ECONOMICS

THE LONG-TERM BEHAVIOUR OF EXCHANGE RATES, PART II: ASPECTS OF EXCHANGE-RATE ECONOMICS

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DISCUSSION PAPER 03.06
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CHAPTER 2

ASPECTS OF EXCHANGE-RATE ECONOMICS

2.1. Introduction

In its most basic form, purchasing power parity (PPP) is rooted in the quantity theory of money. Accordingly, in the longer term, after all adjustments are complete, an increase in the quantity of money leads to an equi-proportional rise in the price level and a depreciation of the currency, other things remaining unchanged. In this sense, in the words of Milton Friedman (1956), “inflation is always and everywhere a monetary phenomenon”, as is the value of a country’s currency. As a theory of the determinants of inflation over longer-term horizons, and especially in high-inflation environments, the quantity theory is by now relatively uncontroversial. The same argument can possibly be made for PPP as a theory of exchange rates over the longer term: The statement that “over the longer term, the currencies of high-inflation countries depreciate”, which has as its basis PPP theory, would now raise only few objections, and even those would probably revolve around disputes about just how long is the “longer term”; what is being held constant in the background of the thought experiment; and the direction of causation, or what causes what -- does inflation cause the exchange rate to depreciate, or is it the other way around? These issues are more of the nature of “questions of clarification”, rather than constituting more fundamental objections to the whole basis of PPP theory.  

The previous paragraph gives a sense that “only money matters” for exchange rates. However, this is not completely true even in the long run, where non-monetary, or “real”, factors can play a significant role in determining the evolution of exchange rates. A leading example of such a real factor is the “productivity-bias” hypothesis introduced

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1 Dornbusch (1987a) also draws parallels between PPP and the quantity theory.
by Balassa (1964) and Samuelson (1964). According to this argument, as cross-country productivity differences are amplified in the production of traded goods (such as agricultural, mineral and manufactured commodities), this leads to the currencies of rich countries being stronger than the values predicted by PPP theory. Conversely, poor countries have currencies that are weaker than their PPP values. This gives rise to important questions regarding one of the most basic issues of economics, how to measure incomes of different countries: If prevailing exchange rates are used to convert GDPs into a common currency, this will have the effect of making rich countries appear to be richer than they really are, and poor countries poorer, so that world income inequality will be overstated. These sorts of measurement issues are of fundamental importance for testing growth theories, as well as for more immediate practical reasons such as the allocation of foreign aid and the amounts that countries should contribute to the cost of running international agencies such as the International Monetary Fund.

Another situation in which a real factor can be important for the long-term value of the exchange rate is when a country experiences a sustained boom in an export industry, which can lead to an appreciation of its currency. This phenomenon, which is known as the “Gregory (1976) thesis” in Australia and the “Dutch disease” elsewhere in the world (The Economist, 26 November 1977, pp. 82-3), can lead to major changes in the industrial structure of the affected country as the stronger value of the currency squeezes other exporters and firms in the import-competing sector. Such an export boom is frequently associated with a resource discovery and some have taken the further step to argue that on balance the country’s economy may actually be hurt by the discovery. This situation has been termed the “resource curse” by Auty (1993) and countries such as Australia, Indonesia and Netherlands are cited as examples.²

² It should be noted that whether the discovery of new resources can in fact end up being a curse is debatable. See, e.g., Davis (1995) and Mikesell (1997). Note also that the resource curse is reminiscent of immiserising growth (Bhagwati, 1958, and Johnson, 1955).
In the short term there is much more scope for professional disagreement regarding the determinants of exchange rates. On a quarter-to-quarter, or even year-to-year, basis, exchange rates are much more volatile than prices, which means that the “drivers” of currency values in the short term must be something other than PPP. The celebrated “overshooting” model, introduced by Dornbusch (1976), is one way of accounting for the excess volatility of exchange rates. In this model, asset markets are taken to clear continuously, while goods prices are sticky. Thus, following a monetary expansion, for a short time the exchange rate depreciates more than the changed monetary stance merits, i.e., the rate overshoots its long-run equilibrium value. This paper by Dornbusch has been one of the most influential in exchange-rate economics in the last three decades as it provides a convincing reason for the observed volatility of exchange rates within a simple and elegant model. In Dornbusch’s model, it is the sticky prices that are the real factor that plays a key role in determining exchange rates in the short term.

Exchange-rate economics has been an active area of research during the past three decades. It includes a wide range of topics such as theories of exchange-rate determination, the efficiency of foreign-exchange markets, official intervention in foreign-exchange markets, target-zone models and foreign-exchange market microstructure; for recent surveys, see Taylor (1995a, 1995b) and Isard (1995). Exchange rates have interconnections with many other macroeconomic variables such as money, income, interest rates, prices, etc., as summarised in the quantity theory of money, the Fisher effect, the various versions of interest-rate parity, purchasing power parity theory, and so on. This chapter does not intend to provide a comprehensive survey of all of the above issues, but places emphasis on aspects of exchange-rate economics that revolve around PPP. Though not all encompassing, PPP is still an extremely

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3 While exchange rates are highly volatile, it is appropriate to note that they tend to be less volatile than stock prices; see, e.g., Frenkel and Mussa (1980). Accordingly, the volatility of exchange rates should not be overstated.

4 According to Rogoff (2001), this paper has been cited 917 times, an extraordinary number, and twenty-five years after its publication, is still referred to on reading lists of graduate courses in international finance in all major economics departments in the US.

5 Section 3.2 gives quantitative evidence on the growth in the research on exchange rates.
important aspect of exchange-rate economics. Dornbusch and Krugman (1976), for example, state that “under the skin of any international economist lies a deep-seated belief in some variant of the PPP theory of the exchange rate.” (p. 540). Though this quote is a quarter of a century old, it clearly continues to be corroborated by intense on-going academic interest in the issue; see Chapter 3 for a review of the recent literature on PPP.

In this chapter we emphasise the interplay of real and monetary factors in determining exchange rates in both the short and the long run. To fix ideas, we start in Section 2.2 with three versions of the relationship between exchange rates and prices. Then, in Section 2.3 we examine the evolution of exchange rates and prices over the past thirty years in a large number of countries. From this broad sweep of experience of the world economy, there emerges substantial prime facie evidence in favour of PPP as a long-run proposition. The evidence presented in this section also illustrates again the fact that in the short run exchange rates are more volatile than prices. Section 2.4 examines the long-run link between exchange rates and prices by setting out the relationship between PPP and the quantity theory of money. Later sections of the chapter then deal with the role of various real factors in determining exchange rates -- enhanced productivity in the production of traded goods in rich countries, sticky prices, which lead to overshooting in the short run, and the case of a booming export sector. Finally, Section 2.9 presents a comprehensive algebraic formulation of previously-discussed factors, followed by a summary and conclusion in Section 2.10.

2.2. Purchasing Power Parity: The Connection between Exchange Rates and Relative Prices

The relationship between exchange rates and national price levels was first recognized by Spanish scholars in the sixteenth century and was summarised in the theory of purchasing power parity by Cassel in the 1920s. The most important feature of PPP is that it specifies a link between exchange rates and prices, but the limitation is that it goes no further in providing details of the link (Frenkel, 1978); see Dornbusch (1987a), Manzur (1993) and
Officer (1982) for more history of thought on PPP. This section presents a graphical framework of PPP by setting out three versions of the link between exchange rates and prices.

A natural starting point of the traditional (or classical) view of PPP is its central building block -- the law of one price, which states that the price of an identical good in two countries should be equal when converted to a common currency. The basic mechanism is arbitrage -- buying in those countries where the price is low and selling where it is high -- will eliminate the price differentials, at least over the medium term. Applying this idea to the price of a market basket at the time \( t \), we have \( P_t = S_t P^*_t \), where \( P_t \) is the domestic price level, \( P^*_t \) the foreign price level, and \( S_t \) the exchange rate. This implies the absolute version of PPP, i.e., \( S_t = P_t / P^*_t \), whereby the absolute PPP of a currency is determined by the ratio of the domestic price level to the foreign price level. In terms of natural logarithms, we have

\[
(2.1) \quad s_t = p_t - p^*_t .
\]

Given the difficulties with the construction of an appropriate common basket of goods for implementing absolute PPP, a weaker version of PPP is often considered. This relative version of PPP is based on price movements, which are measured by changes in price indices relative to a base period. Relative PPP allows for a constant gap in absolute PPP described in equation (2.1),

\[
(2.2) \quad s_t = p_t - p^*_t - k ,
\]

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6 A review of recent developments of PPP theory is given in Section 3.1.

7 Another view of PPP is the efficient-markets view; see Section 3.4 for a brief review.
where $k$ is a constant. Note that the exponential of $k$, $e^k$, is the wedge between $S$ and $P/P^*$, also taken to be a constant. Let expression (2.2) hold for the base period, 0, so that $s_0 = p_0 - p_0^* - k$. We can then subtract this equation from (2.2) to yield

\begin{equation}
\Delta s_t = \Delta p_t - \Delta p_t^*,
\end{equation}

where $\Delta s_t = s_t - s_0$ is the change in the exchange rate, $\Delta p_t = p_t - p_0$ and $\Delta p_t^* = p_t^* - p_0^*$ are changes in price levels of the domestic and foreign country respectively, with the same base starting-point, i.e., $\Delta p_t$ and $\Delta p_t^*$ are inflation rates at home and abroad respectively. Expression (2.3) is the usual presentation of relative PPP in textbooks. It states that the change in the exchange rate should offset the inflation differential.\(^8\) To allow for stochastic deviations from relative PPP, we add a stationary error term to equation (2.2), $e_t$,

\begin{equation}
s_t = p_t - p_t^* - k + e_t. \tag{2.4}
\end{equation}

We draw on the conceptual framework of MacDonald and Stein (1999) to illustrate the above three versions of the traditional view of PPP. Figure 2.1 plots the nominal exchange rate, $s$, against the relative price, $r = p - p^*$. Panel A presents the case where $k = e = 0$, so that the 45° line passing through the origin corresponds to absolute PPP. Any combination of $s$ and $r$ that lies above the line implies an undervaluation of the home country currency, while points below represent

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\(^8\) Equations (2.2) and (2.3) are two presentations of relative PPP. Note that relative PPP expressed in (2.2) includes absolute PPP as a special case where $k = 0$. It can only be tested with price levels, whereas (2.3) can be tested directly with price indices, or price indices computed from price levels.

\(^9\) Maeso-Fernández (1998) points out a mistake that researchers frequently make when formulating stochastic deviations from relative PPP -- adding a stationary error term to (2.3). In such a case, the real exchange rate follows a random walk and relative PPP does not hold.
FIGURE 2.1
EXCHANGE RATES AND RELATIVE PRICES

A. Absolute PPP

B. Relative PPP

C. Stochastic deviations from relative PPP
overvaluation. Panel B allows $k \neq 0$ and $e = 0$, which is relative PPP. Here the 45° line does not pass through the origin, but an increase in the relative price still leads to an equi-proportional depreciation of the currency, as is illustrated by the movement from point A to B, whereby $s_2 - s_1 = r_2 - r_1$.

The central line in Panel C corresponds to relative PPP and is the centre-of-gravity relationship when there are stochastic shocks in the short run. Suppose for simplicity that $e$ is a discrete random variable and that $e_1 < 0$ and $e_2 > 0$ are its only possible values. When the shock is $e_1 < 0$, we obtain a new, lower 45-degree line, which has an intercept of $-k + e_1$; similarly, $e_2 > 0$ results in the upper line in Panel C. Consider the situation in which $s$ is the exchange rate and $r_1$ is the relative price, so that we are located at the point $W$ in Panel C. If there is now the same increase in the relative price as before, so that $r$ rises from $r_1$ to $r_2$, then, in the presence of the shock $e_1$, we move to the point $X$ with the rate depreciating to $s_0$. With the shock $e_2$, the same relative price $r_2$ leads to an exchange rate of $\bar{s}$, as indicated by the point $Y$. More generally, if relative prices change within the range $[r, r_2]$ and if the shocks can now vary continuously within the range $[e_1, e_2]$, then the exchange-rate/relative-price point lies somewhere in the shaded parallelogram $WXYZ$. It is to be noted that as the height of this parallelogram exceeds its base, the possible range of the exchange rate, $\bar{s} - s$, exceeds that of prices, $r_2 - r_1$. This “overshooting” accords with the idea that in the short run exchange rates are considerably more variable than relative prices. This contrasts with the situation in Panels A and B where the exchange rate is proportional to prices and illustrates the importance of stochastic shocks to the PPP relationship. More will be said about overshooting later in Section 2.7.

What determines the value of the equilibrium exchange rate $k$ in equation (2.4)? From equation (2.2), we know that the exponential of $k$ is $K = P/SP^*$, which is the ratio of the cost of a market basket at home to the cost of a foreign market basket, both in
terms of the domestic currency; in other words, $K$ is a relative price of an identical or a similar basket of goods. Many studies identify factors determining this relative price.\footnote{Some examples are De Grauwe and Grimaldi (2001) who examine the long-run relationship between exchange rates, prices and money; Alexius and Nilsson (2000), Marston (1986) and Strauss (2000) in which productivity differentials are a major determinant of real exchange rates; In and Menon (1996), who investigate the long-run relationship between the real exchange rate and the terms of trade; Engel and Rogers (2001), who find a significant role of price stickiness in influencing relative prices of disaggregated goods; and Mark and Choi (1997), who find that productivity differentials, real interest rate differentials, per capita income and monetary variables are all important in predicting real exchange rates.} Such factors include differential growth in sectoral productivities, the expansion of an exporting sector, real interest rate differentials between countries, pricing to market, changes in government spending, current account imbalances, preference shocks, etc. Before reviewing several of the more important factors, in the next section we, (i) present empirical evidence on the cross-country relationship between exchange rates and prices in 68 countries, and (ii) develop an analytical framework.

2.3. 

\textbf{Exchange Rates and Prices in 68 Countries}

This section provides some informal empirical evidence on the relationship between exchange rates and prices in a large number of countries over the post-Bretton Woods period. Similar analyses have been given in Lothian (1985) and Obstfeld (1995). Formal econometric modelling will be presented later in Chapters 4 and 5.

Relative PPP (2.3) shows that the change in the exchange rate from the previous to the current year $t$, $\Delta s_t$, equals the inflation differential $\Delta p_t - \Delta p_t^*$. Thus with $c$ indexing the country, if we plot $\Delta s_{ct}$ against $\Delta p_{ct} - \Delta p_t^*$, then the points should be scattered around a 45 degree line which passes through the origin. Panel A of Figure 2.2 gives such a plot of annual changes for $c = 1, ..., 68$ countries and $t = 1974 - 2000$ with the US as the base country. This figure contains $68 \times (2000 - 1974 + 1) = 1,836$ data points. It is obvious that many of the points are a considerable distance from the
FIGURE 2.2
CHANGES IN EXCHANGE RATES AND RELATIVE PRICES FOR 68 COUNTRIES, 1974-2000
(Annualised logarithmic changes)

A. Short run (all 68 countries, 27 years)

B. Short run (moderate-inflation observations)

C. Long run (68 countries)

Note: See Appendix 2.1 for computational details.
45° line. Panel B is a blow up of the “moderate” inflation observations and the conclusion remains that for annual changes, PPP leaves much to be desired. This could well be due to the prominent role of stochastic shocks hitting foreign-exchange markets in the short run, as was discussed in the previous section.

