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ADVERTISING ECONOMICS

by

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* I would like to thank Professor Kenneth W. Clements for his valuable comments.
A NEW WAY OF MEASURING THE EFFECTS OF ADVERTISING ON CONSUMPTION

by

E. Antony Selvanathan*
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Abstract: This paper extends Theil's (1967) methodology of Divisia moments to analyse the effects of advertising on consumption. The new extensions are applied to U.K. alcohol data.

* I would like to thank Professor Kenneth W. Clements for his valuable comments and Ms. Shirley-Ann Poulton for her assistance.
1. Divisia Indexes

We consider a group of commodities, to be denoted by \( g \), consisting of \( n_g \) goods. The Divisia volume and price indexes are

\[
\begin{align*}
DQ_{gt} &= \sum_{i=1}^{n_g} \bar{w}_{it}q_{it}, \\
DP_{gt} &= \sum_{i=1}^{n_g} \bar{w}_{it}p_{it}
\end{align*}
\]

and the Divisia variances and covariance are

\[
\begin{align*}
K_{gt} &= \sum_{i=1}^{n_g} \bar{w}'_{it}(Dq_{it} - DQ_{gt})^2, \\
\Pi_{gt} &= \sum_{i=1}^{n_g} \bar{w}'_{it}(DP_{it} - DP_{gt})^2, \\
\Gamma_{gt} &= \sum_{i=1}^{n_g} \bar{w}'_{it}(DP_{it} - DP_{gt})(Dq_{it} - DQ_{gt})
\end{align*}
\]

where \( \bar{w}_{it} \) is the arithmetic average of the conditional budget share of commodity \( i \); \( q_{it} \) is the quantity consumed of \( i \) in year \( t \); \( p_{it} \) is the price of \( i \); and \( Dx_{it} = \log x_{it} - \log x_{i,t-1} \) is the log-change in \( x \).

The Divisia price-quantity correlation is

\[
\rho_{gt} = \frac{\Gamma_{gt}}{\sqrt{K_{gt}}}.
\]

The above measures are simple summary measures of prices and quantities.

The Divisia indexes defined by (1.1) for U.K. alcoholic beverages (consisting of beer, wine and spirits, so that \( n_g = 3 \)) are given in columns 2 and 3 of Table 1. The data are from Selvanathan (1987a) and
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McGuinness (1980, App. 2). On average, per capita alcohol consumption increased by about 3.3 percent per annum while alcohol prices rose by 5.1 percent. The corresponding variances are presented in columns 5 and 6, while the price-quantity correlation is given in column 8. The correlation is negative in 16 cases out of 20 with an average of -.4. This reflects the tendency of the drinker to move away from those beverages having above-average price increases.

2. Extensions for Advertising

Let \( a_{1t}, a_{2t} \) and \( a_{3t} \) be the volume of advertising of beer, wine and spirits in year \( t \), and \( D_{1t}, D_{2t} \) and \( D_{3t} \) be the corresponding log-changes. We define the Divisia volume index of advertising of alcohol as

\[
DA_{gt} = \sum_{i=1}^{3} \bar{w}_{it} D_{it}.
\]

This is a weighted first-order moment of \( D_{1t}, D_{2t} \) and \( D_{3t} \) which measures the change in the advertising of alcohol as a whole. The corresponding second-order moment is the Divisia advertising variance,

\[
a_{gt} = \sum_{i=1}^{3} \bar{w}_{it} (D_{it} - DA_{gt})^2.
\]

This measures the extent to which advertising on the three beverages changes disproportionately. Columns 4 and 7 of Table 1 present these two measures. On average, per capita advertising of alcohol increases by about 6.1 percent per annum. Looking at column 7 we see that in some years there is a great deal of variation in advertising among the three beverages.
The Advertising Elasticities of Demand

We use $\mu = (0.45 + 0.36)/2 = 0.405$, the average of the two mean values of $\mu$ in Table 1, and (3.1) to compute the advertising elasticities (at sample means). The own-advertising elasticities of beer, wine and spirits are 0.12, 0.24 and 0.20, respectively.

