√</td>
<td>4.81</td>
<td>1.47</td>
<td>1.38</td>
<td>√</td>
<td>6.32</td>
<td>1.56</td>
<td>1.35</td>
<td>√</td>
<td>14.46</td>
</tr>
<tr>
<td>5. Denmark Dkr</td>
<td>6.95</td>
<td>7.61</td>
<td>√</td>
<td>-9.07</td>
<td>7.88</td>
<td>7.08</td>
<td>√</td>
<td>10.71</td>
<td>8.36</td>
<td>6.78</td>
<td>√</td>
<td>20.95</td>
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<td>6. France Ffr</td>
<td>6.13</td>
<td>6.49</td>
<td>√</td>
<td>-5.71</td>
<td>6.94</td>
<td>6.15</td>
<td>√</td>
<td>12.08</td>
<td>7.35</td>
<td>5.94</td>
<td>x</td>
<td>21.30</td>
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<td>7. Germany DM</td>
<td>1.83</td>
<td>1.80</td>
<td>√</td>
<td>1.65</td>
<td>2.07</td>
<td>1.73</td>
<td>√</td>
<td>17.94</td>
<td>2.19</td>
<td>1.69</td>
<td>√</td>
<td>25.92</td>
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<td>8. Hong Kong HK$</td>
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<td>√</td>
<td>0.13</td>
<td>7.79</td>
<td>7.73</td>
<td>√</td>
<td>0.77</td>
<td>7.80</td>
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<td>9. Italy Lire</td>
<td>1.810</td>
<td>1.725</td>
<td>√</td>
<td>4.81</td>
<td>2.049</td>
<td>1.627</td>
<td>√</td>
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<td>2.169</td>
<td>1.573</td>
<td>x</td>
<td>32.13</td>
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<td>10. Japan ¥</td>
<td>120</td>
<td>164</td>
<td>√</td>
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<td>149</td>
<td>√</td>
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<td>120</td>
<td>√</td>
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<td>2.11</td>
<td>√</td>
<td>-2.40</td>
<td>2.33</td>
<td>2.00</td>
<td>√</td>
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<td>1.59</td>
<td>√</td>
<td>7.28</td>
<td>1.71</td>
<td>1.52</td>
<td>√</td>
<td>11.78</td>
<td>1.81</td>
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<td>18.79</td>
</tr>
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<td>√</td>
<td>-2.30</td>
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<td>1,114</td>
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<td>√</td>
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<td>4.57</td>
<td>√</td>
<td>181.55</td>
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<td>√</td>
<td>177.31</td>
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<td>142</td>
<td>√</td>
<td>9.40</td>
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<td>134</td>
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<td>129</td>
<td>x</td>
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<td>7.82</td>
<td>√</td>
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<td>10.20</td>
<td>7.52</td>
<td>x</td>
<td>30.48</td>
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</table>

Weighted mean of algebraic errors

17. Russia included -0.16 10.57 23.67
18. Russia excluded -2.59 6.49 20.00

Weighted mean of absolute errors

20. Russia excluded 6.88 15.69 20.00

Weighted RMSE of absolute errors

21. Russia included 18.57 26.92 25.82
22. Russia excluded 9.35 10.01 10.27

Notes: 1. The actual exchange rates in columns 2, 6 and 10 refer to monthly averages of the exchange rates in April of the corresponding year and are obtained from the University of British Columbia web site (http://pacific.commerce.ubc.ca/xr/data.html).
2. When computing the weighted means and RMSEs, we use trade (imports plus exports) shares as weights, with the underlying data from International Monetary Fund International Financial Statistics. These weights were also used in Appendix 5.1.
mean error by about 4 percentage points in 2000 and 2001; (2) the actual rates are, on average, fairly close to their forecasts for the year 1999, but the discrepancies widen in the subsequent two years; and (3) the errors for 2001 are all positive, indicating that the 16 currencies are all worth less than forecasted. This is clear evidence that the US dollar continued to be overvalued into 2001. Averaging the algebraic values of the errors has the disadvantage of allowing the positive and negative errors to cancel, which can lead to a false sense of accuracy. An alternative that avoids this problem is to use absolute values of the errors and compute their weighted mean, \( \tilde{d} = \sum w_c |d_c| \). Rows 19 and 20 of Table 6.5 contain the two versions of \( \tilde{d} \), and as can be seen, they also get larger over time, so that the forecasts become more inaccurate in the more distant future. For the year 2001, \( \tilde{d} \) is the same as \( \bar{d} \) as all the errors are positive in that year. Rows 21 and 22 of Table 6.5 measure the dispersion of the absolute errors in the form of weighted root-mean-squared errors (RMSE), \( \sqrt{\sum w_c (|d_c| - \tilde{d})^2} \). These show that when Russia is excluded, the dispersion of the errors is between 9 and 10 percent; the inclusion of Russia increases the dispersion by a factor greater than two.

The Random Walk Model

It is well known that the prediction of exchange rates is a notoriously difficult task. In a highly influential article, entitled “Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?”, Meese and Rogoff (1983) demonstrated that several structural exchange-rate models were unable to outperform the simple prediction that the exchange rate would not change at all, i.e., it follows a random walk. This article significantly altered the economics profession’s understanding about the behaviour of exchange rates. The random walk model is still the standard metric by which to judge the out-of-sample forecast accuracy of empirical exchange-rate models.\(^{10}\) Even though

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\(^{10}\) In recognition of the lasting importance of this article, a conference was held in 2001 at the University of Wisconsin-Madison to celebrate the forthcoming 20th anniversary of its publication. A collection of the conference papers will be published as a special issue of the Journal of International Economics in February 2003.
there are some findings of models whose out-of-sample performance beats that of a random walk (see, e.g., Chinn and Meese, 1995, MacDonald and Taylor, 1993, MacDonald and Marsh, 1997, and Mark, 1995), the consensus still supports the random walk model over fundamental-based models (see, e.g., Frankel and Rose, 1995, and Rogoff, 1999).

In view of the prominence of the random walk (RW) model, we next carry out an out-of-sample comparison of our forecasts from the FE model with the no-change forecasts from the RW model. Columns 2, 5 and 8 of Table 6.6 reproduce the logarithmic errors from Table 6.5, pertaining to the FE model. The forecast exchange rates of country $c$ from the RW model for the three years are all equal to the actual value of country $c$’s nominal rate in 1998. We then compute the logarithmic forecast error of the RW model for the year $1998+j$ as

\[ \Delta \log \text{RW}_{1998+j} = \log(\text{actual}_{1998+j}/\text{RWforecast}_{1998+j}) = \log(S_{c,1998+j}/S_{c,1998}), \quad \text{where } j = 1, 2, 3. \]

Columns 3, 6 and 9 of Table 6.6 give $d'_{c}$ for the 16 countries for each year. The last few rows of these columns present the weighted average of the logarithmic errors $\bar{d}'$, that of the absolute errors $\bar{d}'$, and the weighted root-mean-squared errors. It can be seen from rows 17 to 20 that only when Russia is included, the weighted mean errors of the FE model is smaller (in absolute value) than that of the RW model in 1999; but in all other cases the mean errors for the FE model are larger. Rows 21-22 show that in all cases (except in 1999 with Russia included) the RMSEs of the RW model are smaller than those of the FE model. In summary, these results indicate that the RW model has smaller forecast errors and variances than does the FE model.