Next, as a way to eliminate much of the randomness in the data, we use the average of the annual changes over the entire 27 years, \( \Delta s_c^{(27)} = (1/27) \sum_{t=1974}^{2000} \Delta s_{ct} \), and similarly for the inflation differential. This average change is identical to the annualised change over the 27-year period, i.e., \( \Delta s_c^{(27)} = (1/27) (s_{c,2000} - s_{c,1973}) \). Panel C of Figure 2.2 gives the corresponding plot, with 68 points, one for each country. Visually, now there seems to be impressive support for PPP. As we can interpret the 27-year changes as reflecting the long run, when all adjustments are complete, it can be seen why it is that many authors have emphasised that PPP only applies over the longer term, not on a year-to-year basis.

To measure the dispersion of the points around the 45° line in a given panel of Figure 2.2, we use the root-mean-squared error (RMSE), which has the property that if all points lie on the 45° line, the RMSE is zero. As can be seen from Panel A, which corresponds to annual changes, the RMSE is as high as 10 percent. When high inflation observations are excluded, the RMSE reduces to about 7 percent (Panel B). The changes over the entire 27-year period (Panel C) gives the lowest RMSE of 1 percent. This reinforces the discussion in the previous paragraph that PPP holds better in the long run.

Next, we define the real exchange rate as \( q_c = \log(P_c / s_c P^*) \), where \( P_c \) is the price of a basket of goods and services in country \( c \) in terms of the domestic currency, \( P^* \) is the corresponding price in the US in $US, and \( S_c \) is the exchange rate, defined as the domestic currency cost of $US1. A real appreciation (depreciation) of the home country’s currency corresponds to an increase (decrease) in \( q_c \). In term of changes, we have \( \Delta q_c = (\Delta P_c - \Delta P^*) - \Delta s_c \), with \( \Delta q_c > 0 \) \((-<0\)) meaning a real appreciation.
(depreciation). Then, the annualised change in the real exchange rate over the 27-year period is \( \Delta q_{c}^{(27)} = (1/27)(q_{c,2000} - q_{c,1973}) = (\Delta p_{c} - \Delta p_{c}^{*})^{(27)} - \Delta s_{c}^{(27)} \); see Appendix 2.1 for details. A positive value of \( \Delta q_{c}^{(27)} \) means that the inflation differential exceeds the nominal depreciation of the exchange rate, which amounts to a real appreciation over the 27-year period. Accordingly, \( \Delta q_{c}^{(27)} > 0 \) corresponds to a point below the 45° line in Panel C of Figure 2.2. Figure 2.3 give the frequency distribution of \( \Delta q_{c}^{(27)} \) for the 68 countries. It can be seen that out of the 68 currencies, 53 depreciate over the 27-year period, while 15 appreciate. The cross-country mean of \( \Delta q_{c}^{(27)} \) is about -.5 percent p.a., which is not significantly different from zero. This indicates that on average for the 68 countries, changes in nominal exchange rates are of the same magnitude as those in inflation differentials. This finding is, of course, just another way of expressing the result in the previous paragraph that PPP holds in the long run.

**FIGURE 2.3**

**FREQUENCY DISTRIBUTION OF CHANGES IN REAL EXCHANGE RATES, 1974-2000**

(Annualised logarithmic changes)

![Histogram showing frequency distribution of changes in real exchange rates](image)
Thus far, we have compared exchange rates and prices over one- and 27-year horizons. Now consider an m-year horizon and define the annualised m-year changes as $\Delta s_{ct}^{(m)} = (1/m)(s_{ct} - s_{c,t-m})$ and $(\Delta p_{ct} - \Delta p_{t}^*)^{(m)} = (1/m)\left[(p_{ct} - p_{c,t-m}) - (p_{t}^* - p_{t-m}^*)\right]$. It then follows that if relative PPP holds, we have $\Delta s_{ct}^{(m)} = (\Delta p_{ct} - \Delta p_{t}^*)^{(m)}$ for any value of $m$, the length of the differencing span. If we average over countries and time to obtain $\text{var}[\Delta s^{(m)}]$ and $\text{var}[(\Delta p - \Delta p^*)^{(m)}]$, we have that $\text{var}[\Delta s^{(m)}] = \text{var}[(\Delta p - \Delta p^*)^{(m)}]$. We write the ratio of these two variances as $\phi(m) = \text{var}[\Delta s^{(m)}]/\text{var}[(\Delta p - \Delta p^*)^{(m)}]$; for brevity, we call this the “variance ratio”. Relative PPP implies that $\phi(m) = 1$ for any value of $m$. However, as can be seen from Panels A and B of Figure 2.2, the variance of exchange rates seems to be larger than that of relative prices with annual changes, which corresponds to a differencing span of $m = 1$. By contrast, in Panel C of the figure ($m = 27$) the two variances seems to be more or less the same. This indicates that in the short run $\phi(1) > 1$, while for the long run $\phi(27) \approx 1$. As we know the two “end points” for $\phi(m)$, it is natural to ask, what is the nature of the transition path in going from the short run to the long run? One way to investigate this is to analyse the variance ratio $\phi(m)$ for $1 < m < 27$. Figure 2.4 plots $\phi(m)$ against $m$. Here the variances underlying $\phi(m)$ are averages over the 68 countries and time; see Appendix 2.1 for computational details. As the value of $\phi(m)$ is in the vicinity of 2 for $m = 1$, on average, exchange rates are about twice as volatile as relative prices on a year-to-year basis. The ratio decreases considerably as $m$ increases, and gradually approaches one and wanders around that value after $m$ reaches 5 or 6 years. This result is consistent with the idea that exchange rates overshoot relative prices in the short run, while the two variables have more or less the same variability after 5 or 6 years. We will come back to this in Section 2.7.

11 This variance ratio is to be distinguished from the variance ratio proposed by Cochrane (1988), which is used to detect a unit root in a time series. According to Cochrane, if the ratio of the variance of the $k^{th}$ to the $1^{st}$ differences is greater than one, there exists a unit root.
To summarise, the behaviour of exchange rates and prices in a large number of countries shows that PPP holds better in the long run than on a year-to-year basis. We also found that in the short run exchange rates are more volatile than prices, so that overshooting is evident. As exchange rate movements in the short run cannot be solely explained by the evolution of prices, later sections of this chapter examine the impacts of non-monetary (or real) factors on exchange rates.

2.4. Exchange Rates, Money and Prices

In this section, we examine the PPP theory from the perspective of the quantity theory of money and present a geometric analysis of the relationships among exchange rates, money and prices.\textsuperscript{12}

\textsuperscript{12} This material mainly draws on the lecture notes of International Finance 415 by Ken Clements at The University of Western Australia. See also Clements (1981).
The quantity theory of money (QTM) is built on the equation of exchange, which shows the relationship between the money supply, velocity, prices and volume of transactions. The transactions version of the QTM is $MV = PT$, where $M$ is the supply of money balances, $V$ is the velocity or rate of circulation of these money balances, $P$ is the general level of prices of all transactions, and $T$ is the number of transactions. In many versions of the QTM, $V$ and $T$ are fixed, so that an increase in the supply of money will lead to a proportionate increase in the price level, i.e., $\dot{P} = \dot{M}$, where a circumflex ("\(^\wedge\)") denotes percentage change. Figure 2.5 shows this relationship and illustrates that a doubling of the money stock leads to a doubling of prices.

To relate the QTM to PPP, we decompose the goods that make up the general price level $P$ into traded and nontraded goods. We then have the price level function

\[ P = \phi M, \quad \text{where} \quad \phi = \frac{V}{T} = \text{constant} \]

---

13 The modern version of the quantity theory of money is due to Friedman (1956), who attributes much of what he presents to the “Chicago oral tradition”. Like many things associated with Friedman, this attribution is controversial; see Johnson (1971) and Patinkin (1969).
which is homogeneous of degree one. This function is plotted as the downward sloping convex curve AA in Figure 2.6 and is called the “absolute price schedule” along which the price level is a constant. The distance of the absolute price schedule from the origin measures the price level. Appendix 2.2 examines in some detail the properties of the absolute price schedule. Suppose that the relative price \( \alpha = \frac{P_T}{P_N} \) is constant, so that the two nominal prices must lie somewhere along the ray OP. For the economy to simultaneously satisfy monetary equilibrium and for the relative price to be \( \alpha \), the overall equilibrium must be located at the point \( E \) where the two curves intersect. A doubling of the money supply doubles the price level, moves the absolute price schedule from AA to A′A′, and with the relative price unchanged, the new equilibrium is at the point \( E' \). The homogeneity of the price level function \( P = P(P_T, P_N) \) means that the effect of doubling the quantity of money is to double both sectoral prices \( P_T \) and \( P_N \).

FIGURE 2.6
TRADED AND NONTRADED GOODS PRICES
Figure 2.7 combines the above two figures with the money stock added to the lower quadrant of Figure 2.6. PPP theory comes into play with the additional assumption that it holds for traded goods, i.e., $P_T = S P_T^*$, where $P_T^*$ is the foreign price of traded goods. The left quadrant of Figure 2.7 shows this relationship. As before, a doubling of the money stock doubles the general price level and the domestic price of traded goods also doubles. With the price of the foreign traded goods unchanged at $P_T^*$, the exchange rate depreciates by 100 percent, i.e., $S_1 = 2 S_0$, as can be seen from the left-hand panel of Figure 2.7.

FIGURE 2.7
EXCHANGE RATES, MONEY AND PRICES
What are the implications for the real exchange rate in Figure 2.7? Recall from Section 2.3 that the real exchange rate is defined as \( q = \log(P/SP^*) \). In Figure 2.7, the increase in the price level leads to a proportional increase in the nominal exchange rate. As these two effects cancel each other out, the monetary expansion leaves the real exchange rate unchanged under the assumption that the foreign price level \( P^* \) remains unchanged. Here, monetary shocks have no transitory or permanent effects on the real exchange rate. It is to be noted that this conclusion is more plausible in the long run, when prices tend to be more flexible; recall that Figure 2.2 revealed that only in the long run do exchange rate changes and relative prices move closely together. In Section 2.7, we shall extend this analysis to the situation where prices are sticky in the short run.

2.5. The Productivity Bias I: Theory

Section 2.4 reviewed the relationship between exchange rates, money and prices. An important case of the interaction between real and monetary phenomena in this area is the “productivity-bias” hypothesis of Balassa (1964) and Samuelson (1964). This section examines how it influences the nominal and real exchange rates.

Consider a three-country world, comprising a rich country, a poor one and the rest of the world. The term “productivity bias” refers to higher productivity in the production of traded goods in the rich country relative to that in the poor country. In this case, the price of traded goods in terms of nontraded goods is cheaper in the rich country in comparison to the poor country. If \( \alpha \) is this relative price (as before) and if \( R \) and \( P \) denote the rich and poor countries, then according to the productivity-bias hypothesis, \( \alpha^R < \alpha^P \).

Figure 2.8 explores the implications of this difference in the structure of relative prices for the nominal exchange rate. To isolate the impact of the different structure of
relative prices, we assume that the rich and poor countries both share the same absolute price schedule and face the same foreign prices. The left-hand panels of Figure 2.8 applies PPP to traded goods and they show that $S^R < S^P$, or that the currency of the rich country is worth more than that of the poor country. As both the rich and poor countries share the same absolute price schedule, they have the same price levels ($P^R = P^P$).
as they both face the same foreign price level $P^*$ and as the rich country’s exchange rate is lower ($S^R < S^P$), it then follows that $q^R - q^p = \log(P^R / S^R P^*) - \log(P^p / S^P P^*) = s^R - s^R > 0$. In words, in this case the difference in the real exchange rates is equal to the negative of its nominal counterpart, so that the nominal appreciation in moving from the poor to the rich country corresponds to an equi-proportional appreciation of the real exchange rate. Therefore, countries where traded goods are relatively expensive tend to have cheap currencies in both nominal and real terms, and vice versa.

The implications of the above analysis are as follows:

(i) Application of PPP to price levels indicates that the currencies of the two countries should have the same value.

(ii) PPP for traded goods only indicates that the currency of the rich country is worth more.

(iii) If currencies are in fact priced according to traded-goods PPP, whereas one values them according to price-level PPP, (i) and (ii) above jointly imply that the currency of the rich (poor) country is over (under) valued. This is the productivity-bias hypothesis.

Evidence on the extent of the productivity bias has also been provided by a number of studies using the Penn World Table. These have been summarised by Samuelson (1994, p. 201) as follows:

“…[Kravis et al., 1978, 1983] and Summers and Heston (1991) have documented in repeated Penn studies a fundamental economics fact. This K-H-S effect – or Penn Effect – states that a rich country, in comparison with a poor one, will be estimated to be richer than it really is if you pretend that the simplified Cassel version of purchasing-power parity (PPP) is correct and if you use crude exchange-rate conversions to deflate the nominal total per capita incomes of the two countries. The greater their per capita-income differentials truly are, the greater tends to be the resulting coefficient of bias.”
The economic rationale under the “Penn Effect” is well summarised in Kravis et al. (1978, p. 9):

“… the ratio of real GDP per capita to exchange-rate converted GDP per capita … falls as per capita GDP rises. This phenomenon can be explained in terms of what may be referred to as a ‘productivity-differential’ model, which has been offered at various times by Ricardo, Viner, Harrod, and Balassa. The model turns on the impact of differences in the productivity gap between high- and low-income countries for traded and nontraded goods. International trade tends to drive the prices of traded goods, mainly commodities (but occasionally services), towards equality in different countries. With equal or nearly equal prices, wages in traded goods industries in each country will depend on productivity. Wages established in the traded goods industries within each country will prevail in the country’s nontraded goods industries. In nontraded goods industries, however, international differences in productivity tend to be smaller.

Consequently, in a high-productivity country high wages lead to high prices of service and other nontraded goods; whereas in a low-productivity country low wages produce low prices. The lower a country’s income, the lower will be the prices of its home goods and the greater will be the tendency for exchange-rate conversions to underestimate its real income relative to that of richer countries.”

Finally, it is to be noted that the productivity bias is a non-monetary determinant of exchange rates. The systematic relationship between the relative price of traded goods and incomes is a factor that has to be considered in addition to the simple proportionality between exchange rates and the price level. As Samuelson (1994, p. 223, endnote 2) forcefully notes:

“The Penn effect is a clear violation of the crude form of the purchasing-power parity doctrine of Gustav Cassel (1916, 1918, 1983). It is a clear rebuttal of universal correctness of the Law of One Price for each and every good in a perfectly competitive, frictionless, geographical market. If transport costs and trade impediments were zero, and the exchange rate between the at Home and Abroad currencies were freely at $E$, then competitive arbitrage would enforce the one-price relation $E_j P_j^* / P_j = 1$ and $P_j^* / P_j \equiv P_k^* / P_k$, where $P_j$ is the Home price, $P_j^*$ the Abroad price. Under
autarky “1” becomes anything from 0 to infinity. With low transport costs, we can count on \(1 + \varepsilon\) or \(1 - \varepsilon\), where \(\varepsilon\) is a small positive number.

Cassel made more than one error, along with his basically correct insight that after a dramatic balanced inflation there would be some tendency for the \((P_j / P_k, P^*_j / P^*_k)\) ratios to settle back near the preinflation magnitudes. The Penn effect refutes the Cassel contention that a dollar spent everywhere (after being converted to the currency of that place) buys the same level of well-being (which is equivalent to denying that PPP enforces equal costs of living everywhere as a condition of exchange-rate equilibrium). The Penn effect documents that ‘living seems cheaper in poor countries than in rich.’ ‘Earn in rich countries and retire to spend it in poor’ makes some sense for frugal rentiers.”

The next section will provide some empirical evidence on the productivity-bias hypothesis.

2.6. The Productivity Bias II: Empirical Evidence

In this section, we demonstrate the “Penn effect” using data for 77 countries from the Penn World Table (PWT)\(^{14}\) and review briefly some empirical evidence on the productivity-bias hypothesis that has appeared in the literature.

