ECONOMICS

THE CARDINAL ATTRIBUTES OF UTILITY IN PARETO’S “SUNTO”
- A CRITICAL NOTE ON SOME RECENT RESEARCH

by

Michael McLure

DISCUSSION PAPER 03.01
THE CARDINAL ATTRIBUTES OF UTILITY IN PARETO’S “SUNTO”
- A CRITICAL NOTE ON SOME RECENT RESEARCH

by

Michael McLure

University of Western Australia
Business School
Economics
Crawley, WA 6009

DISCUSSION PAPER 03.01

THE UNIVERSITY OF WESTERN AUSTRALIA
BUSINESS SCHOOL
ECONOMICS
CRAWLEY, WESTERN AUSTRALIA 6009
The Cardinal Attributes of Utility in Pareto’s “Sunto”
– A Critical Note on Some Recent Research
by
Michael McLure*

1) Introduction
The period since the centenary celebration of Pareto’s 1896-97 *Cours d’Économie Politique* has, among other things, been associated with a renewed interest in the originality and importance of Pareto’s “Sunto di alcuni capitoli di un nuovo trattato di economia pura” (Pareto [1900] 1982). Important in this regard are Marchionatti and Gambino (1997), Weber (2001) and Bruni (2002), who present the “Sunto” as the first study to formally represent a system of ordinal utility¹ within a model of economic equilibrium.² Even though these three studies reached vastly different (and often inconsistent) findings, they all acknowledged the importance of the “Sunto” for its clear specification of an ordinal system of utility.

This interest in the “Sunto” is important because the pioneering 20th Century studies of ordinalism in utility theory, which also tended to comment on whether Pareto was an ordinalist, a cardinalist, inconsistent or confused, referred to either the 1909 *Manuel d’Économie Politique* or the 1911 encyclopaedia entry entitled “Économie Mathématique”, which include both ordinal and cardinal aspects to utility systems.³ ‘Recognition’ of the purely ordinal specification of utility in the “Sunto” has necessitated a re-assessment of Pareto’s approach to utility theory.

The purpose of this critical note is to contest the recent interpretation of the *choice-preference* approach of the “Sunto” by drawing attention to its cardinal restrictions and by highlighting the continuity in Pareto approach to utility theory between the “Sunto”, the 1906

* Research Fellow, University of Western Australia, Department of Economics (although preliminary research into this topic commenced when the author was a Fellow of the Institute for Research into International Competitiveness at Curtin University of Technology). While retaining responsibility for any errors in this paper, the author would like to thank Michael McAleer for his comments.

1 In deference to convention, the term ‘utility’ is used here instead of Pareto’s neologism ‘ophelimity’.
2 The ‘progression towards’ a generalisation of choice theory based on ordinal measures utility is also noted in McLure (2001) but is not considered in this note because of the qualification that “the issue of ordinalism in choice, welfare and value theory *per se* is beyond the scope of this analysis” (McLure 2001, p196).
3 Major studies include Hicks and Allen (1934), Lange (1934), Hicks (1939), Schumpeter (1949) and Samuelson (1974). Even John Chipman’s impressive “The Paretian Heritage” (Chipman [1976] 1999) considers the cardinal and ordinal attributes of Paretian utility theory in reference to the *Manuale* and the *Manuel*, although importantly it also notes the ‘beautiful exposition’ of the “Sunto” on dispensing with the measurement of utility.
Manuale di Economic Politica, and beyond. The paper commences with a review of the most recent literature in Section 2, highlighting both analytical and methodological issues. Some analytical properties of Pareto’s choice-preference approach to the utility system are considered in Section 3. Emphasis is placed on identifying the cardinal attributes of the “Sunto”. In addition, the question of whether Pareto knowingly or unknowingly imposed cardinal restrictions on utility is considered in light of: the numeric example presented in the “Sunto”; and the characteristics of the indices of indifference outlined in the Manuale. The methodological context of Pareto’s choice-preference and utility-as-hypothesis (purely cardinal) specifications of equilibrium theory is considered in Section 4, with the respective roles of induction, deduction and pleasure examined for each specification. In light of these findings, the most recent literature on Pareto’s utility theory is critiqued in Section 5. It is suggested that some misleading inferences have been made as a consequence of not recognising: the cardinal attributes of the indifference curve analytics in the “Sunto”; and/or the methodological context within which the choice-preference approach to pure theory was developed. It is concluded in Section 6 that the limits of ordinalism under Pareto’s choice-preference approach is determined by experimental constraints, and that cardinal conditions continued to apply under that approach when attributes of the utility system (related to movement between indifference curves) were considered on a purely deductive basis.