**An Absolute Error Criterion**

We have two sets of errors, $d_1, ..., d_{16}$ for the FE model and $d'_1, ..., d'_{16}$ for the RW model. For country $c$ consider the difference between the two absolute errors, $|d_c| - |d'_c|$. If $|d_c| - |d'_c| < 0$, then the FE model outperforms the RW model. The
<table>
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<td>Australia</td>
<td>5.26</td>
<td>3.26</td>
<td>2.01</td>
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<td>-.77</td>
<td>-.07</td>
<td>18.73</td>
<td>11.64</td>
<td>7.09</td>
<td>27.76</td>
<td>17.31</td>
<td>10.45</td>
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<td>3.28</td>
<td>-1.65</td>
<td>4.88</td>
<td>4.88</td>
<td>.00</td>
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<td>4.81</td>
<td>0.00</td>
<td>6.32</td>
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<td>2.86</td>
<td>14.46</td>
<td>9.40</td>
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<td>8.07</td>
<td>10.71</td>
<td>11.56</td>
<td>-.85</td>
<td>20.95</td>
<td>17.47</td>
<td>3.48</td>
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<tr>
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<td>-5.71</td>
<td>-.65</td>
<td>5.06</td>
<td>12.08</td>
<td>11.76</td>
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<td>17.50</td>
<td>3.80</td>
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<td>6.16</td>
<td>25.92</td>
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<td>.26</td>
<td>.77</td>
<td>.64</td>
<td>.13</td>
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<td>4.37</td>
<td>23.06</td>
<td>11.96</td>
<td>11.10</td>
<td>32.13</td>
<td>17.65</td>
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<td>5.41</td>
<td>1.87</td>
<td>11.78</td>
<td>5.41</td>
<td>6.37</td>
<td>18.79</td>
<td>11.09</td>
<td>7.70</td>
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<td>-28.00</td>
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<td>15.20</td>
<td>36.59</td>
<td>17.59</td>
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<td>30.48</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Russia included</td>
<td>-1.16</td>
<td>-4.07</td>
<td>-2.91</td>
<td>10.57</td>
<td>.73</td>
<td>-</td>
<td>23.67</td>
<td>8.97</td>
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<tr>
<td>Russia excluded</td>
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<td>-1.15</td>
<td>6.49</td>
<td>2.56</td>
<td>-</td>
<td>20.00</td>
<td>10.96</td>
<td>-</td>
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<tr>
<td>Weighted mean of absolute errors</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
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<td>Russia included</td>
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<td>2.84</td>
<td>19.55</td>
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<td>6.04</td>
<td>23.67</td>
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<td>20.00</td>
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<td>-</td>
<td>26.92</td>
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<td>25.82</td>
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<td>5.09</td>
<td>-</td>
<td>10.01</td>
<td>7.36</td>
<td>-</td>
<td>10.27</td>
<td>5.74</td>
<td>-</td>
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</table>

| Weighted mean of absolute errors |                  |                  |                          |                    |                  |                          |                    |                  |                          |
| Russia included  | 18.57             | 19.16            | -                        | 26.92              | 12.08            | -                        | 25.82             | 10.64            | -                        |
| Russia excluded  | 9.35              | 5.09             | -                        | 10.01              | 7.36             | -                        | 10.27             | 5.74             | -                        |
reason for using the absolute errors here is that we are interested in the distance of the errors away from zero in either direction. The idea is that forecasting errors are costly as they can entail taking a wrong position, and in view of the symmetrical nature of financial instruments (they can be either bought or sold), a positive forecast error is as “bad” as a negative one. Another reason for employing absolute values is that the use of algebraic errors can give rise to misleading inferences. To illustrate, suppose that the FE error is small, while that of the RW model is a large negative value, so that the FE model provides the better forecast. Here, \( d \approx 0 \), while \( d' \to -\infty \). But if we use the sign of the difference between the algebraic errors, \( d - d' \), as the criterion, then as \( d - d' \to \infty \), we would conclude that the FE error is large relative to that of the RW and wrongly favour the RW model. On the other hand, using the absolute criterion yields \( |d| - |d'| \to -\infty \), so that the FE model wins, as it should. To further understand this distinction, consider Figure 6.4 in which the two errors \( d \) and \( d' \) are on the two principal axes. This graph also has two lines corresponding to \( d = d' \) and \( d = -d' \). Thus the entire \( \{d,d'\} \) plane is divided up into eight regions labelled \( C, \ldots, J \). Consider the region \( I \), which is the space defined by \( d > 0 \), \( d' < 0 \) and \( |d| < |d'| \), as it lies below the \( d = -d' \) line. In this region, the FE models wins, which we indicate by the shading. A similar argument establishes that for the other three shaded regions the FE model also wins. The reverse is true for the four regions that are not shaded. If we use as the criterion the value of the difference between the algebraic errors \( f = d - d' \), then only in four out of eight cases in Panel A do we identify the correct model. For example, in region \( I \), as it is shaded, the FE model wins; but as \( f > 0 \), we would wrongly choose the RW model on the basis of the algebraic values of the errors.

In Table 6.6, we have \( 3 \times 16 = 48 \) values for each of the two errors. Each of the 48 pairs \( \{d_{ci}, d'_{ci}\} \) falls in one of the eight regions in Figure 6.4 and the location of the errors is summarised in Table 6.7. It can be seen from column 6 that 6.3 percent of the cases fall into region \( I \), where the criterion \( f \) wrongly favours the RW model.
FIGURE 6.4
TWO FORECASTS AND TWO CRITERIA

A. \( f = d - d' \)

\[ \begin{align*}
  
  & \text{A: } f = d - d' \\
  & \text{B: } d = d' \\
  & \text{C: } d - d' < 0 \\
  & \text{D: } d = d' \\
  & \text{E: } f < 0 \\
  & \text{F: } f < 0 \\
  & \text{G: } f < 0 \\
  & \text{H: } f > 0 \\
  & \text{I: } f > 0 \\
  & \text{J: } f > 0 \\
\end{align*} \]

B. \( g = |d| - |d'| \)

\[ \begin{align*}
  
  & \text{A: } f = d - d' \\
  & \text{B: } d = d' \\
  & \text{C: } d - d' < 0 \\
  & \text{D: } d = d' \\
  & \text{E: } g = -d - d' < 0 \\
  & \text{F: } g = -d - d' > 0 \\
  & \text{G: } g = -d + d' > 0 \\
  & \text{H: } g = -d + d' < 0 \\
  & \text{I: } g = d + d' > 0 \\
  & \text{J: } g = d + d' > 0 \\
\end{align*} \]
TABLE 6.7
CLASSIFICATION OF FORECAST ERRORS

<table>
<thead>
<tr>
<th>Region</th>
<th>Characteristics</th>
<th>Percentage of cases</th>
<th>Better model</th>
<th>Criterion</th>
<th>Model identified by criterion</th>
<th>True signal?</th>
<th>Model identified by criterion</th>
<th>True signal?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d</td>
<td>d’</td>
<td>d − d’</td>
<td>d + d’</td>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>C</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>50.0</td>
<td>RW</td>
<td>RW</td>
<td>√</td>
</tr>
<tr>
<td>D</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>12.5</td>
<td>FE</td>
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<td>-</td>
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<td>-</td>
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<td>-</td>
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<td>RW</td>
<td>□</td>
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<td>I</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>6.3</td>
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<td>RW</td>
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<td>J</td>
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<td>-</td>
<td>+</td>
<td>+</td>
<td>12.5</td>
<td>RW</td>
<td>RW</td>
<td>√</td>
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<tr>
<td>Total</td>
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<td>100</td>
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Criterion \( f \) gives a false signal in regions F, G, H and I (column 9); in total these four regions account for \( 0 + 12.5 + 4.2 + 6.3 = 23 \) percent of the cases (column 6). Thus the use of the algebraic values of the errors as the criterion leads to the wrong inference being made in almost one quarter of the cases, which is a substantial bias. Panel B of Figure 6.4 uses \( g = |d| - |d'| \) as the criterion and column 11 of Table 6.7 shows that this works in identifying the correct model in all cases.