REFERENCES


A SIMPLE METHOD FOR ESTIMATING ADVERTISING ELASTICITIES

by

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Abstract: This paper presents a simple method to obtain estimates of advertising elasticities of demand. The method is applied to U.K. alcohol data.

* I would like to thank Professor Kenneth W. Clements for his valuable comments.
1. Introduction

We consider a group of commodities, to be denoted by $g$, consisting of $n_g$ goods. Consider the conditional demand equation for commodity $i$ of the form

\begin{equation}
(1.1) \quad \overline{w}_{it} (Dq_{it} - DQ_{gt}) = \beta_i^D DQ_{gt} + \sum_{j=1}^{n_g} \pi_{ij}[Dp_{jt} - \mu Da_{jt}],
\end{equation}

where $\overline{w}_{it}$ is the arithmetic average of the conditional budget share of $i$ over the years $t-1$ and $t$; $Dx_{it} = \log x_{it} - \log x_{i,t-1}$ is the log-change in $x$; $q_{it}$ is the quantity consumed of $i$; $DQ_{gt} = \sum_{i=1}^{n_g} \overline{w}_{it} Dq_{it}$; $\beta_i^D$ is a constant such that $\sum_{i=1}^{n_g} \beta_i^D = 0$; $\pi_{ij}$ is the $(i,j)$th Slutsky coefficient satisfying $\sum_{j=1}^{n_g} \pi_{ij} = 0$ and $\pi_{ij}' = \pi_{ji}'$ for $i,j=1,\ldots,n_g$; $p_{it}$ is the price of $i$; $\mu$ is the elasticity of the marginal utility of consumption of $i$ with respect to the advertising of $i$, which is the same for all $i$; and $a_{it}$ is the advertising of $i$. For further details of (1.1), see Selvanathan (1987).

By dividing both sides of (1.1) by $\overline{w}_{it}$, we obtain the conditional price and advertising elasticities as $\eta_{ij}' = \pi_{ij}' / \overline{w}_{it}'$ and $\tau_{ij}' = -\mu \pi_{ij}' / \overline{w}_{it}'$. This shows that the advertising elasticities of consumption are a constant multiple $-\mu$ of the corresponding price elasticities. As we expect $\mu$ to be positive and $\eta_{ii}'$ to be negative, the own advertising elasticities are positive. If $i$ and $j$ are net substitutes so that $\eta_{ij}' > 0$, then $\tau_{ij}' < 0$; and for complements $\tau_{ij}' > 0$. These results make sense since it is reasonable to expect that advertising will depress the sales of products which are substitutes for the good in question; and vice versa for complements.
The parameter \( \mu \) obviously plays an important role in determining the values of the advertising elasticities. In what follows we present a simple procedure for estimating \( \mu \).

2. The Procedure

We define

\[
Dq_{it}^* = Dq_{it} - DQ_{gt} - \frac{\beta_i}{\overline{w}_{it}} DQ_{gt} - \frac{n_i \overline{w}_{ij}}{\sum_{j=1}^{n} \overline{w}_{it}} Dp_{jt}
\]

as the change in \( q_i \) adjusted for income and price changes. Using equation (2.1), we can write equation (1.1) as

\[
\overline{w}_{it} Dq_{it}^* = -\mu \sum_{j=1}^{n} \overline{w}_{ij} (Dq_{jt} - DQ_{gt}),
\]

where we have used \( \sum_{j=1}^{n} \overline{w}_{ij} \overline{w}_{ij} = 0 \) and \( DQ_{gt} = \sum_{i=1}^{n} \overline{w}_{it} Dq_{it} \) is the Divisia index of advertising.