2) The Recent Research Literature
As noted in the introduction, three significant and recent studies have represented the “Sunto” as a clear and consistent ordinal specification of economic theory. Marchionatti and Gambino (1997) demonstrate that the choice-preference approach of the “Sunto” was intended to establish the experimental basis of economics as a science; Weber (2001) finds that, in contrast to the “Sunto”, Pareto’s subsequent works were inconsistent with cardinal restriction imposed on indifference curves established from ordinal preferences; and Bruni (2002) advances the view that the subtlety and complexity of Pareto philosophy of science accommodated both ordinal and cardinal utility systems, but with a lower epistemological status assigned to cardinal systems.

Instead of focusing on the development of Pareto’s pure economic theory as a transition from cardinalism to ordinalism, Roberto Marchionatti and Enrico Gambino, in “Pareto and Political Economy as a Science” (Marchionatti and Gambino 1997), place emphasis on the 1890s as a period of analytical advances and methodological revolution. This period is presented as a time in which Pareto developed a pure economics that is firmly grounded in the experimental method. Even though the choice-preference approach outlined
in the “Sunto” is interpreted as a fully developed ordinal system, Marchionatti and Gambino contend that this as a consequence of Pareto’s conscious adoption of experimental methods to remove extra-economic assumptions from the interpretation of economic facts, and not the logical result of difficulties in measuring utility. In this context, the choice-preference approach is primarily concerned with defining the methodological foundations of economics as a ‘science’⁴, and Pareto’s ordinalist analytics are an advance which facilitated the scientific treatment of economics.

In contrast, Christian E Weber’s “Pareto and the 53 Percent Ordinal Theory of Utility” (Weber 2001) is largely confined to the cardinalist-ordinalist dimension of Pareto’s theoretical analytics, with the methodological context generally ignored. Weber finds that, in stark contrast to the clarity and consistency of the ordinal specification of utility in the “Sunto”, Pareto’s 1909 Manuel d’Économie Politique and 1911 “Économie Mathématique” emphasise and defend unnecessary cardinal restrictions on utility functions while arguing for an ordinal theory of utility. Specifically, he is critical of Pareto’s specification of indifference curves on the basis of preferences (ordinal utility) while concurrently: constraining utility by requiring that second-order own partial derivatives be negative (a cardinal property); and considering the complementarity or substitutability of goods in terms of the sign of second-order cross partial derivatives. Weber also suggests that inconsistencies are evident in the context of both static equilibrium and comparative statics (used to derive the principles of demand), leading him to suggest that “it is easy to suspect that he [Pareto] never realised just what he had and had not achieved.” (Weber 2001, p557).

In Vilfredo Pareto and the Birth of Modern Microeconomics, Luigino Bruni (2002) focuses on the methodological context of Pareto’s utility theory. However, unlike Marchionatti and Gambino (who also emphasise the importance of methodology), Bruni considers developments in Pareto’s utility theory well beyond 1900. He concludes that specification of indifference curves on either a cardinal or ordinal basis are admissible within Pareto’s system, although cardinal specifications have a lower epistemological value. In support of this, he notes that within Pareto’s methodology ‘the empirical’ determines the admissibility of concepts to ‘the theoretical’, and that cardinal utility was retained as an hypothesis with some empirical support.⁵ In this context, Pareto’s concern with utility is not

---

⁴ The methodological context of Pareto’s utility systems are discussed in section 4.