The differences between the absolute errors are contained in columns 4, 7 and 10 of Table 6.6. For the year 1999, it can be seen from column 4 that three differences are close to zero, four are negative while the remaining nine are positive. For the years 2000 and 2001, there are overwhelmingly more positive differences, indicating the superior performance of the random walk, especially over horizons longer than one year.

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Rows 19 and 20 of columns 4, 7 and 10 contain $|\tilde{d} - |\tilde{d}'|$, the differences between the weighted means of absolute errors from the two models. These differences are always positive, a result which also favours the RW model. As these differences increase over time, they also indicate that the performance of the FE model deteriorates relative to the RW model as the forecast horizon becomes more distant. For example, when Russia is excluded, these differences are 3.2, 3.7 and 5.7 percent for the years 1999-2001 respectively. On average, for both models, the currency of Hong Kong has the smallest forecast errors, while Russia has the largest. Overall, the FE model seems to be unable to beat the RW model.

Figure 6.5 plots the absolute errors from the RW model against those from the FE model. The four panels correspond to the three years ahead of 1998 and the average. Each panel contains 15 points, indicated by crosses, which correspond to the 15 countries (Russia is excluded in all four panels) and one diamond, indicating the mean. If a point is located above the 45 degree line, then the FE model beats the RW model on the absolute error basis. When moving from Panel A to B to C, we see that more and more points fall below the 45° line, indicating that the relative performance of the RW model improves as the horizon lengthens. In addition, the centre point moves further out from the origin over time, indicating that the mean absolute error for both models increases. Additionally, as time proceeds, the centre point falls further below from the 45° line, which also indicates that the quality of the random walk forecasts improves relative to the FE model. Panel D shows that on average, there are overwhelmingly more points below than above the 45° line. These results illustrate in another way our previous conclusion about the superiority of the RW model.

To summarise the above results regarding the forecast errors, on average those from the FE model are larger and have a bigger dispersion than those from the RW model. This pattern is amplified as the forecast horizon moves further into the future.
FIGURE 6.5
TWO SETS OF FORECAST ERRORS
(Logarithmic errors × 100)

A. The year 1999

B. The year 2000

C. The year 2001

D. Average 1999-2001
Dollar-Adjusted Forecasts

Recall from Table 6.6 that most of the FE errors in 2000-2001 are positive; this can be attributed to the strength of the US dollar, which continued to be overvalued against the 16 currencies on average. In 2000-2001, as most of the currencies were undervalued against the US dollar according to the FE model, they were predicated to appreciate. As most of the actual exchange rates moved in exactly the opposite direction, the FE errors are mostly large and all greater than zero. The same thing occurs with the RW errors in 2000 and 2001, but they are smaller.

We can interpret the weighted mean of the logarithmic errors, \( \bar{\alpha} \), as reflecting a “strong US dollar effect”. To remove this effect, we subtract the weighted mean from the logarithmic errors. The adjusted forecast errors from the FE model, \( d_c - \bar{\alpha} \), are presented in columns 2, 5 and 8 of Table 6.8. By construction, the weighted means are now zero while the RMSEs remain unchanged; see rows 17 and 18. In order to put the forecasts from the RW model on exactly the same footing, we also adjust these by subtracting their weighted mean errors; columns 3, 6 and 9 of Table 6.8 contain the results. It can be seen from column 4 that in 1999, the adjusted differences (in the absolute errors) are negative (i.e., the FE model wins) in 4 out of 15 cases, while in the subsequent two years, the total number of negative values increases to 7 and 8, respectively. This suggests that after the removal of the US dollar effect, the FE model still cannot beat the RW model when the horizon is as short as one year or two years, but its performance improves slightly when the prediction horizon becomes longer. This finding is consistent with the consensus in the literature that at short horizons the RW model characterises exchange rates better than most fundamental-based models.

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11 The strength of the US dollar has been featured frequently not only in the financial press, but has also been commented on in official US government publications; see, e.g., Economic Trends, February 2002, Federal Reserve Bank of Cleveland, US.

12 In their survey, Frankel and Rose (1995) summarise that “the Meese and Rogoff analysis [of the superiority of the random walk model] at short horizons has never been convincingly overturned or explained…” For other surveys of the relevant literature, see Isard (1995) and Taylor (1995).
TABLE 6.8
SECOND COMPARISON OF FORECAST ERRORS FROM TWO MODELS
(Adjusted logarithmic errors × 100)

<table>
<thead>
<tr>
<th>Country</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed-effects</td>
<td>Random walk</td>
<td>Difference of abs errors (2) - (3)</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>1. Australia</td>
<td>7.86</td>
<td>4.70</td>
<td>3.16</td>
</tr>
<tr>
<td>2. Belgium</td>
<td>3.28</td>
<td>.67</td>
<td>2.61</td>
</tr>
<tr>
<td>3. Britain</td>
<td>4.22</td>
<td>4.72</td>
<td>-.50</td>
</tr>
<tr>
<td>4. Canada</td>
<td>7.40</td>
<td>6.25</td>
<td>1.15</td>
</tr>
<tr>
<td>5. Denmark</td>
<td>-6.48</td>
<td>.44</td>
<td>6.04</td>
</tr>
<tr>
<td>6. France</td>
<td>-3.12</td>
<td>2.33</td>
<td>5.60</td>
</tr>
<tr>
<td>7. Germany</td>
<td>4.24</td>
<td>.89</td>
<td>3.35</td>
</tr>
<tr>
<td>8. Hong Kong</td>
<td>2.72</td>
<td>1.44</td>
<td>1.28</td>
</tr>
<tr>
<td>9. Italy</td>
<td>7.40</td>
<td>1.00</td>
<td>6.40</td>
</tr>
<tr>
<td>11. Netherlands</td>
<td>.19</td>
<td>.95</td>
<td>-.76</td>
</tr>
<tr>
<td>12. Singapore</td>
<td>9.87</td>
<td>6.85</td>
<td>3.02</td>
</tr>
<tr>
<td>14. Russia</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>16. Sweden</td>
<td>2.47</td>
<td>5.48</td>
<td>-3.01</td>
</tr>
</tbody>
</table>

Weighted mean
17. Russia excluded
Weighted RMSE
18. Russia excluded

Note: These forecast errors are derived from Table 6.6 by subtracting the relevant weighted mean error.
Summary

To conclude, our forecasts of nominal exchange rates are not particularly accurate, reflecting the well-known difficulties involved in forecasting exchange rates. Using unadjusted forecasts, our model is not able to outperform the random walk model, especially as the horizon becomes more distant. We attribute this problem in part to the continued strength of the US dollar. After removing the US dollar effect, we arrive at a somewhat different conclusion, viz., although our model cannot still beat the random walk model for short horizons, things improve somewhat for longer horizons.13 Notwithstanding the problems with the forecasts, in the next section we show that our estimated equilibrium exchange rates are quite close to those obtained from other studies that use more complex methodologies.

6.7. Comparison with Other Studies

In this section, we compare our estimates of the speed-of-adjustment parameter and equilibrium exchange rates with those from other studies.