Multiplying both sides of equation (2.2) by \( (Dq_{jt} - DQ_{gt}) \) and summing over \( i = 1, \ldots, n \), we obtain

\[
\sum_{i=1}^{n} \overline{w}_{it} Dq_{it}^*(Dq_{jt} - DQ_{gt}) = -\mu \sum_{i=1}^{n} \sum_{j=1}^{n} (Dq_{jt} - DQ_{gt}) \overline{w}_{ij} (Dq_{jt} - DQ_{gt}).
\]

Denoting the left-hand side of (2.3) by \( B_{gt}^* \) and \( (Dq_{jt} - DQ_{gt}) \) by \( x_{it} \), we rearrange equation (2.3) in the form

\[
\mu = -\frac{B_{gt}^*}{x_{it}^* w_{it}^*},
\]

where \( x_{it} = [x_{it}] \) and \( w = [w_{ij}] \). Thus using (2.4) we can obtain an estimate of \( \mu \) for each period.
3. Results

We apply (2.4) to the U.K. alcohol data (made up of beer, wine and spirits, so that $n = 3$) from Selvanathan (1987). The right-hand side of (2.4) depends on the unknown parameters $\beta_i$ and $\pi_{ij}$. To evaluate these we use two approaches.

The First Approach

We commence by eliminating the unknown parameters by assuming (i) unitary conditional income elasticities and (ii) preference independence. This means that (see Clements, 1987)

\[(3.1) \quad \beta_i' = 0, \quad \pi_{ij}' = \phi \bar{w}_{it}(\delta_{ij} - \bar{w}_{jt}).\]

where $\phi$ is the income flexibility and $\delta_{ij}$ is the Kronecker delta.

We obtain estimates of $\mu$ from (2.4) under (3.1) with $\phi = -.7$ (a value consistent with previous findings; see Selvanathan, 1987). These estimates are presented in row 1 of Table 1. The average value of $\mu$ over the 20 year period is .45. This means that under the stated assumptions, the advertising elasticity of the marginal utility of each beverage is .45. This estimate of $\mu$ is very close to the econometric estimate of $\mu$ of .52 presented in Selvanathan (1987).

The Second Approach

We now abandon assumption (3.1) and evaluate $\mu$ by using econometric estimates of the unknown parameters $\phi$, $\beta_i'$ and $\pi_{ij}'$ from Selvanathan (1987, Table 5.2). These results are presented in row 2 of Table 1. As can be seen, the average value of $\mu$ over the twenty-year period is .36 which is not too different from the .45 value obtained before.
We define the Divisia quantity-advertising covariance as

\[ B_{gt} = \sum_{i=1}^{3} w_{i}^{t}(Da_{it} - DQ_{gt})(Dq_{it} - DA_{gt}). \]

This measures the joint movement of alcohol consumption and advertising. Thus, \( B_{gt} \) is positive when, on average, those beverages with above-average growth rates experience greater than average increases in advertising, and vice versa. The corresponding Divisia correlation coefficient is

\[ V_{gt} = \frac{B_{gt}}{\sqrt{K_{gt} \alpha_{gt}}}. \]

Column 9 of Table 1 gives \( V_{gt} \). As can be seen, there is a great deal of variation in this coefficient and it has an average .2. In 13 out of the 20 cases \( V_{gt} \) is positive.

Finally, we define the Divisia price-advertising covariance and the corresponding correlation as

\[ C_{gt} = \sum_{i=1}^{3} w_{i}^{t}(Dp_{it} - DP_{gt})(Da_{it} - DA_{gt}), \quad R_{gt} = \frac{C_{gt}}{\sqrt{\gamma_{gt} \alpha_{gt}}}. \]

This \( R_{gt} \) measures the extent to which those beverages which experience large increases in advertising also have above-average price increases. Column 10 of Table 1 gives \( R_{gt} \). The values of this correlation also fluctuate a great deal with an average of .05.
3. Corrected Quantity-Advertising Measures

The quantity-advertising covariance (2.1) relates the total change in consumption of the three beverages to advertising. As consumption depends on income, prices and advertising, included in this covariance are the effects of income and price changes, as well as any effects of advertising. In other words, the covariance could be positive for reasons other than advertising. In this section we correct (2.1) and (2.2) by removing the effects of income and relative prices.