Let \(\tilde{Y}_c\) be the GDP per capita for country \(c\) from the PWT, expressed in terms of US dollars, and \(Y'_c\) be the country’s GDP in terms of its domestic currency. The ratio \(Y'_c / \tilde{Y}_c\) is a measure of the PPP exchange rate according to the PWT. This PPP exchange rate is expressed in exactly the same way as the spot exchange rate \(S_c\), viz., the domestic currency cost of one US dollar. We express the PPP rate as a fraction of the

---

\(^{14}\) The Penn World Table is a large international database of national incomes and their compositions denominated in a common set of prices in a common currency. It is the product of the International Comparison Program of the United Nations which began in 1968. A description of several versions of the PWT is given in Summers and Heston (1991). For details, see http://pwt.econ.upenn.edu/.
prevailing exchange rate $S_c$, $(Y'_c/\bar{Y}_c)/S_c$, which we write as $D_c$, the exchange-rate discrepancy:

$$
D_c = \frac{Y_c}{\bar{Y}_c},
$$

where $Y_c = Y'_c/S_c$ is conventionally-defined GDP in US dollars. Equation (6.1) thus defines the discrepancy between the two exchange rates as the ratio of conventional GDP to the GDP according to the PWT. It is to be noted that $D_c$ has the dual interpretation as (i) the ratio of the PPP exchange rate (according to the PWT) to the prevailing rate; and (ii) the ratio of conventionally-computed GDP, expressed in US dollars and obtained using the prevailing exchange rate, to the PWT GDP. Columns 2 and 6 of Table 2.1 present $\bar{Y}_c$ for $c = 1, ..., 77$ countries in 1992, ranked in descending order. Columns 3 and 7 give the corresponding $Y_c$. The exchange-rate discrepancies $D_c$ are contained in columns 4 and 8. These discrepancies for rich countries are usually greater than one, while those for poor countries are less than one. The major exceptions to this rule are Hong Kong ($D_c = .8$), Australia (.9) and New Zealand (.8).

To further enhance understanding of the meaning of the exchange-rate discrepancy $D$, consider its value for Switzerland, 1.63. According to the dual interpretation given below equation (6.1), this value means that (i) the PPP exchange rate is 63 percent above the prevailing rate; and (ii) Switzerland’s GDP obtained using the prevailing rate is 63 percent greater than that obtained from the PWT. Regarding interpretation (i), in 1992 as one US dollar costs 1.41 Swiss francs, $S = 1.41$ for Switzerland. With $D = 1.63$, the PPP exchange rate is $1.63 \times 1.41 = SF 2.30$; that is, on the PPP basis, one US dollar costs SF2.30. If we interpret the PPP rate as the “true” exchange rate, we can conclude that the franc is overvalued by $100 \times (2.30-1.41)/1.41 = 63$ percent. Another example, is given by Sierra Leone (one of the poorest countries), where $D = .20$. As in 1992 for this country,
## Table 2.1
### Two Versions of GDP, 1992

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP per capita ($US) PWT</th>
<th>GDP per capita ($US) Conventional</th>
<th>Exchange-rate discrepancy</th>
<th>Country</th>
<th>GDP per capita ($US) PWT</th>
<th>GDP per capita ($US) Conventional</th>
<th>Exchange-rate discrepancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td></td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
</tr>
<tr>
<td>1. Switzerland</td>
<td>21,631</td>
<td>35,268</td>
<td>1.63</td>
<td>41. Algeria</td>
<td>3,076</td>
<td>1,823</td>
<td>.59</td>
</tr>
<tr>
<td>2. Luxembourg</td>
<td>21,144</td>
<td>33,151</td>
<td>1.57</td>
<td>42. Dominican Republic</td>
<td>2,918</td>
<td>1,205</td>
<td>.41</td>
</tr>
<tr>
<td>3. Hong Kong</td>
<td>21,034</td>
<td>17,322</td>
<td>.82</td>
<td>43. Guatemala</td>
<td>2,888</td>
<td>1,072</td>
<td>.37</td>
</tr>
<tr>
<td>4. Canada</td>
<td>20,970</td>
<td>21,052</td>
<td>1.00</td>
<td>44. Sri Lanka</td>
<td>2,783</td>
<td>557</td>
<td>.20</td>
</tr>
<tr>
<td>5. Germany, West</td>
<td>20,197</td>
<td>31,019</td>
<td>1.54</td>
<td>45. Morocco</td>
<td>2,777</td>
<td>1,086</td>
<td>.39</td>
</tr>
<tr>
<td>6. Japan</td>
<td>19,920</td>
<td>30,653</td>
<td>1.54</td>
<td>46. Paraguay</td>
<td>2,655</td>
<td>1,427</td>
<td>.54</td>
</tr>
<tr>
<td>7. Denmark</td>
<td>18,730</td>
<td>28,452</td>
<td>1.52</td>
<td>47. Peru</td>
<td>2,620</td>
<td>1,613</td>
<td>.62</td>
</tr>
<tr>
<td>8. Australia</td>
<td>18,500</td>
<td>17,475</td>
<td>.94</td>
<td>48. Indonesia</td>
<td>2,601</td>
<td>756</td>
<td>.29</td>
</tr>
<tr>
<td>9. Sweden</td>
<td>18,387</td>
<td>28,526</td>
<td>1.55</td>
<td>49. Congo</td>
<td>2,538</td>
<td>1,207</td>
<td>.48</td>
</tr>
<tr>
<td>10. France</td>
<td>18,232</td>
<td>23,482</td>
<td>1.29</td>
<td>50. El Salvador</td>
<td>2,274</td>
<td>1,090</td>
<td>.48</td>
</tr>
<tr>
<td>11. Belgium</td>
<td>18,091</td>
<td>22,643</td>
<td>1.25</td>
<td>51. Egypt</td>
<td>2,274</td>
<td>766</td>
<td>.34</td>
</tr>
<tr>
<td>12. Netherlands</td>
<td>17,373</td>
<td>21,216</td>
<td>1.22</td>
<td>52. Philippines</td>
<td>2,172</td>
<td>825</td>
<td>.38</td>
</tr>
<tr>
<td>13. Norway</td>
<td>17,094</td>
<td>29,467</td>
<td>1.72</td>
<td>53. Romania</td>
<td>2,130</td>
<td>1,083</td>
<td>.51</td>
</tr>
<tr>
<td>15. Singapore</td>
<td>16,736</td>
<td>17,418</td>
<td>1.04</td>
<td>55. Papua New Guinea</td>
<td>1,972</td>
<td>1,079</td>
<td>.55</td>
</tr>
<tr>
<td>16. Italy</td>
<td>16,724</td>
<td>21,308</td>
<td>1.27</td>
<td>56. Bangladesh</td>
<td>1,908</td>
<td>275</td>
<td>.14</td>
</tr>
<tr>
<td>17. Iceland</td>
<td>16,324</td>
<td>26,658</td>
<td>1.63</td>
<td>57. China</td>
<td>1,838</td>
<td>349</td>
<td>.19</td>
</tr>
<tr>
<td>18. U.K.</td>
<td>16,302</td>
<td>18,451</td>
<td>1.13</td>
<td>58. Pakistan</td>
<td>1,793</td>
<td>409</td>
<td>.23</td>
</tr>
<tr>
<td>19. Finland</td>
<td>15,619</td>
<td>21,540</td>
<td>1.38</td>
<td>59. Honduras</td>
<td>1,792</td>
<td>615</td>
<td>.34</td>
</tr>
<tr>
<td>20. New Zealand</td>
<td>15,502</td>
<td>11,726</td>
<td>.76</td>
<td>60. India</td>
<td>1,633</td>
<td>292</td>
<td>.18</td>
</tr>
<tr>
<td>21. Spain</td>
<td>12,986</td>
<td>14,768</td>
<td>1.14</td>
<td>61. Zimbabwe</td>
<td>1,479</td>
<td>650</td>
<td>.44</td>
</tr>
<tr>
<td>22. Israel</td>
<td>12,783</td>
<td>12,851</td>
<td>1.01</td>
<td>62. Kenya</td>
<td>1,176</td>
<td>320</td>
<td>.27</td>
</tr>
<tr>
<td>23. Ireland</td>
<td>12,259</td>
<td>15,125</td>
<td>1.23</td>
<td>63. Nigeria</td>
<td>1,132</td>
<td>351</td>
<td>.31</td>
</tr>
<tr>
<td>24. Cyprus</td>
<td>11,742</td>
<td>9,614</td>
<td>.82</td>
<td>64. Cameroon</td>
<td>1,122</td>
<td>918</td>
<td>.82</td>
</tr>
<tr>
<td>25. Venezuela</td>
<td>8,449</td>
<td>2,984</td>
<td>.35</td>
<td>65. Mauritania</td>
<td>1,083</td>
<td>534</td>
<td>.49</td>
</tr>
<tr>
<td>26. Mauritius</td>
<td>8,025</td>
<td>2,902</td>
<td>.36</td>
<td>66. Lesotho</td>
<td>1,027</td>
<td>445</td>
<td>.43</td>
</tr>
<tr>
<td>27. Malaysia</td>
<td>7,191</td>
<td>3,180</td>
<td>.44</td>
<td>67. Rwanda</td>
<td>961</td>
<td>275</td>
<td>.29</td>
</tr>
<tr>
<td>28. Uruguay</td>
<td>6,736</td>
<td>4,109</td>
<td>.61</td>
<td>68. Sierra Leone</td>
<td>914</td>
<td>184</td>
<td>.20</td>
</tr>
<tr>
<td>29. Chile</td>
<td>6,326</td>
<td>3,080</td>
<td>.49</td>
<td>69. Mozambique</td>
<td>898</td>
<td>126</td>
<td>.14</td>
</tr>
<tr>
<td>30. St.Kitts&amp;Nevis</td>
<td>6,057</td>
<td>4,329</td>
<td>.71</td>
<td>70. Madagascar</td>
<td>757</td>
<td>242</td>
<td>.32</td>
</tr>
<tr>
<td>32. Belize</td>
<td>5,739</td>
<td>2,435</td>
<td>.42</td>
<td>72. Togo</td>
<td>669</td>
<td>430</td>
<td>.64</td>
</tr>
<tr>
<td>33. Fiji</td>
<td>5,288</td>
<td>2,071</td>
<td>.39</td>
<td>73. Uganda</td>
<td>654</td>
<td>186</td>
<td>.28</td>
</tr>
<tr>
<td>34. Thailand</td>
<td>5,018</td>
<td>1,922</td>
<td>.38</td>
<td>74. Burkina Faso</td>
<td>651</td>
<td>322</td>
<td>.49</td>
</tr>
<tr>
<td>36. Colombia</td>
<td>4,254</td>
<td>1,474</td>
<td>.35</td>
<td>76. Malawi</td>
<td>607</td>
<td>199</td>
<td>.33</td>
</tr>
<tr>
<td>37. Iran</td>
<td>4,161</td>
<td>2,169</td>
<td>.52</td>
<td>77. Chad</td>
<td>504</td>
<td>279</td>
<td>.55</td>
</tr>
<tr>
<td>38. Panama</td>
<td>4,102</td>
<td>2,641</td>
<td>.64</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>39. Tunisia</td>
<td>3,807</td>
<td>1,842</td>
<td>.48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40. Namibia</td>
<td>3,231</td>
<td>1,923</td>
<td>.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Note: Data are available for 80 countries, but three countries are not included in the table: (1) the US, which has a GDP per capita of $23,220 and is the numeraire country; and (2) two African countries, which initial plots established to represent gross outliers.
S = 424, the PPP rate is \( \frac{.2 \times 424}{424} = 84.8 \) and the currency is undervalued by \( \left| 100 \times \frac{(84.8 - 424)}{424} \right| = 80 \) percent.

We plot \( Y_e \) against \( \tilde{Y}_e \) in Panel A of Figure 2.9. Along the 45° line from the origin, the PWT GDP is equal to the conventionally-computed GDP. As can be seen, the low-income countries are all located below the 45° line, while high-income counties are mostly above. This suggests that when incomes are converted into US dollars using prevailing exchange rates, poor countries’ incomes tend to be underestimated (in comparison with those obtained with the PPP exchange rates), and rich countries’ incomes tend to be overestimated. Accordingly, the “world” inequality of income is overstated by using prevailing exchange rates. This can be seen from the last row in the right panel of Table 2.1 which gives the standard deviation of the logarithms of income. The PWT gives a standard deviation of 116 percent, while according to the conventional method, the standard deviation of GDP per capita is 171 percent. In addition, according to the first interpretation of D given below equation (6.1), as the PPP exchange rates of poor countries tend to be much lower than the corresponding prevailing rates, the currencies of poor countries tend to be undervalued; and conversely, rich countries’ currencies are overvalued.

The upper-right corner of Panel A of Figure 2.9 reproduces the scatter plot, but on a smaller scale. Added to the scatter is a sketch of a curve that more or less passes through the centre of gravity. This curve is positively sloped with an increasing slope. One functional form that matches these properties is \( Y = a \tilde{Y}^b \), where \( a \) and \( b \) are constants, with \( a > 0 \) and \( b \) being the elasticity of \( Y \) with respect to \( \tilde{Y} \). The slope takes the form \( a \tilde{Y}^{b-1} \) and the change in the slope is \( a (b - 1) \tilde{Y}^{b-2} \); a positive slope requires that \( b > 0 \), while the second derivative is positive when \( b > 1 \). This argument establishes that the elasticity of \( Y \) with respect to \( \tilde{Y} \), \( b \), is greater than one. Another way of seeing that this elasticity is bigger than one is to note that this requires the
FIGURE 2.9
THE PENN EFFECT, 1992

A. Conventional and PWT GDP

B. Exchange-rate discrepancy and PWT GDP
marginal effect to be greater than the corresponding average; and for any point on the curve, it can be seen that the slope of the curve (the marginal effect) is greater than the slope of the corresponding ray from the origin (the average).\footnote{Using $Y = a \tilde{Y}^b$, the marginal effect is $a b \tilde{Y}^{b-1} = b \frac{Y}{\tilde{Y}}$, while the slope of the ray from the origin is $\frac{Y}{\tilde{Y}}$. The ratio of the former to the latter is $b$, which we established is greater than one.}

Next, we analyse the relationship between the exchange-rate discrepancy and GDP. Similar analyses have been carried out in Balassa (1964) and Kravis et al. (1982, pp. 9-11). In Panel B of Figure 2.9, we plot the logarithmic values of exchange-rate discrepancy $D_c$ against those of the PWT GDP, $\tilde{Y}_c$. It can be seen that there is a significant positive relationship between the logarithm of these two variables. Such a relationship can be expressed in the log form as

\begin{equation}
\log D = \alpha + \beta \log \tilde{Y} + \varepsilon,
\end{equation}

where $\alpha$ and $\beta$ are coefficients and $\varepsilon$ is a disturbance term. The least-squares estimates are

\begin{equation}
(6.3) \quad \hat{\alpha} = -4.06 (.34), \quad \hat{\beta} = .42 (.04),
\end{equation}

where the figures in parentheses are standard errors. Thus the elasticity of $D$ with respect to $\tilde{Y}$ is .42. This indicates that if the we go from a country to another which is 50 percent more affluent according to the PWT, the exchange-rate discrepancy increases by 21 percent. This is consistent with Samuelson’s (1994, p. 201) assessment that “the greater [the per-capita income differentials of two countries] truly are, the greater tends to be the resulting coefficient of bias”.