Panel A of Table 6.9 provides a summary of the estimates of $\beta$ from previous chapters and other Big Mac research. It can be seen that the bias-adjusted estimates from Chapter 5 fall in the range .53 – .62, indicating a half-life of 1.1 – 1.4 years. This relatively short half-life is reassuring as it implies a fairly rapid mean-reversion process. Figure 6.6 plots the four estimates of $\beta$ from this thesis, the one from Cumby (1996), and their corresponding half-lives. The estimate from Cumby is reasonably close to ours and suggests a slightly shorter half-life. One possible reason for the difference is that

---

13 Usually, forecasts of exchange rates from fundamental-based models and the random walk model are viewed as two sets of competing forecasts. One possible future research direction is to analyse the scope for improving exchange rate forecasts by combining the two sets into a composite forecast. The optimal combination of forecasts is discussed in Bates and Granger (1969) and Granger and Newbold (1977, Chap. 8), and has been applied to demand analysis by Barten (1993) and Clements and Lan (2001).
### TABLE 6.9
ESTIMATES OF THE SPEED-OF-ADJUSTMENT PARAMETER

<table>
<thead>
<tr>
<th>Estimate (Standard errors in parentheses)</th>
<th>Source</th>
<th>Estimation method</th>
</tr>
</thead>
<tbody>
<tr>
<td>.42 (.08 - .09)</td>
<td>Chapter 4</td>
<td>Least-squares dummy variable (LSDV)</td>
</tr>
<tr>
<td>.53 (.20) to .62 (.29)</td>
<td>Chapter 5</td>
<td>Bias-adjusted GLS</td>
</tr>
<tr>
<td>.45 (.12)</td>
<td>Cumby (1996)</td>
<td>Bias-corrected LSDV</td>
</tr>
</tbody>
</table>

| .85        | Frankel and Rose (1996) | OLS |
| .79 to .92 | Higgins and Zakrajšek (1999) | Four panel unit root tests |
| .78 and .89 | Lothian and Taylor (1996) | OLS |
| .69 to .78 | Papell (1997) | GLS |
| .85 and .87 | Wei and Parsley (1995) | LSDV |

**Notes:**
1. Estimates of the speed of adjustments in non-Big-Mac research are sometimes reported in terms of the parameter $\rho$ in the equation $\Delta q_{it} = \alpha + \rho q_{t-1} + \epsilon_t$. In these cases, we transform the estimate of $\rho$ into that of $\beta$ using $\beta = \rho + 1$.

2. All adjustment speeds in the table are expressed on an annual basis. In the studies where the underlying data are not annual and the parameter estimated is $\beta$, we compute the speed of adjustment per annum as $\beta^n$, where $n$ is the number of periods per year.

Cumby uses a shorter estimation period. In Panel B of Table 6.9, we present the speed-of-adjustment estimates derived from non-Big-Mac research on PPP. These estimates are usually more than .7, implying a half-life of around two years or more. The difference in adjustment speeds is marked and its explanation warrants further research.

---

14 This information is mainly from Table 3.2, which presents the same information differently.

15 For these estimates of the half-life to be directly comparable, the frequency of underlying data should be the same. See footnote 9 of Chapter 3 for a related discussion. Details of temporal aggregation problems pertaining to PPP half-lives can be found in Taylor (2001).
As summarised in Montiel and Hinkle (1999), there are three approaches to estimating equilibrium exchange rates (EERs):

- A relative PPP-based methodology;
- A trade-equations, or a macroeconomic balance, approach; and
- A general-equilibrium approach.

Our use of the Big Mac data adds a new dimension to the first approach and it raises the question, how close are our estimates to those obtained from more complicated techniques? Table 6.10 makes a comparison of our estimates of the long-run equilibrium
exchange rate, with similar concepts, viz., the fundamental equilibrium exchange rate (FEER) and the behavioural equilibrium exchange rate (BEER). Note that the FEERs in columns 3 and 9 refer to the years 1990 and 1995 respectively, and the BEERs in column 6 correspond to the year 1990. As the years to which the FEERs and BEERs apply are contained in our sample, we can make a direct comparison.

We first compare our Big Mac estimates of the EER (column 2) with the FEERs in Williamson (1994) (column 3) for the German mark and Japanese yen. It can be seen that Williamson’s FEERs for both counties are different from our EER estimates by less than 10 percent (column 5). Our EERs are also very close to the BEERs in Clark and MacDonald (1998) for these two countries (columns 6 and 8). From columns 9 and 11, it can be seen that the FEERs for the five industrial counties of Wren-Lewis and Driver (1998) are less than 11 percent (in absolute value) different from our EERs, except for Japan. Including Japan, the average difference is an apparently modest -5 percent. It is to be noted that as the equilibrium exchange rate is one that removes the effects of all short-run factors, it is reasonable to expect the actual rate to deviate from its equilibrium value in any given year, as columns 4, 7 and 10 show. The important message here is that this comparison with the results from more complex methodologies demonstrates the credible performance of the Big Mac Index in providing a new basis for estimating equilibrium exchange rates.

6.8. Concluding Comments

Based on the conclusion from the last chapter that real exchange rates are stationary, in this chapter we derived long-run equilibrium values of exchange rates. As Chapters 4-6 are intertwined, we recap in Figure 6.7 the issues investigated in these three chapters. This figure is somewhat like Figure 4.1, but includes more technical details.
### TABLE 6.10

**COMPARISON OF DIFFERENT ESTIMATES OF EQUILIBRIUM EXCHANGE RATES**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FEER as at 1990</td>
<td>Percentage difference from Actual rate at 1990</td>
<td>EER</td>
<td>FEER as at 1990</td>
</tr>
<tr>
<td>Britain</td>
<td>0.59</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Canada</td>
<td>1.31</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>France</td>
<td>5.63</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Germany</td>
<td>1.62</td>
<td>1.49</td>
<td>-11</td>
<td>-8</td>
</tr>
<tr>
<td>Italy</td>
<td>1,488</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Japan</td>
<td>129</td>
<td>117</td>
<td>-26</td>
<td>-9</td>
</tr>
<tr>
<td>Mean</td>
<td>-</td>
<td>-</td>
<td>-19</td>
<td>-9</td>
</tr>
</tbody>
</table>

**Notes:**
1. Exchange rates are expressed in terms of the domestic currency cost of $US1.
2. Columns 3 and 6 are calculated from Table 11 of Clark and MacDonald (1998) based on the average of daily actual exchange rates for the first quarter of 1990.
3. Column 9 is taken from Table 5.4 of Wren-Lewis and Driver (1998).
FIGURE 6.7
MORE ON THE STRUCTURE OF CHAPTERS 4 TO 6

Chapter 4

- Preliminary analysis of mean reversion (§ 4.5)
- Big Mac data (§ 4.3)

Chapter 5

- Data-generating process:
  \[ q_{ct} = \alpha_c + \beta q_{c,t-1} + u_{ct} \] (§ 4.5)
- Conventional unit-root tests:
  \[ H_0 : \beta = 1 \text{ against } H_1 : \beta < 1 \] (§ 4.5)

Chapter 6

- Nominal equilibrium rate:
  \[ S^E_c = \exp(\bar{p} - q^E_c) \], where
  \[ \bar{p} = \log(P_{ct}/P^{*}_t) \] (§ 6.5)
- Equilibrium exchange rate:
  \[ q^E_c = \frac{\alpha_c}{1-\beta} \] (§ 6.2)
- Convergence of nominal rate to its equilibrium:
  \[ S^E_{c,1998+j} \rightarrow S^E_c \] (j = 1, ..., 6) (§ 6.5)
- An evaluation of forecasts (§ 6.6)
- Comparison with other studies (§ 6.7)
- The distribution of \( q^E_c \) (§ 6.4)
- A brief comparison of
- Bias-adjustment and standard error correction (§ 5.4)
- Re-examination of tests and test power (§ 5.5)
- Conventional unit-root tests (§ 5.2)
- Multivariate unit-root tests (§ 5.2)
- Test power (§ 5.3)
- Multivariate unit-root tests (§ 5.2)
In this chapter, we clarified the theoretical concept of the equilibrium exchange rate and then applied it to the 16 Big Mac countries. Measures of estimation uncertainty were examined using both asymptotic theory and a Monte Carlo procedure. We analysed, through Monte Carlo methods, the whole distribution of the estimated equilibrium exchange rates and derived the future time paths of the actual exchange rates as they adjust to their long-run equilibrium values. A comparison of our estimated equilibrium exchange rates with others was also made. Surprisingly, our estimates of EER are quite close to those derived from more complex methodologies, which further enhances the appeal of the Big Mac approach.