⁵ Bruni’s argues that ‘the empirical’ determines the admissibility of concepts to ‘the theoretical’ within Pareto’s methodology, and that cardinal utility was retained as an hypothesis with some empirical support. As this view is very similar to the view put in Chipman ([1976] 1999), it is difficult to understand why Bruni notes that neither
presented as a fundamental concern over its existence, but a concern over the uncertainty associated with its measurability. This may explain Pareto’s views on the equivalence of equilibrium equations under both the cardinal and ordinal approaches to utility. Bruni also suggests that the synthetic aspect to successive approximations, which Pareto championed when attention is shifted from the abstract analysis to the concrete reality, implied cardinal-type considerations.

3) Some Analytical Properties of the Utility System outlined in the “Sunto”

Pareto’s analysis of movement between points on indifference curves in the “Sunto” (Pareto [1900] 1982, pp.410-412) is undertaken on the basis that index numbers for the system of indifference curves are continuous, and from this assumption, two properties of the index function \( I(x, y) \) are derived. The first property, a consequence of the requirement that first-order partial derivatives \( \frac{\partial I}{\partial x} \) and \( \frac{\partial I}{\partial y} \) must be both positive, is that the positive index numbers for each indifference curve increase when there is an increase in the quantity of \( x \) (when \( y \) is unchanged) or \( y \) (when \( x \) is unchanged). The second property, a consequence of the condition that second-order own partial derivatives \( I_{xx}(x, y) \) and \( I_{yy}(x, y) \) must be negative, is that the rate of increase in index numbers assigned to indifference curves diminishes for each additional unit of \( x \) or \( y \). This diminishing marginal index numbers requirement is analogous to diminishing marginal utility, both of which require the utility system to demonstrate cardinal characteristics.

For the purposes of this paper, it is important to note that Pareto recognised that his utility system implied cardinal measurement. In the section of the “Sunto” entitled “Soluzione Analytica del Problema”, a numeric example is provided. Specifically, a hypothetical equation defining the system of indifference curves (formula 49)\(^6\) was used to calculate a series of index numbers, \( \zeta \), for the system of indifference curves (Pareto [1900] 1982, pp.419-423). The index numbers used in the example were derived to 4 decimal places, and presented as analogous to page numbers in a book or volume.

---

“…Chipman nor [others]… explain how and above all why Pareto’s position could differ from the one customarily attributed to him” (Bruni 2002, p.128).

\(^6\) Formula (49) is: \( x - \zeta \hat{y}(y - 2 \zeta) = 0.1 \) where \( \zeta \), representing ordered preferences \( \zeta_1, \zeta_2, \zeta_3, \ldots \), defines the parameters of the function \( I(x, y) \). Index numbers were calculated at the point of tangency between formula 49 and the equation that defines the curve of obstacles (i.e. a budget constraint where 2.1 of \( x \) is exchanged for 1 of \( y \)) as an individual’s original endowment (specified entirely in \( x \)) is varied by small amounts. In practice, the indifference curve formula would be estimated through observation.
“The numbers indicated at the top of a page are very useful in themselves. They form a system of indices and they permit us to judge at first glance which of two combinations is preferred. Once on this path it is possible for us to do much more than summarise the facts. All the information indicated in our volume is summarised by the formula (49) where, based on a convenient interpretation, \( \zeta \) can represent the page numbers. When calculating \( \zeta \) we stop at the 4th decimal place assuming that in total \( \zeta \) represents tens of thousands of numbers and that these become the page numbers….To find out which page of the volume contains a certain combination of \( x, y \), we calculate, with equation 49, the value of \( \zeta \) which corresponds to this combination.” (Pareto [1900] 1982, p423, italics added)