Recall from Table 2.1 that $D_c$ for rich countries are usually greater than one, while those for poor countries less than one. It is thus natural to ask, what level of
income divides these two groups of countries, rich and poor? That is, at what value of income does \( Y = \bar{Y} \) hold, so that the PPP exchange rate is equal to the prevailing rate? As poor (rich) countries have \( D < 1 \ (>1) \), the income level which divides countries into two groups, \( \bar{Y}^* \), is that associated with \( D = 1 \) (i.e., \( \log D = 0 \)). Thus it follows from equation (6.2) that \( \alpha + \beta \log \bar{Y}^* = 0 \), or

(6.4) \[
\bar{Y}^* = \exp\left(-\frac{\alpha}{\beta}\right).
\]

A simple way of evaluating this is to use \( \hat{\alpha} = -4.06 \) and \( \hat{\beta} = .42 \) from equation (6.3), so that \( \bar{Y}^* = \exp(4.06/.42) = \$US16,260 \) in 1992. This dividing level of income can be thought of as the “international poverty line” and is indicated in both Panels A and B of Figure 2.9. As can be seen from Figure 2.9, a majority of countries are classified as “poor” on this basis. Table 2.1 shows that this poverty line lies between UK and Finland, and seems to be high compared to the mean over the 77 countries, \$US7,300 (see the third to last row on the right panel). Some qualifications to the international poverty line are in order, of course. First, poverty is defined in an unconventional way, viz., on the basis of the relationship between the PPP exchange rate and the prevailing rate. This can be described as a “revealed preference” approach that uses the productivity-bias hypothesis as a maintained hypothesis. A further qualification is that as \( \bar{Y}^* \) is defined by equation (6.4) as the ratio of two parameters and it is well known that when least-squares estimates are used to evaluate such ratios, there can be econometric problems due to certain moments not existing.\(^{16}\) If one were to feel that the use of the term “international poverty line” is too emotive, the more neutral (if less colourful) term “standard income” could be used to describe \( \bar{Y}^* \).

\(^{16}\) Under normality of the disturbance in the estimating equation, the ratio of two least-squares estimates is not normally distributed and does not have finite moments; see Bewley and Fiebig (1990) and Zellner (1978) for details. To overcome these problems, Efron’s (1979) bootstrap simulation procedure can be used. Using the nonparametric version of this procedure with 1,000 trials, we obtain the mean value of \( \bar{Y}^* \) of \$16,539, with a root-mean-square error (RMSE) of \$3,040. The mean value is not too far away from the point estimate of \$16,260 reported in the text. But the RMSE is sizeable, which serves as a warning that the international poverty line is not estimated too precisely. See Appendix 2.3 for details of the simulation.
To further examine the relationship between the conventionally-computed GDP, \( Y \), and the PWT GDP, \( \tilde{Y} \), we substitute (6.1) into the right-hand side of (6.2) to obtain
\[
\log(Y/\tilde{Y}) = \alpha + \beta \log \tilde{Y} + \varepsilon.
\]
By collecting terms, we then have
\begin{equation}
(6.5) \quad \log Y = \alpha + (1 + \beta) \log \tilde{Y} + \varepsilon.
\end{equation}
If \( \sigma_x \) is the standard deviation of \( \log x \), equation (6.5) then implies that
\begin{equation}
(6.6) \quad \sigma_Y^2 = (1 + \beta)^2 \sigma_{\tilde{Y}}^2 + \sigma_{\varepsilon}^2.
\end{equation}
From equation (6.2), we obtain the least-squares estimate of \( \sigma_{\varepsilon}^2 \), \( \hat{\sigma}_{\varepsilon}^2 \), to be .17. As \( \hat{\beta} = .42 \), \( \sigma_{\varepsilon} = 1.16 \) (from the last row of Table 2.1), we obtain from equation (6.6) that \( \sigma_Y = 1.70 \). This figure agrees rather well with the observed dispersion of \( Y \) of 1.71; see the last row of Table 2.1. The ratio of the standard deviation of conventionally-computed GDP to that of the PWT GDP is \( 1.71/1.16 \approx 1.47 \). Note that if \( \sigma_{\varepsilon}^2 \ll \sigma_{\varepsilon}^2 \), it follows from equation (6.6) that \( \sigma_Y/\sigma_{\varepsilon} \approx 1 + \beta \). As this condition is easily satisfied above \( (\sigma_{\varepsilon}^2 = .17, \sigma_Y^2 = 1.16^2) \), this explains why the above ratio of the dispersion of \( Y \) and \( \tilde{Y} \) is not too different to the estimated value of \( 1 + \beta \), viz., 1.42.\(^{17}\) This illustrates a type of “magnification effect” of using prevailing exchange rates to convert incomes into a common currency -- using prevailing exchange rates has the effect of magnifying the cross-country inequality of income.

Conceptually, the prevailing exchange rate is determined by the price of traded goods, \( P_T \), while the PPP exchange rate reflects the prices of both traded and nontraded goods, \( P_T \) and \( P_N \). Suppose that (i) the PPP exchange rate is proportional to a weighted geometric mean of \( P_T \) and \( P_N \), \( S_{\text{PPP}} = \eta_l P_T^\gamma P_N^{1-\gamma} \), where \( \gamma \) is the weight assigned to traded goods and \( \eta_l \) is a positive constant; and (ii) the prevailing exchange rate is

\(^{17}\) Another way to express this result is to note that (i) from equation (6.6), \( \sigma_Y \geq \sqrt{\sigma_{\varepsilon}^2 - \sigma_{\varepsilon}^2} = (1 + \beta) \sigma_{\varepsilon} \); and (ii) the previous inequality becomes closer to an equality as \( \sigma_{\varepsilon}^2 \) falls relative to \( \sigma_{\varepsilon}^2 \).
proportional to \( P_T \): \( S = \eta_2 P_T \), where \( \eta_2 \) is a positive constant. Thus the exchange-rate discrepancy is \( D = \frac{S_{ppp}}{S} = \eta_1 P_T^\gamma \frac{P_{N}^{1-\gamma}}{\eta_2 P_T} \), or

\[
D = \eta \left( \frac{P_N}{P_T} \right)^{(1-\gamma)}
\]

where \( \eta = \frac{\eta_1}{\eta_2} \) is also a positive constant. Taking logs of the above equation, we have

\[
(6.7) \quad \log D = \log \eta + (1-\gamma) \log \frac{P_N}{P_T}.
\]

Therefore, in this case, \( \log D \) is linearly increasing in \( \log (P_N / P_T) \), the relative price of nontraded to traded goods. Accordingly, as Panel B of Figure 2.9 involves the logarithm of \( D \) on the vertical axis, we can reinterpret this graph as giving the relationship between \( \log (P_N / P_T) \) and \( \log (\bar{Y}) \), except for a positive multiplicative constant which acts merely as a scaling factor. Under this interpretation, the graph shows that nontraded goods tend to be relatively cheaper in poor countries, while they are more expensive in rich countries.

Equation (6.2) gives the relationship between the discrepancy \( D \) and the PWT GDP, \( \bar{Y} \). Combining equations (6.2) and (6.5), we can express \( D \) in terms of \( Y \) as

\[
(6.8) \quad \log D = \alpha' + \beta' \log Y + \varepsilon',
\]

where \( \alpha' = \alpha(1 - \beta') \) is a new intercept; \( \beta' = \beta/(1 + \beta) \) is a new slope coefficient; and \( \varepsilon' = \varepsilon(1 - \beta') \) is a new disturbance term. The above equation shows that the elasticity of \( D \) with respect to \( Y \) is \( \beta' = \beta/(1 + \beta) \), which, using \( \hat{\beta} = .42 \), becomes \( \hat{\beta}' = .42/(1 + .42) = .30 \). Balassa (1964) regresses \( D \) (expressed as a percentage) on
GNP per capita in US dollars and obtains the following equation: \( D = 49.34 + 0.025Y \). At sample means (denoted by a bar), the implied elasticity of \( D \) with respect to \( Y \) is 
\[ [dD/dY] \times (\bar{Y}/\bar{D}) = 0.025 \times 1200/80 = 0.38, \]
which is not too different from our estimate of 0.30.

In another paper along the similar lines, Clements and Semudram (1983) examine the cross-country relationship between the price of haircuts and GDP per capita. The price in domestic-currency terms is \( P_H \), which is then converted to US dollars using the prevailing exchange rate, to yield \( P_H/S \). There are two interpretations of this price \( P_H/S \):

- It may be considered as being proportional to the exchange-rate discrepancy \( D \) on the basis that \( P_H \) has both traded and nontraded components and could be expressed as being proportional to \( P_T^\gamma P_N^{1-\gamma} \), which is the same as \( S_{ppp} \) defined above equation (6.7). Accordingly under this interpretation, \( DS/SS/P PPPH \phi = \phi = \phi \), where \( \phi \) is a new factor of proportionality which drops out then we compute the income elasticity of \( P_H/S \).

- As haircuts are highly labour intensive, they themselves can be considered to be a nontraded good *par excellant*. In other words, the share of traded goods in the “production” of haircuts is very low, approaching zero. Under this interpretation, the price \( P_H/S \) then becomes \( P_N/P_T \).

Clements and Semudram find that the elasticity of \( P_H/S \) with respect to \( Y \) is 0.26. This result can be compared with ours in two ways:

- When the relative price of haircuts is interpreted as being proportional to the exchange-rate discrepancy. In this case, Clements and Semudram’s elasticity of 0.26 is to be compared with \( \beta' = \beta/(1+\beta) \) in equation (6.8), which equals 0.30 for \( \hat{\beta} = 0.42 \). Accordingly, the two estimates of this elasticity are quite close.

- When the price of haircuts is interpreted as being equal to the relative price of nontraded goods, \( P_N/P_T \). From equations (6.7) and (6.8), the elasticity of \( P_N/P_T \) with respect to \( Y \) is
\[ \lambda \equiv \frac{d[\log(P_N/P_T)]}{d(\log D)} \times \frac{d(\log D)}{d(\log Y)} = \frac{1}{1-\gamma} \beta' = \frac{.30}{1-\gamma} \]

for \( \hat{\beta} = .42 \). As the weight \( \gamma \) is a positive fraction, the above elasticity exceeds .3. But if we interpret \( \gamma \) as the cost share of traded goods in the production of haircuts, then \( \gamma \) is small and the above elasticity \( \lambda \) would not be too much above .30. This bound on the value of \( \lambda \) is not too different from the Clements and Semudram’s estimate of this elasticity of .26.

Other recent empirical evidence has shown considerable support for the productivity-bias hypothesis; see, e.g., DeLoach (2001), Habermeier and Mesquita (1999), Heston et al. (1994) and Ito et al. (1997). But all is not completely uncontroversial. Direct negative evidence of the Balassa-Samuelson theory is found in Bahmani-Oskooee and Niroomand (1996) and Duval (2001). One of the key assumptions in the Balassa-Samuelson model, PPP in traded goods, is questioned in Canzoneri et al. (1999) and Engel (1999). Pricing to market (PTM), originally developed by Dornbusch (1987b) and Krugman (1987), is offered as one explanation for the failure of PPP with respect to traded goods. PTM posits that monopolistically competitive firms choose to set different prices between segmented national markets, so that nominal exchange rate movements will not necessarily be fully reflected in prices of individually traded goods. The low pass-through from exchange rates to prices and resultant failure of PPP for traded goods has been an important strand of the literature on exchange-rate economics.18

2.7. Overshooting

Recall from Section 2.4 that a change in the money stock leaves the real exchange rate unchanged. As noted at the end of that section, such an analysis assumes that all prices are flexible in that they vary instantaneously with changes in the money stock.

---

18 See Goldberg and Knetter (1997) for a comprehensive review of the empirical literature on pricing to market.
Accordingly, that analysis should be interpreted as reflecting the long run when prices are likely to be more fully flexible. However empirically, sticky prices are important in the short run, especially with respect to services. The impact of sticky prices can be illustrated using the classification of traded and nontraded goods; typically, the prices of traded goods are more flexible as they tend to relate more to commodities traded in organised markets, whereas the prices of nontraded goods are sticky in the short run (due to contracts, menu costs, etc.).

Consider again the effects of an increase in the money supply. As before, immediately after this increase, the absolute price schedule moves outwards, from AA to A'A' in the right-hand panel of Figure 2.10. Suppose that in the short run, the price of nontraded goods is fixed, which is the extreme case of sticky prices. This means that the price of traded goods has to “do all the adjusting” and this price must increase (in proportional terms) by more than the monetary expansion and the overall price level.
This is illustrated in Figure 2.10 as the price of traded goods increases from $P_T^0$ to $P_T^1$, while the price of nontraded goods remains unchanged at $P_N^0$. Accordingly, the relative price schedule moves anti-clockwise, the relative price of traded goods rises from $\alpha_0$ to $\alpha_1$, and the overall equilibrium moves from the point $E_0$ to $E_1$. With the increase in the price of traded goods from $P_T^0$ to $P_T^1$, the nominal exchange rate thus increases from $S_0$ to $S_1$, as shown in the left-hand panel of Figure 2.10. However, at the point $E_1$, as the relative price of traded goods has increased above its long-run equilibrium value $\alpha_0$, there is an excess demand for nontraded goods. Accordingly, the economy cannot remain at $E_1$ and over time, this excess supply reduces the relative price of traded goods back down to its original level $\alpha_0$. Thus the long-run equilibrium is at the point $E_2$, the intersection of the new absolute schedule $A'A'$, and the original relative price schedule, OR. The movement of the economy from $E_1$ to $E_2$ corresponds to the exchange rate appreciating from $S_1$ to $S_2$, as shown in the left-hand panel of Figure 2.10. To summarise, the monetary expansion causes the exchange rate to initially depreciate from $S_0$ to $S_1$, but over time part of this initial depreciation is offset by a subsequent appreciation to $S_2$. The long-run effect of the monetary expansion is to depreciate the rate as $S_2 > S_0$. Panel A of Figure 2.11 illustrates this time path. Due to sticky prices, the exchange rate initially depreciates by too much, so that it has to appreciate subsequently. This behaviour is consistent with the observed volatility of exchange rates and is known as “overshooting”. This terminology was introduced by Dornbusch (1976) who explained it in a rational-expectations framework in which asset markets clear continuously, while goods prices are sticky. It should be clear that the above rendition of overshooting, which is due to Clements (1981), is a little different to Dornbusch’s, but the result is the same.

The overshooting of the nominal rate can also be understood from the following analytical perspective. Appendix 2.2 shows that the price level can be written in terms of proportional changes as $\hat{P} = \gamma \hat{P}_T + (1 - \gamma) \hat{P}_N$, where the elasticity $0 < \gamma < 1$. Thus if

56
\[ \dot{P}_N = 0 \] (extreme sticky prices), then \[ \dot{P} = \gamma P_T, \] or \[ \dot{P}_T = (1/\gamma) \dot{P} > \dot{P} \] and the price level of traded goods increases more than the price level. That leads to overshooting.