Overall, there are several attractions of our approach to estimating equilibrium exchange rates: (i) It only requires Big Mac prices and nominal exchange rates, both readily available, as inputs; (ii) the economic structure placed on the problem is simple, viz., each real exchange rate has some well-defined long-run equilibrium value; and (iii) the implementation of the approach is relatively easy. The results of Chapters 4-6 lead to the conclusion that the Big Mac approach provides a convenient and valuable way of testing for PPP and estimating equilibrium exchange rates.
Appendix 6.1

Simulating Equilibrium Exchange Rates

In Section 6.2, we defined the equilibrium real exchange rate for country \( c \) as

\[
q_c^E = \frac{\alpha_c}{1-\beta}, \quad c = 1, \ldots, 16,
\]

where \( \alpha_c \) and \( \beta \) are the country-specific intercept and the common speed-of-adjustment parameter of the following fixed-effects AR(1) model,

\[
q_{ct} = \alpha_c + \beta q_{c,t-1} + u_{ct}, \quad c = 1, \ldots, 16, \quad t = 2, \ldots, 10.
\]

To implement equation (A1.1), we use the bias-adjusted GLS estimates given in column 3 of Table 6.1. A Monte Carlo simulation procedure is used to derive the standard errors of the estimated equilibrium exchange rates based on a multivariate normal distribution of the error terms in equation (A1.2). A simulation procedure is also used to derive the future adjustment path of the exchange rate to its long-run equilibrium value. We commence with an overview of the procedure used and then present a detailed statement.

Overview

For a given trial, we start by drawing multivariate-normal error terms from \( \mathcal{N}(0, S) \) for equation (A1.2), where \( S \) is the estimated disturbance covariance matrix based on the trade-weighted common factor model. Then we use this generated error, the simulated real exchange rate in the previous period, and the data-based (bias-adjusted) GLS estimates to form the simulated value of the current-period exchange rate. Note that
the initial value of the real exchange rate is set to be its observed value in the first year of the sample period. Equation (A1.2) is then re-estimated by bias-adjusted GLS to yield the trial s estimates $\alpha_c^{(s)}$ and $\beta_c^{(s)}$. These estimates are then used in equation (A1.1) to yield the estimated equilibrium exchange rate for trial s, $q_c^{E(s)} = \alpha_c^{(s)} / (1 - \beta_c^{(s)})$, and its asymptotic variance, $\text{var} q_c^{E(s)} = (\partial f_c^{(s)} / \partial \theta_c^{(s)})' \Sigma_c^{(s)} (\partial f_c^{(s)} / \partial \theta_c^{(s)})$, where the matrices $\theta_c^{(s)}$ and $\Sigma_c^{(s)}$ are as defined in equation (2.2) in the text, but now are specific for trial s. We perform 1,000 trials and compute the mean $\bar{q}_c^{E}$, the root-mean-squared error (RMSE) and the root-mean-squared asymptotic standard error (RMSASE) as

\[
\begin{align*}
\bar{q}_c^{E} &= (1/1,000) \sum_{s=1}^{1,000} q_c^{E(s)}, \\
\text{RMSE} &= \sqrt{(1/1,000) \sum_{s=1}^{1,000} (q_c^{E(s)} - \bar{q}_c^{E})^2}, \\
\text{RMSASE} &= \sqrt{(1/1,000) \sum_{s=1}^{1,000} \text{var}(q_c^{E(s)})}.
\end{align*}
\]

(A1.3)

In each trial, we also compute the nominal exchange rate based on equation (5.6) for country c:

\[
S_{ct} = e^\bar{p}_c - q_a,
\]

(A1.4)

where $\bar{p}_c$ is the average relative price for country c from 1989 to 1998. The mean and RMSE of the 1,000 nominal rates $S_c^{E(s)}$ are then obtained in the same manner as above.

Next, we derive the time path of each real exchange rate into the future. Let $q_{c,1998}$ be the observed real exchange rate for country c in 1998, and for some future year
1998 + j, let $q^{(s)}_{c,1998+j}$ be the value at trial $s$ of the simulated rate $j$ years ahead. We then have the trial $s$ version of equation (5.1):

\[
(A1.5) \quad q^{(s)}_{c,1998+j} = q^E_{c}^{(s)} + (q^E_{c,1998} - q^E_{c}) \beta^{(s)} + \sum_{\tau=1}^{s} \beta^{(s)}_{j+\tau} u^{(s)}_{c,1998-\tau} + \sum_{\tau=1}^{s} \beta^{(s)}_{j-\tau} u^{(s)}_{c,1998+\tau}.
\]

As convergence to equilibrium is quite rapid, we only examine a 6-year horizon, so that $j = 1, \ldots, 6$. We calculate the mean of $q^{(s)}_{c,1998+j}$ over the $s=1,\ldots,1000$ trials, and denote it by $\overline{q}_{c,1998+j}$. We sort the 1,000 simulated values of $q^{(s)}_{c,1998+j}$ into ascending order, and the 25th and the 975th values define the lower and upper bounds of the 95 percent confidence interval.

**The Procedure in Detail**

The detailed simulation procedure is as follows:

1. **Step 1:** Estimate equation (A1.2) by bias-adjusted GLS using Steps 1-11 described in Appendix 5.4. Denote the estimates by $\overline{\alpha}$ and $\overline{\beta}$.

2. **Step 2:** Generate the simulated exchange rates, $q^s_{ct}$ and $q^s_{c,t-1}$ in the same manner as previously (see, e.g., Steps 3-5 in Appendix 5.4).

3. **Step 3:** Re-estimate equation (A1.2) by bias-adjusted GLS to yield the simulated estimates of the coefficient vector at trial $s$, $\theta^s = [\alpha^s_1, \ldots, \alpha^s_{16}, \beta^s]$, and its 17 $\times$ 17 covariance matrix $\text{var} \theta^s$.

4. **Step 4:** Substitute the bias-adjusted estimates at trial $s$, $\alpha^s_c$ and $\beta^s$, into expression (A1.1) to obtain the equilibrium exchange rate $q^E_{c} = f_c[\theta^s] = \alpha^s_c / (1-\beta^s)$. 

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Step 5: Use the average relative price for country $c$ over the whole sample period (1989-1998), $\bar{p}_c$, to compute the nominal equilibrium rate based on equation (A1.4), i.e., $S^{E(s)}_c = e^{\bar{p}_c - q^{E(s)}_c}$.

Step 6: Pick the $c^{th}$ and $17^{th}$ elements of $\theta^s$, and denote the new vector by $\theta^s_c$, and the intersections of the $c^{th}$ and $17^{th}$ rows and columns of the $\text{var } \theta^s_c$, and denote this $2 \times 2$ sub-matrix by $\Sigma^{(s)}_c$. The asymptotic variance of $q^{E(s)}_c$ is then calculated as $\text{var } q^{E(s)}_c = (\partial f^{(s)}_c / \partial \theta^s_c)' \Sigma^{(s)}_c (\partial f^{(s)}_c / \partial \theta^s_c)$.

Step 7: Compute $q^{(s)}_{c,1998+j}$ according to (A1.5) for $c = 1, \ldots, 16$ and $j = 1, \ldots, 6$.

Step 8: Repeat Steps 2 to 7 1,000 times to yield $s = 1, \ldots, 1,000$ values of $q^{E(s)}_c$ and $q^{(s)}_{c,1998+j}$, for $j = 1, \ldots, 6$.

Step 9: From the 1,000 real equilibrium rates, obtain its mean $\bar{q}^{E}_c$, RMSE and RMSASE using (A1.3). From the 1,000 nominal equilibrium rates $S^{E(s)}_c$, compute the mean and RMSE in the same manner.