Pareto clearly appreciated that, across different systems of indices (e.g. \( \zeta, \beta, \psi \ldots \)), index numbers would vary for the same indifference curve, but nevertheless index numbers within each system would retain their cardinal, or proportionately cardinal, aspect. In the context of Pareto’s book analogy, the scope of index numbers represented by the parameter \( \zeta \) may approximate a sequence that corresponds to every page of a book, whereas the index numbers of \( \beta \) may represent an ordering for every 10 pages and \( \psi \) may represent an ordering of index numbers determined with reference to every 50 pages of the book. The arbitrary function of these parameters would not necessarily be identical to a function of total utility, if it exists, but, it would be proportional to a function of total utility. This interpretation is consistent with the Mathematical appendix to the Italian edition of the 1906 Manuale di Economia Politica, where paragraph 10 (“Properties of indifference curves”) considers the properties which establish the shape of indifference curves in reference to goods exchanged and without any reference to utility, and paragraph 11 (“Characteristics of the indices deduced from those lines of indifference”) deduces, from the information contained in paragraph 10, the first and second-order own partial derivative conditions reported in the “Sunto”.

Of course, even though Pareto was conscious of the cardinal attributes of his choice-preference approach to pure theory, it is still possible that he was inconsistent in his indifference curve analytics. To consider this question, it is necessary to reflect on why Pareto introduced the choice-preference approach to economics.

4) The Methodological Context of Pareto’s Utility Systems
The first paragraph of the “Sunto” sets out the methodological context of different theoretical systems:
“In the social sciences there is no deductive method in contradiction to the inductive method, there are only inductions and deductions that complement each other. Sometimes it is easier to deduce certain consequences from certain facts and then verify them; other times it is preferable to study these consequential facts and work backwards to establish their cause inductively; … It is of little importance whether we commence with deduction or induction as long as we manage to cement fact with idea, which constitutes the goal of any theory. (Pareto [1900] 1982, p368)

The purely cardinal utility-as-hypothesis approach to pure theory outlined in the 1896-97 Cours d’Économie Politique commences with deduction based on the premise that utility is an hypothesis whereas the choice-preference approach in the “Sunto” commences with induction to establish preferences on a purely ordinal basis. These inductive results are followed by deduction, where such deductions imply cardinal attributes to utility. While the importance of the “Sunto” for the development of ordinalist analytics is now appreciated, the “Sunto” is also important for recognising that the “… equations of pure economics and their consequences remain substantially unaltered whether one starts by considering pleasure as a quantity, or whether one limits oneself, as we do, to the fact of choice” (Pareto [1900] 1982, p. 373).

Deriving cardinal attributes from the shape of indifference curves established from purely ordinal ranking of data, as provided for under the choice-preference approach, was an important aspect of the equivalence of the two approaches. Notwithstanding this equivalence, Pareto comments in the “Sunto” that, if one was distracted, it would be possible to interpret the first-order partial derivative condition of the choice-preference approach as indicating that pleasure decreases with additional quantities of $x$ and $y$, and the second-order own partial derivative condition as indicating that there is diminishing marginal utility. However, Pareto’s concern with such a ‘distracted interpretation’ is that it reverses the path to that taken in the choice-preference approach (Pareto [1900] 1982, p.412), not that it implies cardinal attributes. In fact, Pareto implicitly recognises that these conditions of the choice-preference approach require a cardinal link to pleasure when he notes that, “for the rest” one can reason just as well adopting the system of indices and without having to admit that one can measure pleasure” (Pareto [1900] 1982, p.412-413, italics added). In short, Pareto’s objective in introducing the choice-preference approach was to provide a basis for accommodating ordinal data from induction as a first step to general theoretical modelling (in contrast to the utility-as-

---

7 That is, apart from the properties of the index function deduced from the shape if indifference curves.
hypothesis approach which proceeds straight to deduction from hypothetical premise). However, when deduction was introduced to extend and ultimately complete the general specification of economic theory, Pareto had no qualms about retaining cardinal properties of utility.