How does the real exchange rate move in the case of nominal overshooting? We consider this question in two steps. First, nothing happens to the real exchange rate in the long run, as the move from \( E_0 \) to \( E_2 \) in Figure 2.10 simply involves a nominal

**FIGURE 2.11**

EXCHANGE-RATE OVERSHOOTING

A. The nominal rate

<table>
<thead>
<tr>
<th>Time</th>
<th>Depreciates by “too much” initially</th>
<th>Subsequent appreciation</th>
<th>Long run</th>
</tr>
</thead>
<tbody>
<tr>
<td>s₁</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>s₂</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>s₀</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B. The real rate

<table>
<thead>
<tr>
<th>Time</th>
<th>Depreciates in the short run</th>
<th>Subsequent appreciation</th>
<th>Long run</th>
</tr>
</thead>
<tbody>
<tr>
<td>q₀</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q₁</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q₂</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
depreciation equal to the increase in the price level. This is exactly like the situation in Figure 2.7. Therefore, the real exchange rate remains unchanged in the long run, so that if $q_0$ and $q_2$ are the initial and subsequent long-run values of the real exchange rate, then $q_2 = q_0$, as shown in Panel B of Figure 2.11. Second, consider the behaviour of the real exchange rate in the short-run when the equilibrium point moves from $E_0$ to $E_1$ in Figure 2.10. As the nominal rate now depreciates by more than the increase in the price level, this involves a real depreciation, so that the new value of the real rate is $q_1 < q_0$, as illustrated in Panel B of Figure 2.11. From the previous paragraph, we know that in the short run, the increase in the price level is magnified onto the price of traded goods, $\hat{P}_T = (1/\gamma) \hat{P}$, and application of PPP to traded goods implies that the nominal rate moves proportionally with the price of traded goods, $\hat{S} = \hat{P}_T = (1/\gamma) \hat{P}$. The change in the real rate is then $q_1 - q_0 = \hat{P} - \hat{S} - \hat{P}^* = [1 - (1/\gamma)] \hat{P} < 0$, as $(1/\gamma) > 1$. In words, the real rate also depreciates in the short run. All of this real depreciation is offset by a subsequent appreciation in the transition as the economy moves from the point $E_1$ to the steady-state equilibrium point $E_2$, as shown in Panel B of Figure 2.11.

As summarised in Rogoff (2001), the empirical evidence of overshooting comes from two streams:

1. Testing for a positive correlation between the real exchange rate and the real interest differential. The simple overshooting model predicts that an increase in the nominal money supply leads to an increase in the real money supply in the short run (due to sticky prices). It lowers the real interest rate, which depreciates the real exchange rate in the short run, thus leading to the positive correlation. The classic paper in this area in favour of overshooting is Frankel (1979); see Issac and de Mel (2001) for an update and a critique of this stream of work.

2. Examination of the co-movement of forward and spot exchange rates. If there is a once-off unanticipated change in the level of the money supply, the spot exchange rate should move by more than the forward rate regardless of the horizon. Such excess movement is evidence of overshooting. Flood (1981) pioneers such a methodology, but he finds that the tendency is for spot and forward rates to move in tandem, which is inconsistent with overshooting.
There also exists other evidence against overshooting (see, e.g., Wilkinson et al., 2001). One possible reason suggested in the literature for evidence against overshooting is the use of a monetary policy feedback rule. In order to stabilise the nominal or real exchange rates, interest rates or output, a central bank may intervene in response to changes in the price level, thus generating exchange-rate dynamics different to those predicted by the Dornbusch framework (Kempa and Nelles, 1999). It is to be noted that even though the empirical evidence of the overshooting hypothesis is thin (Obstfeld and Rogoff, 1995, p. 644, Eichenbaum and Evans, 1995), the informal evidence presented in Section 2.3 seemed to offer some support. Figure 2.2, which plotted changes in exchange rates and relative prices, showed that the two changes are more or less equal in the long run (see Panel C). But as evidenced by Panels A and B of the figure, this is certainly not the case on a year-to-year basis. In addition, Figure 2.4, which plotted the ratio of the variance of exchange rates to that of relative prices against the length of horizon, illustrated that in comparison to relative prices, exchange rates in the short run are more volatile; but over the longer term, the volatility of the two variables seems to be of the same order of magnitude. This is also indicative of overshooting.

2.8. A Booming Sector

Another situation that may exert influence on the value of the real exchange rate is an expansion of some export sector. To analyse the effects of such a booming sector, we assume that the whole economy is divided into three broad sectors: Exportables, importables and nontraded goods, whose nominal prices are denoted by $P_E$, $P_M$ and $P_N$, respectively. Figure 2.12, from Dornbusch (1974), plots the relative price of importables in terms of nontraded goods, $P_M/P_N$, against that of exportables, $P_E/P_N$. Equilibrium in the market for nontraded goods requires that these two relative prices lie somewhere on the downward-sloping schedule labeled $NN$. To see this, take an arbitrary point on $NN$ such as $X$ and move in a due easterly direction...
FIGURE 2.12
THE EFFECTS OF A BOOMING EXPORT SECTOR

As this move entails a decrease in the price of nontraded goods relative to exportables, and as at X the market for nontraded goods initially cleared, at Y there is an excess demand for nontraded goods. This excess demand can be eliminated through increasing the relative price of nontraded goods in terms of importables by moving in a southerly direction from Y to Z. At a point such as Z, the market for nontraded goods clears again, which establishes that NN must be downward sloping. We assume that the country is small so that it faces given world prices of the two traded goods, \( P_M \) and \( P_E \). The terms of trade are \( P_M / P_E = p_M / p_E \), which for a fixed value, \( (p_M / p_E)_0 \), are given by the slope of the ray from the origin in Figure 2.12 labelled OD. The clearing of both the markets for traded and nontraded goods occurs at the point \( E_0 \), where OD and NN intersect.

In the previous paragraph, we established that points above the NN schedule represent excess demand for nontraded goods. But by Walras’ Law, these points also represent excess supply of traded goods, which implies a surplus in the country’s trade account. This also implies that those points lying below the NN schedule represent a
trade-account deficit. Thus if the economy now experiences a boom in an export sector leading to an exogenous increase in those exports, points that previously constituted a trade-account deficit are now consistent with a zero trade balance, or equilibrium in the market for traded goods. All this means that the expansion of an export sector causes \( NN \) to move inward toward the origin, to \( N'N' \). This causes the overall equilibrium to shift from \( E_0 \) to \( E_1 \), with the result that \( p_M \) and \( p_E \) both fall. Due to the decrease in prices of importables and exportables that are not part of the expanding sector, the profitability of firms in these sectors is squeezed. These effects are known as the “Gregory Thesis” (Gregory, 1976) in Australia and the “Dutch Disease” in Europe and elsewhere.\(^{19}\)

Next, consider the problem of aggregating exportables and importables into “traded goods”. If the shares of importables and exportables in traded goods are \( \beta \) and \( 1-\beta \), respectively, we can define the price of traded goods (relative to nontraded goods) as a weighted average of its two components:

\[
p_T = \beta p_M + (1 - \beta) p_E.
\]

Figure 2.13 gives a geometric presentation of this function and analyses how the price of traded goods changes as a result of a booming sector. Quadrant I of this figure reproduces Figure 2.12. Let us first look at the initial equilibrium point \( E_0 \). It corresponds to an importables price of \( p_{M_0} \) and an exportables price of \( p_{E_0} \). In Quadrant II, we draw a line from the origin \( OF \) which has a slope of \( \beta \) with respect to the vertical axis. Then the distance \( OA_0 \) on the left horizontal axis is \( \beta \) times the initial price of importables \( p_{M_0} \), i.e., it is the first component of the traded goods price, \( \beta p_{M_0} \). Quadrant III simply transfers this importables component to the lower vertical axis \( OP_T \) through a 45° line, so that the distance \( OB_0 \) is of the same length as \( OA_0 \). In quadrant IV, treating \( B_0 \) as the origin, we add a line \( B_0G \) which has a slope of \( (1-\beta) \) with respect to the horizontal axis. Thus the distance \( B_0C_0 \) on the lower vertical axis \( OP_T \) is \( (1-\beta) \) times the

FIGURE 2.13
MORE ON THE EFFECTS OF A BOOMING EXPORT SECTOR

exportable goods price \( p_E^0 \), i.e., it is the second component of the traded goods price \((1 - \beta) p_E^0\). As \( OC_0 = OB_0 + B_0C_0 \), \( OC_0 \) is the weighted average of the importables and exportables prices; i.e., the distance \( OC_0 \) represents the traded goods price \( p_T^0 \) corresponding to the initial equilibrium \( E_0 \). Recall that when there is a booming sector, the home goods schedule \( NN \) shifts down and towards the origin to \( NN' \), and the new equilibrium point is \( E_1 \). Using the same geometric analysis, we obtain the distance \( OC_1 \) on the lower vertical axis \( OP_T \) as the traded goods price \( p_T^1 \) corresponding to the new
equilibrium point $E_1$. It can be seen from the figure that $p^*_T < p^*_S$, meaning that the price of traded goods (relative to nontraded goods) decreases as a result of a booming sector.

How does a booming sector influence the real exchange rate? When an export sector is expanding, the additional activities in this sector cause the nominal rate to appreciate and/or the price of home goods to rise; these cause the real exchange rate to appreciate. This can be seen by noting that the export boom causes the relative price of traded goods to fall. Figure 2.8 in Section 2.5 can be used to show that lower relative price of traded goods corresponds to the appreciated nominal rate. As the expanding export sector has no effect on the overall price level, the nominal appreciation leads to an equi-proportional real appreciation, i.e., $q_1 - q_0 = \log(P/S_T^*) - \log(P/S_S^*) = s_0 - s_1 > 0$. The appreciation of the real exchange rate resulting from a booming sector is shown in Figure 2.14. These results can also be derived algebraically as follows: As in Section 2.7, the change in the price level is a weighted average of the change in the prices of traded and nontraded goods, $\hat{P} = \gamma \hat{P}_T + (1 - \gamma) \hat{P}_N$, which can be rewritten as $\hat{P} = \hat{P}_T - (1 - \gamma)(\hat{P}_T - \hat{P}_N)$. Recall that the relative price of traded goods

---

**FIGURE 2.14**

THE REAL EXCHANGE RATE AND A BOOMING SECTOR

Logarithm of real exchange rate

$q_1$

$q_0$

Appearance of a booming sector

Long run

Time
is $\alpha = P_T / P_N$, or in terms of proportional changes, $\hat{\alpha} = \hat{P}_T - \hat{P}_N$. As it is assumed that the overall price level does not change, $\hat{P} = 0$, it follows that $\hat{P}_T = (1-\gamma) \hat{\alpha}$. The application of PPP to traded goods implies that the nominal rate moves proportionally, $\hat{S} = \hat{P}_T = (1-\gamma) \hat{\alpha}$. Thus, given that $(1-\gamma)$ is a positive fraction, the decrease in the relative price of traded goods ($\hat{\alpha} < 0$) caused by the export boom leads to the appreciation of the nominal rate ($\hat{S} < 0$). The change in the real rate is $q_1 - q_0 = \hat{P} - \hat{S} - \hat{P}^* = -\hat{S} = (1-\gamma) \hat{\alpha} > 0$, indicating a real appreciation.

Empirically, the experience with export booms in many countries is consistent with the above theory. The supporting evidence comes from studies of both developed and developing countries. Some examples are:

1. The natural gas boom in the Netherlands in the 1960s (which inspired the name “Dutch Disease”). This caused an appreciation of the real exchange rate and a subsequent squeeze of other export industries (Kremers, 1986).

2. The Australian gold rushes in the 1850s. Cairnes (1859) found wages doubled nationwide as a result. The extension of the agricultural industry and every industry other than gold mining “suffered a real check” (p. 59). On the other hand, the nontradables or service industries generally benefited from the gold discoveries (Doran, 1984).

3. The oil boom in Saudi Arabia from the mid-1970s to the early 1980s. This led to an appreciation of the real exchange rate throughout the period, a contraction of the non-oil traded goods sector, and an expansion of the nontraded goods sector (Al-Mabrouk, 1991).

The undesirable consequences of a booming sector are usually associated with the “resource curse thesis”, which holds that resource-abundant countries tend to perform more poorly than resource-scarce countries. However, it is argued that an export boom is only a partial explanation of the relatively slow growth of resource-abundant countries, and many other factors also play a role, such as characteristics of a country and its policy response to a resource boom (Davis, 1995, Mikesell, 1997, and Neary and van Wijnbergen, 1986). Governments, private companies and civil society each play important but different roles in helping avoid the problems caused by booming sectors,
though well-designed government policies are critical (Eggert, 2001). Usui (1997) gives a comparison of Mexico and Indonesia in response to their respective oil booms. Indonesia avoided the Dutch disease by accumulating budget surpluses and devaluing its exchange rate. Mexico devaluated the peso immediately after the oil boom, but its subsequent adoption of an expansionary policy appreciated exchange rates and encouraged substantial foreign borrowing, which eventually triggered a capital flight. Therefore, the Dutch disease literature concludes that an export boom is a mixed blessing to resource-abundant countries and appropriate policy adjustments are crucial to ensure a cure for the ‘disease’.

2.9. An Algebraic Formulation

In this section we draw together the material in previous sections by way of an algebraic presentation of the results. We are concerned with the relationships among the overall price level at home and abroad \((P \text{ and } P^*)\), traded and nontraded goods prices at home \((P_T \text{ and } P_N)\) and abroad \((P_T^* \text{ and } P_N^*)\), and the nominal and real exchange rates \((S \text{ and } R)\). The following five equations summarise these relationships in terms of proportional changes:

\[
\begin{align*}
\text{The price level at home:} & \quad \hat{P} = \gamma \hat{P}_T + (1-\gamma)\hat{P}_N \\
\text{The price level abroad:} & \quad \hat{P}^* = \gamma \hat{P}_T^* + (1-\gamma)\hat{P}_N^* \\
\text{The relative price of traded goods:} & \quad \hat{P}_T - \hat{P}_N = \hat{\alpha} \\
\text{The nominal exchange rate:} & \quad \hat{S} = \hat{P}_T - \hat{P}_T^* \\
\text{The real exchange rate:} & \quad \hat{R} = \hat{P} - \hat{S} - \hat{P}^*. 
\end{align*}
\]

For simplicity, we assume that the weight accorded to traded goods in the price level function \((\gamma)\) is the same for the home and foreign countries. The relaxation of this assumption does not qualitatively change the results.
As there are nine variables and five equations in the above model, four variables have to be exogenous in order to arrive at the solution. As we shall make two different assumptions regarding the exogenous/endogenous classification of variables, it is convenient to write the above model in matrix form as \( AZ = 0 \), where

\[
Z = [\hat{P}_T \hat{P}_N \hat{S} \hat{R} \hat{P}^* \hat{\alpha} \hat{P}_T^* \hat{P}_N^*]',
\]

and

\[
A = \begin{bmatrix}
\gamma & 1-\gamma & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & \gamma & 1-\gamma \\
1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & -1 & 0 & 0 & 0
\end{bmatrix}.
\]

We have two versions of the above model: (1) The flexi-price version which treats \( \hat{P}, \hat{\alpha}, \hat{P}_T^* \) and \( \hat{P}_N^* \) as exogenous, while \( \hat{P}_T, \hat{P}_N, \hat{S}, \hat{R} \) and \( \hat{P}^* \) are endogenous; and (2) the sticky-price version where \( \hat{\alpha} \) and \( \hat{P}_N \) are swapped, so that \( \hat{P}, \hat{P}_N, \hat{P}_T^* \) and \( \hat{P}_N^* \) are exogenous and \( \hat{P}_T, \hat{\alpha}, \hat{S}, \hat{R} \) and \( \hat{P}^* \) are endogenous. The flexi-price version of the model is used to analyse the long-run response to a monetary expansion, a productivity enhancement and a booming sector. The sticky-price version is used to analyse the short-run effects of a monetary expansion. Once we make the choice of exogenous variables, the above model \( AZ = 0 \) can be rewritten as \( BY + CX = 0 \), where \( Y \) is a vector of the five endogenous variables, \( X \) is a vector of the four exogenous variables, and \( B \) and \( C \) are \( 5 \times 5 \) and \( 5 \times 4 \) coefficient matrices corresponding to \( Y \) and \( X \), respectively. That is, \( A = [B : C] \) and \( Z = [Y' : X']' \). Provided that \( B \) is invertible, the solution of the model is then \( Y = -B^{-1}CX \). We now derive the solutions for both versions of the model and then apply them to the shocks considered earlier in this chapter.\(^{20}\)

\(^{20}\) This style of solving economic models, with the switching of endogenous and exogenous variables, is introduced by Johansen (1960) and extended by Dixon et al. (1982).
The Flexi-Price Version