Step 10: Compute the mean of future real rates $\bar{q}^{(s)}_{c,1998+j} = (1/1,000) \sum_{s=1}^{1,000} q^{(s)}_{c,1998+j}$.

Step 11: Sort the 1,000 values of $q^{(s)}_{c,1998+j}$. The 25th and 975th percentiles are the lower and upper bounds of $\bar{q}^{(s)}_{c,1998+j}$ respectively, denoted by $q^{L}_{c,1998+j}$ and $q^{U}_{c,1998+j}$.

Step 12: Convert $q^{L}_{c,1998+j}$ and $q^{U}_{c,1998+j}$ into their nominal counterparts by using equation (A1.4). Repeat Steps 10-12 for $j = 1, \ldots, 6$. 
Appendix 6.2

The Signal Extraction Technique

In Section 6.5, we used the signal extraction approach to forecast future exchange rates. This appendix sets out the technical details of the signal extraction technique (SET) and its applications to two problems: (1) The extraction of information regarding the price level from a noisy observation on the price of an individual product (Lucas, 1973); and (2) forecasting prices at home and abroad, which is used to derive future exchange rates. The first section of this appendix is based on Enders (1995, Chap. 3).

SET Fundamentals

Signal extraction issues arise when we wish to find the optimal forecast of the component of a time series. Examples of the decomposition of a series into its components are the “random walk plus drift” model, the “general trend plus irregular” model and the “local linear trend” model. As a simple example of SET, consider a series consisting of two independent white-noise components:

\[(A2.1) \quad y_t = \varepsilon_t + \eta_t,\]

where \(E(\varepsilon_t) = E(\eta_t) = E(\varepsilon_t \eta_t) = 0\), \(E(\varepsilon_t^2) = \sigma^2_\varepsilon\) and \(E(\eta_t^2) = \sigma^2_\eta\). The value of \(y_t\) is observed, but not the components \(\varepsilon_t\) and \(\eta_t\). The problem is to find the optimal forecast of \(\hat{\varepsilon}_t\), conditional on the information available in the form of the value of \(y_t\). The conditional expectation is written as \(E(\varepsilon_t | I_t)\), where \(I_t\) represents the set of information available as at time \(t\), which comprises \(y_t, \sigma^2_\varepsilon\) and \(\sigma^2_\eta\).
A linear forecast of $\varepsilon_t$ takes the form

\begin{equation}
(A2.2) \quad \hat{\varepsilon}_t = a + b y_t,
\end{equation}

where $a$ is an intercept and $b$ is a slope coefficient. As $E(\varepsilon_t) = E(\eta_t) = 0$ and $E(y_t) = E(\varepsilon_t) + E(\eta_t) = 0$, the intercept $a$ is zero, and thus $\hat{\varepsilon}_t = b y_t$. We define the optimal forecast by choosing the value of the slope coefficient $b$ that minimises the mean squared error (MSE):

$$
MSE = E(\varepsilon_t - \hat{\varepsilon}_t)^2 = E(\varepsilon_t - b y_t)^2.
$$

Using equation (A2.1) we have

$$
MSE = E[(1-b)\varepsilon_t - b \eta_t]^2 \quad \text{and, since} \quad E(\varepsilon_t, \eta_t) = 0,
$$

$$
MSE = (1-b)^2 E(\varepsilon_t^2) + b^2 E(\eta_t^2) = (1-b)^2 \sigma_\varepsilon^2 + b^2 \sigma_\eta^2.
$$

The first-order condition to minimise the MSE with respect to $b$ is then

$$
-2(1-b)\sigma_\varepsilon^2 + 2b\sigma_\eta^2 = 0,
$$

so that

$$
b = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2}.
$$

It can be seen that the slope term $b$ is determined by the relative variance of $\varepsilon_t$. Consider the case in which most of the variability of $y_t$ is attributed to its component $\varepsilon_t$; then, $\text{var } y \approx \text{var } \varepsilon$, or $\sigma_\varepsilon^2 + \sigma_\eta^2 \approx \sigma_\varepsilon^2$, which implies that $\sigma_\eta \approx 0$. In this situation, $b = \sigma_\varepsilon^2 / (\sigma_\varepsilon^2 + \sigma_\eta^2) \approx 1$ and a given realisation of $y$ is interpreted as providing much information on $\hat{\varepsilon}$. Conversely, when $\sigma_\varepsilon^2 \approx 0$, $b \approx 0$, and a reading on $y$ provides minimal information on $\hat{\varepsilon}$. 
The Lucas Model

Suppose a firm in a competitive industry experiences an increase in the nominal price of its product. This leads to two alternatives: First, if the price increases more than prices in general, then there has been a relative price rise and the firm should expand output. Second, if the firm’s price just increases at the same rate as all prices, then the relative price is constant and the firm’s output should remain unchanged. Clearly, the comparison of the firm’s price with the price level is of critical importance for the behaviour of the firm. Due to information asymmetries, indexes of the price level are typically available only after a considerable lag. This means that the firm faces a major problem: It can observe its price, but the same is not true for the price level. It has to form an estimate, or an expectation, of the index of all prices.

Another way of describing the above problem is to state that the firm’s nominal price is a noisy reading on its relative price, i.e., it is made up of both the relative price and the price level. Denoting the relative price by \( z \) and the general price level by \( P \), the firm’s actual price \( P(z) \) is the sum of these two components:

\[
(A2.3) \quad P(z) = P + z.
\]

The problem is to extract the signal of the general price level, \( \hat{P} \), from the noisy observation on the firm’s price \( P(z) \). In a prominent paper that helped him win the Nobel Prize in 1995, Lucas (1973) proposed that this situation could be analysed using the SET.
Assume that (1) the historical information on the general price level is summarised in its mean $\bar{P}$ and variance $\sigma_P^2$; and (2) the firm’s relative price is independent of $P_t$, with mean zero and variance $\sigma_z^2$. A linear forecast of $P_t$ is

(A2.4) \[ \hat{P}_t = \alpha + \beta P_t(z), \]

where $\alpha$ and $\beta$ are both constants. Taking the expectation of equation (A2.3), we have $E[P_t(z)] = E(P_t) = \bar{P}$ as $E(z) = 0$. Using this information in the expectation of equation (A2.4), we obtain $\bar{P} = E(\alpha) + \beta \bar{P}$. As $\alpha$ is a constant, it can be expressed as

(A2.5) \[ \alpha = E(\alpha) = (1-\beta) \bar{P}. \]

Consider the MSE defined as $E(P_t - \hat{P}_t)^2$. Combining (A2.4) and (A2.3), we have $\hat{P}_t = \alpha + \beta P_t + \beta z$, so that $\text{MSE} = E[(1-\beta)P_t - \alpha - \beta z]^2$. Expanding the square on the right-hand side yields

(A2.6) \[
\text{MSE} = (1-\beta)^2 E(P_t^2) + \alpha^2 + \beta^2 E(z^2) \\
- 2\alpha (1-\beta) E(P_t) - 2\beta (1-\beta) E(P_t z) + 2\alpha \beta E(z).
\]

As for any variables $x$ and $y$, $\text{var}(y) = E[y - E(y)]^2 = E(y^2) - E^2(y)$ and $\text{cov}(x,y) = E(x y) - E(x)E(y)$, we have $E(P_t^2) = E^2(P_t) + \text{var}(y) = \bar{P}^2 + \sigma_P^2$, $E(z^2) = E^2(z) + \text{var}(z) = \sigma_z^2$, and $E(P_t z) = E(P_t)E(z) = 0$ since $E(z) = 0$. Thus, equation (A2.6) can be simplified to

\[
\text{MSE} = (1-\beta)^2 (\bar{P}^2 + \sigma_P^2) + \alpha^2 + \beta^2 \sigma_z^2 - 2\alpha (1-\beta) \bar{P}.
\]
Substituting (A2.5) into the above equation, we have \( \text{MSE} = (1-\beta)^2 \sigma_p^2 + \beta^2 \sigma_z^2 \). Minimising the MSE as before yields

(A2.7) \[ \beta = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_z^2}. \]

Therefore, the optimal forecast of the general price level is:

\[ \hat{P}_t = (1-\beta)\bar{P} + \beta P_t(z), \]

with \( \beta \) defined by (A2.7). If the change in nominal price is mainly caused by market-specific, but not economy-wide shocks, then \( \sigma_z^2 \gg \sigma_p^2 \) and the slope coefficient \( \beta \approx 0 \). The optimal forecast of the price level in this case is just (approximately) equal to its historical mean: \( \hat{P}_t \approx \bar{P} \). This makes intuitive sense as here a realisation of the firm’s price provides little information on the price level. On the other hand, when most of the variability of the firm’s nominal price results from shocks common to the whole economy, \( \sigma_z \approx 0 \), and \( \beta \approx 1 \). The predicted price level is \( \hat{P}_t \approx P_t(z) \), so that the general price level mimics the movements of the firm’s nominal price.