As Pareto presented the choice-preference and utility-as-hypothesis approaches as two quite distinct approaches, it is appropriate to consider when he regarded the choice-preference specification as best cementing ‘fact with ideas’ and when this would be achieved through the utility-as-hypothesis approach. It would appear that the choice-preference approach was mainly used to consider the conditions of static equilibrium. In the “Sunto” it is noted that:

“It must be observed that we do not need to know these [indifference] lines to their full extension, it is enough to know them close to the point where equilibrium is actually established. Within these limits it is possible to have an approximate notion of the lines of indifference and it is desirable that statisticians spend time on such work.” (Pareto [1900] 1982, p398)

When deductions pertaining to virtual movements were considered in commodity space, Pareto largely utilised the utility-as-hypothesis specification of economics. For example, the 1902 “Di un nuovo errore nello interpretare dell’economia matematica”, a classic study of the welfare implication of exchange, is undertaken without reference to index numbers, relying entirely on the utility-as-hypothesis specification (with the economic meaning of the abstract theoretical analysis cemented to fact by the numeraire good and the definition of its relationship to marginal utility). As a consequence, Chapter 3 of the 1906 Manuale and 1909 Manuel, entitled “General Notion of Economic Equilibrium”, treats the pure economics of equilibrium from a choice-preference perspective whereas Chapter 4, entitled “Tastes”, considers indifference curves and their relationship to utility and index numbers.

The difference between the “Sunto” and Chapter 4 of the Manuel (and Manuale) does not concern the introduction of cardinal restrictions on choice preference systems, but a reduction in the strict demarcation between the choice-preference and the utility-as-hypothesis approaches to pure economics. In short, the careful specification of the two approaches in the Mathematical appendix to the Manuale, where the choice-preference approach is defined without reference to utility and the utility-as-hypothesis approach is referenced to utility, is followed by a synthesis of the two approaches. In this regard, paragraph 13 of the appendix of the 1906 Manuale di Economia Politica (Characteristics of curves of indifference deduced from curves of ophelimity) commences with the assumption that utility is known such that $I =$
Φ, and with derivatives of the system of indexes \((\partial I/\partial x = \psi_x)\) under the choice-preference specification replaced by marginal utility \((\varphi)\). It goes on to express the second-order own partial derivative conditions, which were deduced in the “Sunto” and in paragraph 11 of the Manuale without explicit reference to utility or pleasure, but with explicit reference to utility \((\varphi_{xx}<0; \varphi_{yy}<0)\); an acknowledgement that these condition require decreasing marginal utility and that, as a consequence, index numbers are cardinaly referenced to pleasure.

The initial separate treatment of the choice-preference approach and the utility as hypothesis approach followed by a synthesis of the two approaches is clearest in the appendix of the 1906 Italian Manuale. The appendix to the 1909 Manuel, which was restructured by Pareto in response to Vito Volterra’s review of the Manuale, truncates the process and applies the synthesis without first carefully considering the choice-preference and utility-as-hypotheses approaches in isolation.

5) Critique of Recent Studies

Once the analytical and methodological rationales for the cardinal properties of Pareto’s system of indifference curves are recognised, clear deficiencies in the three recent studies emerge.

The merit of Marchionatti and Gambino (1997) and Bruni (2002) is that they illustrate the importance of Pareto’s methodology to his system of utility. Importantly, their different findings (with Marchionatti and Gambino suggesting that Pareto was an ‘ordinalist’, and Bruni finding that Pareto was an ‘ordinalist and cardinalist’) are only partly due to the different reference periods of the studies. The fundamental source of the inconsistency in their findings is that neither study gave due regard to the analytics of indifference curves under Pareto’s choice-preference approach. However, their findings are not inconsistent when their references to Pareto’s development of an ordinalist system are re-read as references to Pareto’s limited ordinalist system that is subject to cardinal constraints.