In the flexi-price version, \( \hat{P}, \hat{\alpha}, \hat{P}_T^* \) and \( \hat{P}_N^* \) are exogenous, \( \hat{P}_T, \hat{P}_N, \hat{S}, \hat{R} \) and \( \hat{P}^* \) are endogenous. We write the above solution for this case as \( Y_i = -B_i^{-1} C_i X_i \), where the coefficient matrices \( B_i \) and \( C_i \) are just the left- and right-parts of the matrix \( A \), i.e., \( A = [ B_i : C_i ] \) and \( Z = [ Y'_i : X'_i ]' \):

\[
B_i = \begin{bmatrix}
\gamma & 1-\gamma & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 \\
1 & -1 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 \\
\end{bmatrix}, \quad Y_i = \begin{bmatrix}
\hat{P}_T \\
\hat{P}_N \\
\hat{S} \\
\hat{R} \\
\hat{P}^* \\
\end{bmatrix}, \quad C_i = \begin{bmatrix}
-1 & 0 & 0 & 0 \\
0 & 0 & \gamma & 1-\gamma \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
-1 & 0 & 0 & 0 \\
\end{bmatrix}, \quad X_i = \begin{bmatrix}
\hat{P} \\
\hat{\alpha} \\
\hat{P}_T^* \\
\hat{P}_N^* \\
\end{bmatrix}.
\]

Solving for \( Y_1 \), we have

\[
Y_1 = -B_1^{-1} C_1 X_1 = \begin{bmatrix}
1 & 1-\gamma & 0 & 0 \\
1 & -\gamma & 0 & 0 \\
1 & 1-\gamma & -1 & 0 \\
0 & -(1-\gamma) & 1-\gamma & -(1-\gamma) \\
0 & 0 & \gamma & 1-\gamma \\
\end{bmatrix} \begin{bmatrix}
\hat{P} \\
\hat{\alpha} \\
\hat{P}_T^* \\
\hat{P}_N^* \\
\end{bmatrix}.
\]

Therefore, the solutions are:

(9.1) \( \hat{P}_T = \hat{P} + (1-\gamma) \hat{\alpha} \),
(9.2) \( \hat{P}_N = \hat{P} - \gamma \hat{\alpha} \),
(9.3) \( \hat{S} = \hat{P} + (1-\gamma) \hat{\alpha} - \hat{P}_T^* \),
(9.4) \( \hat{R} = -(1-\gamma) \hat{\alpha} + (1-\gamma) \hat{P}_T^* - (1-\gamma) \hat{P}_N^* \),
(9.5) \( \hat{P}^* = \gamma \hat{P}_T^* + (1-\gamma) \hat{P}_N^* \).
Consider the effects of a monetary expansion, which we measure by an increase in the general price level, so that $\hat{P}>0$ and the other exogenous variables are held unchanged ($\hat{\alpha} = \hat{P}_T^* = \hat{P}_N^* = 0$). It then follows from equations (9.1) to (9.3) that the prices of traded and nontraded goods and the nominal exchange rate all increase proportionally with the price level ($\hat{P}_T = \hat{P}_N = \hat{S} = \hat{P} > 0$). However, equations (9.4) and (9.5) reveal that the real exchange rate and the foreign price level remain unchanged ($\hat{R} = \hat{P}^* = 0$).

Next, consider the productivity-bias hypothesis according to which traded goods are relatively more expensive in poor countries in comparison to rich countries. For the poor country here, we have $\hat{\alpha} > 0$ and $\hat{P} = \hat{P}_T^* = \hat{P}_N^* = 0$. Remembering that the weight accorded to traded goods in the overall level $\gamma$ is a positive fraction, equations (9.1) and (9.2) show that this leads to an increase in the nominal price of traded goods, $\hat{P}_T = (1-\gamma)\hat{\alpha} > 0$ and a fall in the nominal price of nontraded goods, $\hat{P}_N = -\gamma \hat{\alpha} < 0$. This contrast in the movement of these two nominal prices occurs as the overall price level is held constant by assumption:

$$\hat{P} = \gamma \hat{P}_T + (1-\gamma)\hat{P}_N = \gamma (1-\gamma)\hat{\alpha} - (1-\gamma)\gamma\hat{\alpha} = 0.$$ 

In addition, as can be seen from equations (9.3) and (9.4), both the nominal and real exchange rates depreciate, i.e., $\hat{S} = -\hat{R} = (1-\gamma)\hat{\alpha} > 0$.

Finally, in the case of a booming sector, the relative price of traded goods falls ($\hat{\alpha} < 0$), while the price level remains unchanged ($\hat{P} = 0$). Thus it follows from equation (9.2) that $\hat{P}_N = -\gamma \hat{\alpha} > 0$, indicating that the price of nontraded goods increases. From equations (9.1), (9.3) and (9.4), we have $\hat{P}_T = \hat{S} = -\hat{R} = (1-\gamma)\hat{\alpha} < 0$, so that the price of traded goods decreases and both the nominal and real exchange rates appreciate.
The Sticky-Price Version

In the sticky-price version, \( \hat{P}, \hat{P}_N, \hat{P}_T^* \) and \( \hat{\alpha} \) are endogenous, and \( \hat{P}_T, \hat{\alpha}, \hat{S}, \hat{R} \) and \( \hat{P}^* \) are exogenous. We then interchange columns 2 and 7 of the matrix \( A \) to obtain a new matrix characterising the model, which we denote by \( A_* \). In this case, the new coefficient matrices \( B_2 \) and \( C_2 \) are just left- and right-parts of \( A_* \), i.e., \( A_* = [ B_2 : C_2 ] \) and \( Z_2 = [ Y_2' : X_2' ]' \):

\[
B_2 = \begin{bmatrix}
\gamma & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 \\
1 & -1 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1
\end{bmatrix}, \quad Y_2 = \begin{bmatrix} \hat{P}_T \\ \hat{\alpha} \\ \hat{S} \\ \hat{R} \\ \hat{P}^* \end{bmatrix}, \quad C_2 = \begin{bmatrix} -1 & 1 - \gamma & 0 & 0 \\
0 & 0 & \gamma & 1 - \gamma \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
-1 & 0 & 0 & 0 \end{bmatrix}, \quad X_2 = \begin{bmatrix} \hat{P} \\ \hat{P}_N \\ \hat{P}_T^* \\ \hat{\alpha} \end{bmatrix}.
\]

Thus, we can express this version of the model as \( B_2 Y_2 + C_2 X_2 = 0 \). Solving for \( Y_2 \), we have

\[
Y_2 = -B_2^{-1} C_2 X_2 = \begin{bmatrix}
1/\gamma & 1 - (1/\gamma) & 0 & 0 \\
1/\gamma & -1/\gamma & 0 & 0 \\
1/\gamma & 1 - (1/\gamma) & -1 & 0 \\
1 - (1/\gamma) & -1 + (1/\gamma) & 1 - \gamma & -(1-\gamma) \\
0 & 0 & \gamma & 1 - \gamma
\end{bmatrix} \begin{bmatrix} \hat{P} \\ \hat{P}_T \\ \hat{P}_N^* \\ \hat{\alpha} \end{bmatrix}.
\]

Therefore, the solutions are:

\[
\hat{P}_T = \frac{1}{\gamma} \hat{P} + \left( 1 - \frac{1}{\gamma} \right) \hat{P}_N, \quad (9.6)
\]

\[
\hat{\alpha} = \frac{1}{\gamma} \hat{P} - \frac{1}{\gamma} \hat{P}_N, \quad (9.7)
\]
The above can be used to analyse exchange-rate overshooting in three stages -- the effects in the short, intermediate and long run. Note that in all stages, foreign prices are held constant (\(P \equiv P^* \equiv 0\)).

(1) **Short run.** There is an increase in the money supply (\(\dot{P} > 0\)) with the price of nontraded goods fixed (\(\dot{P}_N = 0\)). According to equations (9.6) to (9.8), \(\dot{\hat{P}} = \dot{\hat{S}} = (1/\gamma) \hat{P} > \hat{P}\), indicating that the price of traded goods and the nominal exchange rate increase more than the price level. It can be seen from equation (9.9) that \(\dot{R} = [1-(1/\gamma)] \hat{P} < 0\), so that the real exchange rate depreciates in the short run.

(2) **The transition path.** In the short run, the relative price of traded goods increases, but this increase cannot be sustained over time if no real factors (the ultimate determinants of all relative prices) have changed. Accordingly, this high relative price generates an excess supply of traded goods, or, what amounts to the same thing, an excess demand for nontraded goods. Accordingly in the medium term, the relative price of nontraded goods gradually increases (or the relative price of traded goods falls) to offset the excess demand for nontraded goods. This occurs with an increase in the nominal price of nontraded goods (\(\dot{P}_N > 0\)) when the price level is held constant (\(\dot{P} = 0\)). Then, \(\dot{\alpha} = -(1/\gamma) \dot{\hat{P}}_N < 0\), from equation (9.7), indicating that the relative price of traded goods decreases. In addition, it can be seen from equations (9.6), (9.8)
and (9.9) that $\hat{P}_T = \hat{S} = -\hat{R} = [1 - (1/\gamma)] \hat{P}_N < 0$, so that the nominal and real exchange rates both appreciate.

Note that the decrease in the relative price of traded goods is dynamic in nature and what follows is a brief exploration of the adjustment path. Some attractively simple results emerge if we suppose that the relative price of traded goods, $\alpha$, is mean reverting to its long-run value $\alpha_0$ according to

$$\frac{d\alpha(t)}{dt} = \phi[\alpha(t) - \alpha_0], \quad \alpha(0) = \alpha_1,$$

where $\phi$ is the speed of adjustment, which the stability condition requires to be negative. Solving the above equation, we have $\alpha(t) = \alpha_0 + (\alpha_1 - \alpha_0) e^{\phi t}$, which is plotted in Figure 2.15. As the proportional change of any variable $x$ is $\dot{x} = dx/dx$, it follows that

$$\dot{\alpha}(t) = \phi \left[ 1 - \frac{\alpha_0}{\alpha_0 + (\alpha_1 - \alpha_0) e^{\phi t}} \right].$$

**Figure 2.15**

**THE TRANSITION PATH OF THE RELATIVE PRICE**

\[ \alpha(t) = \alpha_0 + (\alpha_1 - \alpha_0) e^{\phi t} \]
Suppose that during the transition phase, the price level does not change, so that \( \hat{P} = \gamma \hat{P}_T + (1 - \gamma) \hat{P}_N = 0 \). Recall that the relative price of traded goods is defined as \( \hat{\alpha} = \hat{P}_T - \hat{P}_N \). Solving these two equations jointly for \( \hat{P}_T \) and \( \hat{P}_N \) at time \( t \), we have
\[
\hat{P}_T(t) = (1 - \gamma) \hat{\alpha}(t) \quad \text{and} \quad \hat{P}_N(t) = -\gamma \hat{\alpha}(t).
\]
Substituting these into equations (9.8) and (9.9) and noting that \( \hat{P} = \hat{P}_T^* = \hat{P}_N^* = 0 \), we have \( \hat{S}(t) = (1 - \gamma) \hat{\alpha}(t) \) and \( \hat{R}(t) = - (1 - \gamma) \hat{\alpha}(t) \). Finally, substituting equation (9.11) into the above expressions for \( \hat{P}_T(t) \), \( \hat{S}(t) \) and \( \hat{R}(t) \), we obtain:
\[
\hat{P}_T(t) = \varphi (1 - \gamma) \left[ 1 - \frac{\alpha_0}{\alpha_0 + (\alpha_1 - \alpha_0) e^{\phi t}} \right], \quad \hat{\alpha}(t) = \varphi \left[ 1 - \frac{\alpha_0}{\alpha_0 + (\alpha_1 - \alpha_0) e^{\phi t}} \right],
\]
\[
\hat{S}(t) = \varphi (1 - \gamma) \left[ 1 - \frac{\alpha_0}{\alpha_0 + (\alpha_1 - \alpha_0) e^{\phi t}} \right], \quad \hat{R}(t) = -\varphi (1 - \gamma) \left[ 1 - \frac{\alpha_0}{\alpha_0 + (\alpha_1 - \alpha_0) e^{\phi t}} \right],
\]
\[
\hat{P}^*(t) = 0.
\]

As \( \hat{P}_T(t) \), \( \hat{S}(t) \) and \( \hat{R}(t) \) are the relative price function \( \hat{\alpha}(t) \) times a constant, their transition paths in the intermediate run are similar to that in Figure 2.15.

(3) **Long run.** In the long run, the increase in the price of nontraded goods matches that in the price level, so that \( \hat{P} = \hat{P}_N > 0 \). Thus from equations (9.7) and (9.9), we have \( \hat{R} = \hat{\alpha} = 0 \). This indicates that in the new equilibrium state, the real exchange rate and the relative price of nontraded goods return to their previous levels. In addition, equations (9.6) and (9.8) show that the price of traded goods and the nominal exchange rate have the same proportional increase as the increase in the general price level \( \hat{S} = \hat{P}_T = \hat{P} > 0 \).
To summarise, in the presence of sticky prices, a monetary expansion causes the nominal exchange rate to depreciate by too much initially, necessitating a subsequent appreciation. The real exchange rate depreciates first and then appreciates until it returns to its original value.

Summary

Table 2.2 summarises this section by tabulating the responses of each endogenous variable to changes in each exogenous variable for the two versions of the model. The entries in the upper and lower panel of the table correspond to elements in the $5 \times 4$ solution matrices, $-B_1^1C_1$ and $-B_2^1C_2$. As the model is expressed in terms of proportional changes, an entry in either panel of the table is interpreted as the elasticity of the endogenous variable listed at the far left of the corresponding row with respect to the exogenous variable given at the top of the corresponding column. For example, the second entry in the first row of the flexi-prices model, $(1-\gamma)$, tells us that when the relative price of traded goods $\alpha$ changes by 1 percent, the price of traded goods will change by $(1-\gamma)$ percent. The reason for this result is that the price level is being held constant, which means that a weighted average of changes in the prices of traded and nontraded goods must be zero. That this is indeed the case can be seen from the second entry of the second row, $-\gamma$, which is the percent change in the price of nontraded goods in response to a 1 percent change in $\alpha$. Thus the weighted average of these two price changes is $\gamma \hat{P}_N + (1-\gamma)\hat{P}_N = \gamma(1-\gamma)\hat{\alpha} + (1-\gamma)(-\gamma)\hat{\alpha} = 0$, which accords with the constancy of the overall price level, $\hat{P} = 0$. 


<table>
<thead>
<tr>
<th>Exogenous variable</th>
<th>Endogenous variable</th>
<th>Price level at home</th>
<th>Relative price of traded goods</th>
<th>Foreign prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of traded goods</td>
<td>$P_T$</td>
<td>1</td>
<td>$1 - \gamma$</td>
<td>$0$</td>
</tr>
<tr>
<td>Price of nontraded goods</td>
<td>$P_N$</td>
<td>1</td>
<td>$-\gamma$</td>
<td>0</td>
</tr>
<tr>
<td>Nominal exchange rate</td>
<td>$S$</td>
<td>1</td>
<td>$1 - \gamma$</td>
<td>$-1$</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>$R$</td>
<td>0</td>
<td>$-(1 - \gamma)$</td>
<td>$1 - \gamma$</td>
</tr>
<tr>
<td>Foreign price level</td>
<td>$P^*$</td>
<td>0</td>
<td>0</td>
<td>$\gamma$</td>
</tr>
</tbody>
</table>

**Note:** $\gamma$ is the elasticity of price level $P$ with respect to the traded goods price $P_T$, i.e., $\gamma = \frac{\partial \log P}{\partial \log P_T}$. 