**Application to Exchange Rates**

The real exchange rate \( q_t \) (in logarithmic form) is made up of two components:

(A2.8) \[ q_t = p_t - s_t, \]
where \( p_t \) is the logarithm of the ratio of home to foreign prices and \( s_t \) is the logarithmic nominal exchange rate. In Section 6.5, we derived forecasts of future values of the real exchange rate \( q_t \). What we need to do is to find the optimal predictor of \( p_t \), \( \hat{p}_t \), based on the available information \( I_t \), which consists of \( q_t \), \( \bar{p} \) and \( \bar{s} \). That is, we have to obtain \( E (p_t | I_t) = E (p_t | q_t, \bar{p}, \bar{s}) \) to infer the future values of the nominal exchange rate \( s_t \).

We make three assumptions: (1) That \( p_t \) has mean \( \bar{p} \) and variance \( \sigma_p^2 \); (2) that \( s_t \) has mean \( \bar{s} \) and variance \( \sigma_s^2 \); and that (3) \( p_t \) and \( s_t \) are orthogonal, i.e., \( \text{cov}(p_t, s_t) = 0 \). The justification for the third assumption is that over the short term, exchange rates seem to be more or less unrelated to relative prices. Dropping the time subscript for convenience and letting \( x = -s \), the real exchange rate in (A2.8) can then be expressed as the sum of its two components:

\[
(A2.9) \quad q = p + x.
\]

The linear forecast of \( p \) has the form

\[
(A2.10) \quad \hat{p} = \eta + \lambda q
\]

with \( \text{MSE} = E (p - \hat{p})^2 \). Substituting the right-hand side of equation (A2.10) for \( \hat{p} \), we have \( \text{MSE} = E (p - \eta - \lambda q)^2 = E [(1-\lambda) p - \eta - \lambda x]^2 \), where the second step is based on (A2.9). Expanding the square on the right-hand side gives

\[
(A2.11) \quad \text{MSE} = (1-\lambda)^2 E(p^2) + \eta^2 + \lambda^2 E(x^2) - 2 \eta (1-\lambda) \bar{p} + 2 \eta \lambda \bar{x} - 2 \lambda (1-\lambda) E(p x).
\]
To simplify this expression, we use (i) \( E(p^2) = \bar{p}^2 + \sigma_p^2 \), and \( E(x^2) = \bar{x}^2 + \sigma_x^2 \); and (ii) \( E(px) = E(p)E(x) = \bar{p}\bar{x} \), which follows from \( \text{cov}(p, x) = E(px) - E(p)E(x) \).

Equation (A2.11) then becomes

\[
\text{MSE} = (1 - \lambda)^2 (\bar{p}^2 + \sigma_p^2) + \eta^2 + \lambda^2 (\bar{x}^2 + \sigma_x^2) - 2 \eta (1 - \lambda) \bar{p} + 2 \eta \lambda \bar{x} - 2 \lambda (1 - \lambda) \bar{p} \bar{x}.
\]

(A2.12)

The problem now becomes to choose the values of \( \eta \) and \( \lambda \) to minimise the MSE. We differentiate the right-hand side of equation (A2.12) with respect to \( \eta \) and \( \lambda \) to yield:

\[
\frac{\partial (\text{MSE})}{\partial \eta} = 2 \eta - 2(1 - \lambda) \bar{p} + 2 \lambda \bar{x}, \quad \text{and}
\]

\[
\frac{\partial (\text{MSE})}{\partial \lambda} = -2(1 - \lambda)(\bar{p}^2 + \sigma_p^2) + 2 \lambda (\bar{x}^2 + \sigma_x^2) + 2 \eta \bar{p} + 2 \eta \bar{x} - 2 p \bar{x} + 4 \lambda \bar{p} \bar{x}.
\]

Setting these derivatives equal to zero yields

\[
\lambda = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_x^2}, \quad \eta = (1 - \lambda) \bar{p} - \lambda \bar{x}.
\]

The optimal forecast of \( p \) is \( \hat{p} = E(p \mid I_t) = E(p \mid q_t, \bar{p}, \bar{x}) \). Substituting the above expressions into (A2.10), the optimal forecast takes the form:

\[
\hat{p} = (1 - \lambda) \bar{p} + \lambda (\bar{x} + q_t),
\]
where $\lambda = \sigma_p^2 / (\sigma_p^2 + \sigma_s^2)$. When $\sigma_p^2$ is small, $\lambda \approx 0$ and $\hat{p} \approx \bar{p}$. Given an estimate of $q_i$ for some year in the future, the predicted value of $s_i$ from equation (A2.8) is $\hat{s}_i = \bar{p} - q_i$. In words, when the variability of relative prices is small relative to that in $q$ (as seems to be the case), SET provides a simple solution to the problem of forecasting the nominal exchange rate: It is equal to the mean of the relative prices minus the forecast of the real exchange rate.
References


In his classic paper advocating a system of flexible exchange rates, Friedman (1953) envisaged that floating rates would adjust fairly smoothly over time in moving from one equilibrium to another. A major mechanism underlying this smooth adjustment process was, according to Friedman, speculation that would have stabilising effects. However, such has not been the case with floating exchange rates since the breakdown of the Bretton-Woods system in 1973. During this period, an enduring characteristic of exchange rates has been their volatility. Consider, for example, the values of British pound, the German mark and the Japanese yen relative to the US dollar. Over the past 30 years, about 30 percent of the monthly changes (in absolute value) in these three currencies are greater than 3 percent. By contrast, the corresponding relative prices (the price level at home relative to that abroad) are much less volatile with 30 percent of their monthly absolute changes greater than only 1.3 percent. Over the longer term, however, the exchange rate is less volatile and tends to revert to some kind of mean or trend level that reflects relative prices. This implies that a version of purchasing power parity holds in the long run. Though still controversial, PPP is now finding increasing support in the literature as a theory explaining the long-term behaviour of exchange rates.

The thesis made three main contributions to the existing literature on exchange rates: (i) It synthesised aspects of exchange-rate economics that revolve around PPP; (ii) it used enhanced econometric procedures in novel tests of PPP using the Big Mac prices published in The Economist magazine; and (iii) it introduced a new methodology to derive the long-term equilibrium values of currencies. This chapter summarises the six major chapters of the thesis and discusses the significant implications of the study.
7.1. A Summary of the Thesis

Chapter 1 commenced with a discussion of currency values and argued that for many purposes the PPP exchange rate is a more appropriate measure to use than the prevailing rate. Based on the post-Bretton Woods experience of three major currencies, the chapter demonstrated that while the short-run variability of exchange rates is substantial, their long-term behaviour can be well understood when seen through the lens of PPP. However, there still exist large and persistent deviations from parity -- an empirical regularity that remains a puzzle within a strict PPP framework.