Weber (2001) carefully considered the analytics of the utility system, but only in the 1909 French Manuel and the “Économie Mathématique”. The deficiency of the study is that it did not consider either the cardinal analytics of the “Sunto”, the analytics and synthesis of the choice-preference and utility-as-hypothesis approaches in the mathematical appendix of the 1906 Manuale or the methodological context of Pareto’s developments in utility theory. As a consequence, Weber was unable to identify either the fundamental consistency, or the subtle differences, between the “Sunto” and later works.

---

8 Marchionatti and Gambino did not extend their investigation beyond 1900, whereas Bruni did, including discussion on the 1909 Manuel.
Weber’s specific concern is that the 1909 *Manuel* and “Économie Mathématique”: impose a second-order own partial derivative condition (a cardinal attribute equating to diminishing marginal utility); utilise second-order cross partial derivative conditions to test the complementarity and substitutability of goods (which is also predicated on cardinal measurement); and fail to successfully derive demand principles from the synthesis of the “purely ordinal approach to utility” of the “Sunto” and the comparative statics analysis from Part 5 of the 1893 “Considerazioni sui principii fondamentali …” (Pareto [1983](1982)). However, this critique cannot be sustained once the cardinal attributes of the choice-preference approach are acknowledged and the methodological context of changes to Pareto’s utility system is recognised.

The fundamental issue that Pareto considered was whether economists should commence theoretical deductions from the assumption that pleasure is a quantity or only after making inductive observations concerning preferences. If the latter is selected (for reasons of scientific rigour), then Pareto recognised that the utility system must accommodate variations in index numbers as the parameters of the index system are altered. However, as data are collected at or near equilibrium, he saw little advantage in a pure and general ordinalist system. Instead, the index numbers within each utility system were deduced from the shape of indifference curves (which can be established from observation of choices) and remained proportionate to a cardinal measure of pleasure.

Pareto’s many comments on the difficulty of measuring utility, not only in the “Sunto” but also in the *Manuale*, therefore concern: (i) the essentially insurmountable problems that consumers’ experience when making assessments based on pleasure as a quantity; (ii) the implications for experimental economics of these insurmountable problems (i.e. data can be collected on preferences, including the order of preference for consecutive acquisitions, but data cannot be collected for units of pleasure); and (iii) the appropriateness of founding pure economic analysis of equilibrium on the assumption that utility is a quantity in light of (i) and (ii). However, once induction has been used to establish a preference basis for pure theory (i.e. pure theory is specified in a manner that can accommodate index numbers for indifference curves), mathematical deductions can be contemplated even when this involves the identification of cardinal properties on the utility function; Pareto’s only consideration being whether the outcome managed to “cement fact with idea”.

11
6) Conclusion

The ordinalist attributes of Pareto’s choice-preference approach is generally limited to points of equilibrium or points near equilibrium for the purpose of establishing the shape of indifference curves over a relevant range at, or near, equilibrium. When laws or regularities associated with movement between different indifferent curves were considered, Pareto relied on mathematical deductions from the shape of indifference curves to assign index numbers that were proportionate to utility as a quantity.

In arriving at this conclusion, it was necessary to consider Pareto’s indifference curve analytics and his methodological objectives. The former reveals the cardinal attributes of utility systems developed under Pareto’s choice-preference approach and the latter establishes that the scope of ordinalism under Pareto’s choice-preference approach is knowingly limited by experimental constraints.

The findings of Marchionatti and Gambino (1979) and Bruni (2002) were incomplete because they did not give adequate consideration to the analytics of Pareto’s utility systems. The finding of Weber (2001) was incomplete and misleading because he did not consider the methodological context of the choice-preference approach, which established the goal of Pareto’s work on utility theory, and did not identify or consider the cardinal attributes of the choice-preference approach to pure economics specified in the “Sunto”.

References