---

<table>
<thead>
<tr>
<th>Exogenous variable</th>
<th>Endogenous variable</th>
<th>Price level at home</th>
<th>Price of nontraded goods</th>
<th>Foreign prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of traded goods</td>
<td>$P_T$</td>
<td>$1/\gamma$</td>
<td>$1 - (1/\gamma)$</td>
<td>0</td>
</tr>
<tr>
<td>Relative price of traded goods</td>
<td>$\alpha$</td>
<td>$1/\gamma$</td>
<td>$-1/\gamma$</td>
<td>0</td>
</tr>
<tr>
<td>Nominal exchange rate</td>
<td>$S$</td>
<td>$1/\gamma$</td>
<td>$1 - (1/\gamma)$</td>
<td>$-1$</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>$R$</td>
<td>$1 - (1/\gamma)$</td>
<td>$-1 + (1/\gamma)$</td>
<td>$1 - \gamma$</td>
</tr>
<tr>
<td>Foreign price level</td>
<td>$P^*$</td>
<td>0</td>
<td>0</td>
<td>$\gamma$</td>
</tr>
</tbody>
</table>

---

Note: $\gamma$ is the elasticity of price level $P$ with respect to the traded goods price $P_T$, i.e., $\gamma = \frac{\partial \log P}{\partial \log P_T}$. 

---

TABLE 2.2
RESPONSES OF ENDOGENOUS VARIABLES TO CHANGES IN EXOGENOUS VARIABLES
(Elasticities)
2.10. **Summary and Conclusion**

This chapter examined several aspects of exchange-rate economics that revolve around the theory of PPP. First, it presented a geometric framework for analysing the relationship between exchange rates and relative prices. Through an informal examination of exchange rates and relative prices in 68 countries, we illustrated that PPP is not a theory of exchange-rate determination in the short run, but rather a long-run equilibrium relationship between exchange rates and prices. Then the chapter analysed several important factors that influence the nominal and real exchange rates, focusing on the relationship between exchange rates, money and prices. Four kinds of shocks that may change nominal and real exchange rates were examined: (1) A monetary expansion, (2) the productivity bias, (3) a monetary expansion with sticky prices, and (4) a booming export sector. An algebraic formulation of the analysis of the four shocks was also provided. Table 2.3 summarises their influences on the nominal and real exchange rates.

| TABLE 2.3 |
| RESPONSE OF EXCHANGE RATES TO VARIOUS SHOCKS |

<table>
<thead>
<tr>
<th>Shock</th>
<th></th>
<th>Exchange rate</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Nominal</td>
<td>Real</td>
</tr>
<tr>
<td>1. Monetary expansion with</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- flexible prices</td>
<td></td>
<td>Depreciation</td>
<td>No change</td>
</tr>
<tr>
<td>- sticky prices</td>
<td></td>
<td>Initial depreciation and partial appreciation subsequently (overshooting)</td>
<td>Initial depreciation and subsequent appreciation, but no change in the long run</td>
</tr>
<tr>
<td>2. Increase in relative price of traded goods (productivity bias)</td>
<td></td>
<td>Depreciation</td>
<td>Depreciation</td>
</tr>
<tr>
<td>3. A booming sector</td>
<td></td>
<td>Appreciation</td>
<td>Appreciation</td>
</tr>
</tbody>
</table>
Subsequent chapters of the thesis first provide a review of the recent literature on PPP and then focus on the empirics of PPP. We will apply PPP to the Big Mac data and formally test for PPP. Using a PPP-type of framework, we finally introduce a new approach for the long-run pricing of currencies.
Appendix 2.1

More on Exchange Rates and Relative Prices
in 68 Countries

In Section 2.3, we analysed changes in exchange rates and relative prices and ratios of the variances of these two changes. This appendix sets out the data sources and computational details.

We used the data obtained from the International Financial Statistics CD ROM (December 2001), published by the International Monetary Fund, which contains annual data over the period of 1973-2000 for 68 countries. The 68 countries were selected for analysis on the basis of availability of data. In three cases, missing values were interpolated or extrapolated, which pertained to the inflation rates of Botswana in 1974, Dominica in 1979 and Rwanda in 1994.

The average annualised logarithmic change of the exchange rate $S$ for country $c$ from the year $t-m$ to $t$ is $\Delta s_{ct}^{(m)} = (1/m)(\log S_{ct} - \log S_{c,t-m})$, where the variable $m$ is the length of the underlying change with $m = 1, \ldots, 27$ years. The change $\Delta s_{ct}^{(m)}$ is defined for $t = 1973 + m, \ldots, 2000$, $c = 1, \ldots, 68$. To assist with visualising the nature of $\Delta s_{ct}^{(m)}$, Table A2.1 gives the details for one country.

Similar to $\Delta s_{ct}^{(m)}$, the variable $(\Delta p_{ct} - \Delta p_t^{*})^{(m)}$ is the average annualised logarithmic change of the price level of country $c$ relative to that of the US, from the year $t-m$ to $t$, i.e., $(\Delta p_{ct} - \Delta p_t^{*})^{(m)} = (1/m) [\log (P_{c,t} / P_{c,t-m}) - \log (P_t^{*} / P_t^{*})]$. As the data are published in the form of annual inflation rates, $\pi_{ct} = P_{c,t} / P_{c,t-1} - 1$, we express $(\Delta p_{ct} - \Delta p_t^{*})^{(m)}$ in terms of inflation as
TABLE A2.1
CHANGES IN EXCHANGE RATES FOR VARIOUS LAG LENGTHS

<table>
<thead>
<tr>
<th>Year</th>
<th>Length of the change m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1974</td>
<td>Δs_{c,1974}^{(1)}</td>
</tr>
<tr>
<td>1975</td>
<td>Δs_{c,1975}^{(1)}</td>
</tr>
<tr>
<td>⋮</td>
<td>⋮</td>
</tr>
<tr>
<td>1999</td>
<td>Δs_{c,1999}^{(1)}</td>
</tr>
<tr>
<td>2000</td>
<td>Δs_{c,2000}^{(1)}</td>
</tr>
</tbody>
</table>

Panel A of Figure 2.2 in Section 2.3 presented a scatter plot of \( \Delta s_{c,t}^{(m)} \) against \((\Delta p_{ct} - \Delta p_t^*)^{(m)}\) for \( m = 1 \). The exchange rates here for country \( c \) correspond to the column for \( m = 1 \) in Table A2.1. As \( c = 1, \ldots, 68 \) and \( t = 1974, \ldots, 2000 \), we had in total \( 68 \times 27 = 1,836 \) data points in Panel A. Panel B gave the same plot as Panel A, but only included 1,814 observations pertaining to “moderate” inflation. Panel C presented the same scatter plot for \( m = 27 \), and corresponds, for country \( c \), to the last column of Table A2.1, and thus had one data point for each of the 68 countries.
The frequency distribution in Figure 2.3 of Section 2.3 referred to the average annualised changes in the 68 real exchange rates over the period of 1974-2000. For country \(c\), this was computed as \(\Delta q_c^{(27)} = (\Delta p_c - \Delta p^*)^{(27)} - \Delta s_c^{(27)}\). The \(\Delta s_c^{(27)}\), \((\Delta p_c - \Delta p^*)^{(27)}\) and \(\Delta q_c^{(27)}\) are contained in Table A2.2. The last three rows on the right panel give the cross-country means, standard deviations and the standard errors of the means.

The ratio of the variance of exchange-rate changes to that of relative-price changes is \(\phi(m) = \text{var}[\Delta s^{(m)}]/\text{var}[(\Delta p - \Delta p^*)^{(m)}]\). We estimate the variance in the numerator by

\[
\text{var}[\Delta s^{(m)}] = \frac{1}{[2000 - (1973 + m) + 1] \times 68} \sum_{c=1}^{68} \sum_{t=1973+m}^{2000} (\Delta s_c^{(m)} - \Delta s^{(m)})^2,
\]

where \(\Delta s^{(m)}\) is the mean of the \([2000 - (1973 + m) + 1] \times 68 = (28 - m) \times 68\) observations over countries and time of \(\Delta s_c^{(m)}\), i.e.,

\[
\Delta s^{(m)} = \frac{1}{[2000 - (1973 + m) + 1] \times 68} \sum_{c=1}^{68} \sum_{t=1973+m}^{2000} \Delta s_c^{(m)}.
\]

Note that for the transition from the year 1973 + m to 2000, there are \(2000 - (1973 + m) + 1\) defined values of \(\Delta s_c^{(m)}\) for each country \(c\); for example, when \(m = 26\), which corresponds to the second last column of Table A2.1, the transition period goes from 1999 to 2000. Here there are only \(2000 - (1973 + m) + 1 = 2000 - 1999 + 1 = 2\) defined values of \(\Delta s_c^{(m)}\): \(\Delta s_c^{(26)}\) and \(\Delta s_c^{(26)}\). Therefore, we add one in the square brackets in equations (A2.1) and (A2.2). The variance of relative-price changes in the denominator of \(\phi(m)\) is estimated in a similar manner.
<table>
<thead>
<tr>
<th>Country</th>
<th>Logarithmic changes in ( S_c )</th>
<th>( P_c / P_{US} )</th>
<th>Real exchange rate</th>
<th>Country</th>
<th>Logarithmic changes in ( S_c )</th>
<th>( P_c / P_{US} )</th>
<th>Real exchange rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Argentina</td>
<td>38.15</td>
<td>37.77</td>
<td>-0.38</td>
<td>37. Mexico</td>
<td>10.68</td>
<td>10.71</td>
<td>0.03</td>
</tr>
<tr>
<td>2. Australia</td>
<td>1.59</td>
<td>0.69</td>
<td>-0.90</td>
<td>38. Morocco</td>
<td>1.46</td>
<td>0.70</td>
<td>-0.76</td>
</tr>
<tr>
<td>3. Barbados</td>
<td>-0.05</td>
<td>0.75</td>
<td>0.80</td>
<td>39. Myanmar</td>
<td>0.47</td>
<td>4.20</td>
<td>3.72</td>
</tr>
<tr>
<td>4. Bolivia</td>
<td>20.39</td>
<td>20.23</td>
<td>-0.15</td>
<td>40. New Zealand</td>
<td>1.89</td>
<td>1.25</td>
<td>-0.65</td>
</tr>
<tr>
<td>5. Botswana</td>
<td>3.34</td>
<td>2.29</td>
<td>-1.05</td>
<td>41. Niger</td>
<td>1.76</td>
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<td>-1.67</td>
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<tr>
<td>6. Burundi</td>
<td>3.69</td>
<td>2.79</td>
<td>-0.90</td>
<td>42. Nigeria</td>
<td>8.23</td>
<td>6.86</td>
<td>-1.36</td>
</tr>
<tr>
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<td>0.07</td>
<td>-0.59</td>
<td>43. Norway</td>
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<td>0.35</td>
<td>-0.35</td>
</tr>
<tr>
<td>8. Chile</td>
<td>10.68</td>
<td>12.01</td>
<td>1.33</td>
<td>44. Pakistan</td>
<td>2.85</td>
<td>1.68</td>
<td>-1.17</td>
</tr>
<tr>
<td>9. Colombia</td>
<td>7.21</td>
<td>6.65</td>
<td>-0.56</td>
<td>45. Panama</td>
<td>0.00</td>
<td>-0.84</td>
<td>-0.84</td>
</tr>
<tr>
<td>10. Costa Rica</td>
<td>6.22</td>
<td>5.16</td>
<td>-1.06</td>
<td>46. Paraguay</td>
<td>5.36</td>
<td>4.58</td>
<td>-0.78</td>
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<tr>
<td>11. Côte d'Ivoire</td>
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<td>-0.52</td>
<td>47. Peru</td>
<td>29.48</td>
<td>30.43</td>
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<tr>
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<td>0.14</td>
<td>-0.72</td>
<td>48. Philippines</td>
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<td>2.62</td>
<td>-0.60</td>
</tr>
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<td>13. Denmark</td>
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<td>0.26</td>
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<td>49. Poland</td>
<td>15.17</td>
<td>10.92</td>
<td>-4.25</td>
</tr>
<tr>
<td>14. Dominica</td>
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<td>0.35</td>
<td>-0.08</td>
<td>50. Rwanda</td>
<td>2.65</td>
<td>2.21</td>
<td>-0.44</td>
</tr>
<tr>
<td>15. Ecuador</td>
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<td>9.96</td>
<td>-1.15</td>
<td>51. Samoa</td>
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</tr>
<tr>
<td>16. Egypt</td>
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<td>2.99</td>
<td>-0.45</td>
<td>52. Saudi Arabia</td>
<td>0.09</td>
<td>-0.58</td>
<td>-0.67</td>
</tr>
<tr>
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<td>3.40</td>
<td>1.39</td>
<td>53. Senegal</td>
<td>1.76</td>
<td>0.60</td>
<td>-1.17</td>
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<td>18. Ethiopia</td>
<td>2.24</td>
<td>0.99</td>
<td>-1.25</td>
<td>54. Seychelles</td>
<td>0.14</td>
<td>0.31</td>
<td>0.17</td>
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<tr>
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<td>0.55</td>
<td>-1.05</td>
<td>55. Sierra Leone</td>
<td>12.17</td>
<td>12.22</td>
<td>0.04</td>
</tr>
<tr>
<td>20. Ghana</td>
<td>14.05</td>
<td>12.12</td>
<td>-1.93</td>
<td>56. Singapore</td>
<td>-0.58</td>
<td>-0.89</td>
<td>-0.31</td>
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<td>3.84</td>
<td>-0.20</td>
<td>57. Swaziland</td>
<td>3.90</td>
<td>3.03</td>
<td>-0.87</td>
</tr>
<tr>
<td>22. Guatemala</td>
<td>3.29</td>
<td>2.93</td>
<td>-0.36</td>
<td>58. Sweden</td>
<td>1.18</td>
<td>0.47</td>
<td>-0.71</td>
</tr>
<tr>
<td>23. Haiti</td>
<td>2.42</td>
<td>2.92</td>
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<td>59. Switzerland</td>
<td>-1.10</td>
<td>-0.89</td>
<td>0.21</td>
</tr>
<tr>
<td>24. Honduras</td>
<td>3.26</td>
<td>2.79</td>
<td>-0.46</td>
<td>60. Syrian Arab Rep.</td>
<td>1.74</td>
<td>3.14</td>
<td>1.40</td>
</tr>
<tr>
<td>25. Hungary</td>
<td>2.91</td>
<td>2.98</td>
<td>0.07</td>
<td>61. Tanzania</td>
<td>7.65</td>
<td>6.68</td>
<td>-0.97</td>
</tr>
<tr>
<td>26. Iceland</td>
<td>7.42</td>
<td>7.01</td>
<td>-0.41</td>
<td>62. Thailand</td>
<td>1.17</td>
<td>0.40</td>
<td>-0.77</td>
</tr>
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<td>27. India</td>
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<td>1.38</td>
<td>-1.42</td>
<td>63. Togo</td>
<td>1.76</td>
<td>0.71</td>
<td>-1.05</td>
</tr>
<tr>
<td>28. Iran, I.R. of</td>
<td>5.65</td>
<td>5.59</td>
<td>0.06</td>
<td>64. Turkey</td>
<td>17.32</td>
<td>16.67</td>
<td>-0.65</td>
</tr>
<tr>
<td>29. Israel</td>
<td>14.75</td>
<td>14.64</td>
<td>-0.12</td>
<td>65. United Arab Emirates</td>
<td>-0.14</td>
<td>-2.18</td>
<td>-2.04</td>
</tr>
<tr>
<td>30. Jamaica</td>
<td>6.29</td>
<td>5.85</td>
<td>-0.44</td>
<td>66. UK</td>
<td>0.71</td>
<td>0.99</td>
<td>0.28</td>
</tr>
<tr>
<td>31. Japan</td>
<td>-1.43</td>
<td>-0.69</td>
<td>-0.74</td>
<td>67. Uruguay</td>
<td>15.30</td>
<td>15.77</td>
<td>0.47</td>
</tr>
<tr>
<td>32. Kenya</td>
<td>3.90</td>
<td>3.42</td>
<td>-0.48</td>
<td>68. Venezuela</td>
<td>8.20</td>
<td>8.11</td>
<td>-0.09</td>
</tr>
<tr>
<td>33. Korea</td>
<td>1.86</td>
<td>1.58</td>
<td>-0.28</td>
<td>Mean</td>
<td>5.12</td>
<td>4.67</td>
<td>-0.45</td>
</tr>
<tr>
<td>34. Malaysia</td>
<td>0.70</td>
<td>-0.40</td>
<td>-1.10</td>
<td>Standard deviation</td>
<td>7.01</td>
<td>7.05</td>
<td>0.99</td>
</tr>
<tr>
<td>35. Malta</td>
<td>0.20</td>
<td>-0.53</td>
<td>-0.73</td>
<td>Standard error of mean</td>
<td>0.85</td>
<td>0.87</td>
<td>0.13</td>
</tr>
<tr>
<td>36. Mauritius</td>
<td>2.54</td>
<td>2.01</td>
<td>-0.53</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE A2.2**

CHANGES IN EXCHANGE RATES AND PRICES FOR 68 COUNTRIES: 1974-2000

(Average annualised logarithmic changes × 100)
Figure 2.4 in Section 2.3 plotted $\phi(m)$. Note that only 50 countries were used in this plot. We excluded 18 countries which fixed their exchange rates against the US dollar for part of the period of 1974-2000, as their $\Delta s^{(m)}_{ct}$ were sometimes zero.
Appendix 2.2

More on the Absolute Price Schedule

The absolute price schedule shown in Figure 2.6 of Section 2.4 is the locus of \( \{P_T, P_N\} \) such that the overall price level is constant. This appendix analyses further the properties of this schedule and applies it to four commonly-used indices: Laspeyres’, Paasche, Fisher’s and Cobb-Douglas.