Chapter 2 considered aspects of exchange-rate economics that revolve around PPP. It presented in a geometric framework three versions of the relationship between exchange rates and prices and introduced a new type of variance ratio to analyse the “excess” volatility of exchange rates (in relation to prices) in a large number of countries over the post-Bretton Woods period. The chapter then focused on the interplay of monetary and non-monetary (or real) factors in determining exchange rates in both the short and the long run. It contained a detailed investigation of the effects on the exchange rate of (i) a monetary expansion; (ii) the productivity bias, which deals with sectoral differences in productivity across countries; (iii) a monetary expansion with sticky prices; and (iv) a booming export sector. A comprehensive model was introduced that dealt with these four factors in a unified manner.

Chapter 3 gave an overview of the initial demise and then resurrection of research interest in PPP over the course of the past three decades. It documented the substantial growth in the PPP literature and the ubiquity of a popular PPP metric, the Big Mac index. The concept of the half-life as a measure of deviations from parity was clarified and a summary of its estimates reported in the PPP literature was presented. Finally, the chapter provided a summary of recent empirical findings pertaining to PPP from a wide variety of countries using newly-developed econometric techniques.
Chapter 4 first reviewed the evolution of exchange rates and macroeconomic conditions in a number of key countries. It then examined the important features of the Big Mac data for 16 countries from The Economist magazine. A case study was provided that showed that recent movements of the Australian dollar seemed to be in agreement with PPP theory. This chapter also carried out a preliminary analysis regarding Big Mac real exchange rates, the nominal rates adjusted for prices at home and abroad. The results indicated that each real exchange rate has a tendency to revert to some well-defined value, so that real exchange rates are “mean reverting”, or “stationary”. This implies that a weaker version of PPP, “relative” PPP, holds.

To account for the co-movement of different exchange rates, Chapter 5 performed multivariate unit-root tests by using generalised least-squares methods in the seemingly unrelated regression setting. A new methodology based on Monte Carlo techniques was introduced to tackle the problems of bias and estimation uncertainty associated with a small sample in models with lagged dependent variables. The results from the chapter again lend support to the idea of the stationarity of real exchange rates, which amounts to (relative) PPP holding in the long run. Monte Carlo simulation results showed that our tests have desirable power properties.

The important implication of long-run PPP is that movements in the exchange rate compensate for the shifts in the price level over the long term, despite possible short-run deviations. Thus real exchange rates have a distinct tendency to return to some equilibrium values in the long run. Using this idea of a long-run equilibrium value, Chapter 6 derives, through Monte Carlo simulations, the whole distribution of equilibrium exchange rates for each of the 16 countries and their adjustment paths as the actual rates converge to equilibrium. We evaluated the performance of our forecasts by comparing them with those derived from the popular alternative of no-change, according to which the exchange rate follows a random walk. Even though our forecasts do not beat the random walk model, especially over a short time horizon, our estimates of equilibrium exchange rates are quite similar to those obtained from more complex methodologies.
7.2. Implications of the Study

The simplicity and ease of implementation of Big Mac PPP makes it attractive for international comparisons of incomes, earnings and costs of living, etc., especially for emerging and developing countries that can have data of limited quality and availability. Our estimates of equilibrium exchange rates are particularly useful for several purposes: (i) To forecast exchange rates in the future and to have some knowledge of the nature of the possible path of adjustment to equilibrium. This use has been demonstrated in detail in Chapter 6 of the thesis. (ii) To enable governments identify exchange-rate misalignments that may require corrective action. (iii) To help make corporate investment decisions. We now discuss some of these issues in some more detail.

International comparisons of incomes, earnings, costs of living, etc. are of great interest and importance, so much so that they are arguably among the most important issues in all of economics. Academically, researchers are interested in the measurement and determinants of cross-country real income differences and improvements in living standards over time. Additionally, the cross-country measurement of incomes has important practical implications. International organisations need reliable real income estimates, so that member nations are treated equitably in terms of their financial contributions to the organisations and their access to loans and other forms of assistance under favourable conditions. Multinational enterprises also require similar information about international prices, costs and incomes for the purpose of carrying out investment analysis, profitability comparisons, cost management, determination of expatriate salaries, etc. Finally, individuals need this sort of information when making decisions regarding where to locate internationally. Over the last three decades, many attempts have been made to estimate real incomes across countries. However, there are still several unresolved problems and it seems to be widely accepted that real-income estimates for developing countries in particular are not very reliable. Chapter 1
emphasised that international comparisons based on prevailing exchange rates are extremely unreliable.

In their monumental work, the International Comparison Program (ICP) uses the approach of collecting and combining the prices of numerous individual commodities to make cross-country income comparisons.\(^4\) One can only admire this work for its breadth and painstaking attention to detail; no doubt these are the major reasons why it has become so popular and influential. While this research is impressive, it still may not be appealing to everyone. The ICP approach is quite costly to implement and can involve substantial delays in collecting all the required data. Moreover, as the detailed methodology is quite complex, it may not be readily understood and accepted by non-specialists. By contrast, our approach requires as inputs only Big Mac prices and exchange rates, both of which are freely available instantaneously. In addition, the economic structure placed on the problem is simple and its implementation is quite straightforward. Such attractions thus make the PPP methods advocated in the thesis a feasible “shortcut” approach to carrying out international comparisons of prices, costs and incomes.

In many countries, exchange rates are managed by governments. In the course of managing exchange rates, it is of extreme importance to identify when and by how much exchange rates are misaligned, i.e., when there are large and persistent departures from the equilibrium exchange rate. As Edwards (1988, p. vii) puts it, “The misalignment of the real exchange rate has been a source of serious economic distress for a number of developing countries. In fact the ills of many nations in the past fifteen years or so have been related, in one way or another, to inappropriate exchange rate policies.” Although written in 1988, this statement is equally true today. The approaches to estimating equilibrium exchange rates are many and varied, ranging from the simplest methods based on crude versions of PPP, to single-equation reduced form estimates, and simulations of large-scale macroeconomic models (Montiel and Hinkle, 1999).

Regarding the accuracy of the resulting equilibrium exchange rates estimates, Williamson, one of the leading lights in the area, concludes that “no one has any confidence that such estimates can be made at all precisely, but precision is not needed to provide useful guidance, given the size of swings that unmanaged exchange rates have exhibited.” (Williamson, 2000, p. 7). The new methodology for estimating equilibrium exchange rates introduced in this thesis serves to make the problem much simpler. Big Mac equilibrium exchange rates could be especially convenient for assessing exchange-rate misalignment in developing countries, where data, time and professional capacity may be in limited supply. As estimation uncertainty is made explicit in our framework, it is possible to say exactly how much confidence can be placed in our estimates, so that policymakers can act in an informed manner with the appropriate degree of caution.

With increasing globalisation, more and more companies are engaged in world trade, investment and raising capital in off-shore markets. They have to make critical decisions on strategies such as whether to use a local or overseas supplier; whether to make an investment which yields income denominated in domestic currency, but which is financed by foreign-currency loans; how to assess an international investment that will be serviced from domestic cash flow, etc. The use of prevailing exchange rates as a currency conversion factor is misleading in many instances as the high volatility of rates in the short run can lead to misleading inferences about the long-term economics of proposed corporate decisions. Equilibrium exchange rates are thus useful in such circumstances, especially when investment horizons are long. In addition, firms often use a variety of hedging strategies to manage exposure to foreign-exchange risks, so as to reduce the impact of adverse currency fluctuations on profitability. In such cases, estimates of equilibrium exchange rates can also help CFOs by providing them with benchmark rates. Our approach can be employed by corporations in all the above instances to give quick estimates of equilibrium exchange rates with known degrees of uncertainty.