Recall that the price level function is \( P = P(P_T, P_N) \), where \( P_T \) and \( P_N \) are prices of traded and nontraded goods, respectively. This function can be thought of as an “aggregator function”, or a price index. Fisher (1922) evaluates various forms of price indices by their ability to satisfy ten key tests. One such test is proportionality whereby if all prices are multiplied by a positive constant \( \lambda \), then the new value of the index should be equal to the old index multiplied by \( \lambda \). In other words, the index should be homogeneous of degree one. This means that the price level can be written in terms of proportional changes as \( \hat{P} = \gamma \hat{P}_T + (1 - \gamma) \hat{P}_N \), where a circumflex (“^”) denotes proportional change and \( \gamma = \partial (\log P) / \partial (\log P_T) \) is the elasticity of price level \( P \) with respect to the price of traded goods \( P_T \).

A further desirable property of any price index is that it increases in its arguments, the individual prices. This implies that the elasticity \( \gamma \) is a positive fraction. To see this, write \( \hat{P} = \gamma_T \hat{P}_T + \gamma_N \hat{P}_N \), where \( \gamma_T = \partial (\log P) / \partial (\log P_T) \) and \( \gamma_N = \partial (\log P) / \partial (\log P_N) \). As \( P \) increases with \( P_T \) and \( P_N \), both \( \gamma_T \) and \( \gamma_N \) are positive. As proportionality implies that \( \gamma_T + \gamma_N = 1 \), it follows that \( \gamma_T \) and \( \gamma_N \) are both positive fractions. It is to be noted that \( \gamma_T \) here equals \( \gamma \) above. The relationship between the elasticities
\( \gamma_i \) and the slope of the absolute price schedule in Figure A2.1 is as follows. Using
\[ P = P(T, P_N) \text{ and } \gamma_i = (\partial P / \partial P_T) P_i / P, \]
the slope of this schedule is

\[
(A2.1) \quad \frac{dP_T}{dP_N} \bigg|_{AA} = -\frac{\gamma_N P/P_N}{\gamma_T P/P_T} = -\frac{1 - \gamma_T}{\gamma_T} \frac{P_T}{P_N},
\]

as \( \gamma_T + \gamma_N = 1 \). As \( \partial P / \partial P_T > 0 \) and \( \partial P / \partial P_N > 0 \) are both positive, the absolute price schedule is downward sloping.

The convexity of the absolute price schedule is analysed in Clements and Semudram (1983) as follows. Consider the two points \( X \) and \( Y \) on AA in Figure A2.1. As \( P_T / P_N \) is higher (or \( P_N / P_T \) is lower) at \( X \) relative to the point \( Y \), it is expected that the consumption of nontraded goods is greater at the point \( X \) relative to \( Y \). Accordingly at \( X \), nontraded goods received a larger weight in the overall price index relative to at the point \( Y \). This means that a given increase in \( P_N \) causes the overall price level to rise by more at \( X \) than at \( Y \). To offset this rise in \( P_N \), in order to keep

---

**FIGURE A2.1**

**THE ABSOLUTE PRICE SCHEDULE**
the overall price level constant, \( P_T \) needs to decrease by a larger amount at X, relative to Y. Therefore, the (absolute) slope of \( AA \) declines as we move down the schedule, i.e., the absolute schedule \( AA \) is convex to the origin. Although this is a satisfactory argument, below we show that for a certain popular price index, the absolute price schedule is not convex.

Consider four explicit functional forms for the price level function, \( P = P(P_T, P_N) \), (1) Laspeyres’ index, (2) the Paasche index, (3) Fisher’s ideal index, and (4) the Cobb-Douglas index. These are all commonly-used price index formulae. For convenience, denote by \( P_1 \) and \( P_2 \) the prices of traded and nontraded goods. These indices compare prices in the current period, \( P_{11} \) and \( P_{21} \), with those in the base period, \( P_{10} \) and \( P_{20} \). The corresponding quantities are \( Q_{11} \), \( Q_{21} \), \( Q_{10} \) and \( Q_{20} \). As a matter of notation, let \( M_{st} = \sum_{i=1}^{2} P_{is} Q_{it} \) be the total cost of the basket purchased in period \( t \) valued at prices in period \( s \), where \( s, t = 0, 1 \); and let \( w_{ist} = P_{is} Q_{it}/M_{st} \) be the share of good \( i \) in this total cost. This notation can be set out in tabular form as follows:

<table>
<thead>
<tr>
<th>Period ( t )</th>
<th>Price of good</th>
<th>Quantity of good</th>
<th>Total cost of basket purchased in ( t ) at prices in period</th>
<th>Share of good ( i ) in the total cost of basket in period</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 )</td>
<td>( P_{10} )</td>
<td>( Q_{10} )</td>
<td>( M_{00} ) ( M_{01} )</td>
<td>( w_{100} ) ( w_{101} )</td>
</tr>
<tr>
<td>( 1 )</td>
<td>( P_{11} )</td>
<td>( Q_{11} )</td>
<td>( M_{10} ) ( M_{11} )</td>
<td>( w_{110} ) ( w_{111} )</td>
</tr>
</tbody>
</table>

**Laspeyres’ index.** The cost of the base-period basket, \( Q_{10} \) and \( Q_{20} \), at current-period prices is \( M_{10} = \sum_{i=1}^{2} P_{i1} Q_{i0} \). Laspeyres’ index is the ratio of this cost to the observed total expenditure in the base period \( M_{00} = \sum_{i=1}^{2} P_{i0} Q_{i0} \): \( P_L = M_{10}/M_{00} \). The implied elasticities then take the form \( \gamma_i^L = P_{i1} Q_{i0}/M_{10} = w_{i10} \), which is the share of good \( i \) in the
cost of the basket valued at current-period prices. From equation (A2.1), the slope of the absolute price schedule is \(-\gamma_P T P / \gamma_P N_P\), which in the Laspeyres’ case takes the form \(-P N Q N_0 P T / P T N Q T_0 P N\). If we interpret \(P T\) in the numerator of this expression as \(P T_1\), and \(P N\) in the denominator as \(P N_1\), then the slope simplifies to \(-Q N_0 / Q T_0\). As this slope is independent of the two prices, in the Laspeyres’ case the absolute price schedule is a straight line, as shown in Panel A of Figure A2.2.

The Paasche index is obtained by replacing \(Q_{i0}\) in the numerator and denominator of the Laspeyres’ index with \(Q_{i1}\). This means that the Paasche index takes the form \(P_P = M_{i1} / M_{01}\), where \(M_{i1} = \sum_{i=1}^2 P_{i1} Q_{i1}\) is the observed expenditure on good \(i\) in the current-period basket and \(M_{01} = \sum_{i=1}^2 P_{i0} Q_{i1}\) is the cost of the current-period basket at base-period prices. The implied elasticities then take the form \(\gamma_i^p = P_{i1} Q_{i1} / M_{i1} = w_{i11}\), which is the observed expenditure on good \(i\) in the current-period prices as a proportion of the expenditure on all goods. This proportion is also known as the “ith budget share”. The slope of the absolute price schedule in the Paasche case takes the form \(-P_T P N_1 Q N_1 / P_N P T_1 Q T_1\), which can be simplified to \(-Q N_1 / Q T_1\). Thus in the Paasche case the absolute price schedule is also a straight line, as shown in Panel B of Figure A2.2.

Fisher’s ideal index is the geometric average of Laspeyres’ and the Paasche indices, \(P_F = \sqrt{P_L P_P}\). It can be shown that the elasticities of Fisher’s index are the arithmetic average of those of its two component indices, \(\gamma_i^f = \frac{1}{2}(\gamma_i^l + \gamma_i^p)\). Accordingly, the slope of the absolute price schedule is \(-P_T \gamma_N^f / P_N \gamma_T^f\). Interpreting \(P_T\) as \(P_T_1\), and \(P_N\) as \(P N_1\) as before, this can be written as \([-1 + 2(P_{N1} / P_{T1})k_1] / [1 + 2(P_{T1} / P_{N1})k_2]\), where \(k_1 = Q N_0 Q N_1 / (Q T_1 Q N_0 + Q T_0 Q N_1)\) and \(k_2 = Q T_0 Q T_1 / (Q T_1 Q N_0 + Q T_0 Q N_1)\) are positive constants. When \(P_T\) decreases and \(P_N\) increases along the absolute price
schedule (i.e., $P_N/P_T$ increases), the slope strictly decreases in absolute value. This can be seen by writing the slope as $f(x) = -(1 + 2k_1x)/(1 + 2k_2/x)$ with $x = P_{NI}/P_{TI} > 0$. As $f'(x) = -2k_1(1 + 2k_2/x) + (1 + 2k_1x)(-k_2/x^2)/(1 + 2k_2/x)^2 < 0$, it follows that the slope $f(x)$ strictly decreases in $x$. Since $f(x) < 0$, the slope gets steeper with increasing $x$. Therefore, in Fisher's case the absolute price schedule is concave to the origin, as shown in Panel C of Figure A2.2.
Finally, consider the *Cobb-Douglas index* \( P = (P_{T1} / P_{T0})^{w_{T0}} (P_{N1} / P_{N0})^{w_{N0}} \), where \( w_{i0} \) is the share of good \( i \) in the cost of the basket in the base period, i.e., \( w_{i0} = P_{i0} Q_{i0} / M_{00} \). Then the implied elasticities take the form \( \gamma_i = w_{i0} P_{i0} \). Therefore, the slope of the absolute price schedule is \( -w_{N0} P_T / w_{T0} P_N \). With \( P_T \) decreasing and \( P_N \) increasing along the absolute price schedule, this slope decreases monotonically in absolute value, so that the absolute price schedule is convex toward the origin, as shown in Panel D of Figure A2.2.

In summary, we have examined the properties of absolute price schedules for four functional forms of the price level function, \( P = P(T, N) \). In the cases of Laspeyres’ and the Paasche indices, the absolute price schedules are both straight lines (with different slopes), which are special cases of a convex curve. When the price level is formulated in the Cobb-Douglas form, we obtain a convex absolute price schedule. However, in Fisher’s case, the absolute price schedule is concave to the origin. It is to be noted that what really matters for the absolute price schedule is not convexity, but the negative slope, which serves to keep the overall price level constant.
Appendix 2.3

Bootstrapping the International Poverty Line

In Section 2.6, we regressed the exchange-rate discrepancy, \( D \), on the PWT GDP, \( \tilde{Y} \). The regression equation was given by equation (6.2), to which we add a country subscript \( c \):

\[
\log D_c = \alpha + \beta \log \tilde{Y}_c + \epsilon_c, \quad c = 1, \ldots, 77.
\]

The least-squares estimates are \( \hat{\alpha} = -4.06 (.34) \) and \( \hat{\beta} = .42 (.04) \). We showed that the international poverty line is the value of \( \tilde{Y} \) corresponding to \( D = 1 \), which we denoted by \( \tilde{Y}^* \). The expression for \( \tilde{Y}^* \) was given by equation (6.4), also reproduced below:

\[
\tilde{Y}^* = \exp\left( -\frac{\alpha}{\beta} \right).
\]

As discussed in the text, \( \tilde{Y}^* \) involves the ratio of two parameters. It is well known that when least-squares estimates are used to evaluate such ratios, there can be econometric problems due to non-normality and certain moments not existing. To overcome these problems, we use Efron’s (1979) bootstrap simulation procedure.

The bootstrap simulation technique is as follows:

Step 1: Estimate equation (A3.1) by OLS and denote the estimates by \( \hat{\alpha} \) and \( \hat{\beta} \).

Step 2: Obtain the least-squares residual vector of equation (A3.1), \( \hat{\epsilon} = [\hat{\epsilon}_1, \ldots, \hat{\epsilon}_{77}]' \), where \( \hat{\epsilon}_c = \log D_c - \hat{\alpha} - \hat{\beta} \log \tilde{Y}_c, \quad c = 1, \ldots, 77 \).

Step 3: Draw the bootstrap error vector \( \epsilon = [e_1, \ldots, e_{77}]' = [\hat{\epsilon}_{t_1}, \ldots, \hat{\epsilon}_{t_{77}}]' \), where \( t_c \ (c = 1, \ldots, 77) \) is a random number uniformly drawn from 1 to 77.
Step 4: Use the data-based least-squares estimates, $\hat{\alpha}$ and $\hat{\beta}$, the observed value of the vector $\tilde{Y} = [Y_c]$ and the error vector $e$ to form the simulated value of the dependent vector $\log D' = \log [D'_c]$ based on equation (A3.1).

Step 5: Use $\log D'$ and $\tilde{Y}$ to re-estimate equation (A3.1) by least-squares to yield $\alpha^{(s)}$ and $\beta^{(s)}$.

Step 6: Substitute $\alpha^{(s)}$ and $\beta^{(s)}$ into equation (A3.2) to obtain the simulated value of $\tilde{Y}^*$ in trial $s$, denoted by $\tilde{Y}^{*(s)}$.

Step 7: Repeat steps 3-6 1,000 times to yield $\tilde{Y}^{*(1)}, \ldots, \tilde{Y}^{*(1,000)}$. Then compute the mean, $\tilde{Y}^*_{\text{Mean}} = (1/1,000) \sum_{s=1}^{1,000} \tilde{Y}^{*(s)}$, and the root-mean-squared error (RMSE),

$$\sqrt{\frac{1}{1/1,000} \sum_{s=1}^{1,000} (\tilde{Y}^{*(s)} - \tilde{Y}^*_{\text{Mean}})^2}.$$  

Using this procedure, the mean value of $\tilde{Y}^*$ is $16,539$ with a RMSE of $3,040$. These values are given in footnote 16 of this chapter.
References


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