Following Selvanathan (1987b), we express the demand equation for beverage $i$ as

$$DQ_{it} - DQ_{gt} = \sum_{j=1}^{3} \beta_{ij} DQ_{jt} - \sum_{j=1}^{3} \pi_{ij} Dp_{jt},$$

where $\sum_{i=1}^{3} \beta_{i} = 0$, $\sum_{j=1}^{3} \pi_{ij} = 0$ and $\sum_{j=1}^{3} \lambda_{ij} = 0$; and $\pi'_{ij} = \pi_{ij}$, $i,j = 1,2,3$. The term in brackets in this equation is the log-change in the consumption of $i$ after allowing for the effects of income and prices. We denote this term by

$$(3.1) \quad Dq^{*}_{it} = Dq_{it} - DQ_{gt} - \sum_{j=1}^{3} \frac{\beta_{ij}}{w_{it}} DQ_{jt} - \sum_{j=1}^{3} \frac{\pi_{ij}}{w_{it}} Dp_{jt},$$

For short, we shall refer to $Dq^{*}_{it}$ as the corrected quantity change of $i$.

Using $\sum_{i=1}^{3} \frac{\beta_{i}}{w_{it}} = 1$, $\sum_{i=1}^{3} \beta_{i} = 0$ and $\sum_{i=1}^{3} \pi_{ij} = 0$, it follows that the Divisia mean of the corrected quantity changes is zero,

$$(3.2) \quad DQ^{*}_{gt} = \sum_{i=1}^{3} \frac{3}{w_{it}} Dq^{*}_{it} = 0.$$
In view of (3.2), the Divisia covariance between the corrected quantity changes and advertising is

\[ 3 \sum_{i=1}^{\infty} \bar{w}_{it} Dq_{it}^*(Da_{it} - DA_{it}) = B_{gt}^\times \text{(say)}. \]

We shall call \( B_{gt}^\times \) the corrected quantity-advertising covariance. The corrected quantity variance is

\[ k_{gt}^\times = 3 \sum_{i=1}^{\infty} \bar{w}_{it} \left[Dq_{it}^\times\right]^2. \]

Finally, the corrected quantity-advertising correlation is

\[ \nu_{gt}^\times = \frac{B_{gt}^\times}{\sqrt{k_{gt}^\times} \alpha_{gt}}. \]

This is the corrected version of (2.2).

4. **Estimates of the Corrected Quantity-Advertising Correlation**

The measures (3.3) and (3.4) depend on the unknown parameters \( \beta_i^t \) and \( \pi_{ij}^t \). To evaluate these measures we use two approaches.

**The First Approach**

We commence by eliminating the unknown parameters by assuming (1) unitary conditional income elasticities and (ii) preference
independence. This means that (see Clements, 1987)

\[(4.1) \quad \beta'_i = 0, \quad \tau'_{ij} = \phi\bar{w}'_{it}(\delta_{ij} - \bar{w}'_{jt}),\]

where \(\phi\) is the income flexibility and \(\delta_{ij}\) is the Kronecker delta.

Under (4.1), the corrected quantity change of \(i\) defined by (3.1) becomes

\[
Dq^*_it = (Dq_{it} - Dq_{gt}) - \phi(Dp_{it} - Dp_{gt}),
\]

where \(DP_{gt} = \sum_{i=1}^{3} \bar{w}'_{it}Dp_{it}\). The corrected quantity-advertising covariance (3.3) then becomes

\[(4.2) \quad B^*_gt = B_{gt} - \phi C_{gt},\]

where we have used (2.1) and (2.3). Thus the corrected quantity-advertising covariance is the difference between the uncorrected version and \(\phi\) times the Divisia price-advertising covariance. As \(\phi < 0\) we can write \(B^*_gt - B_{gt} = |\phi|C_{gt}\). Therefore \(B^*_gt > B_{gt}\) when the relative prices of those beverages with high advertising intensities increase. This is simply because these increases in relative prices cause consumption of these beverages to grow less rapidly; and this slower growth is included in the uncorrected covariance \(B_{gt}\), making it smaller.

Under (4.1) the corrected quantity variance (3.4) becomes

\[(4.3) \quad K^*_gt = K_{gt} + \phi^2\Pi^*_{gt} - 2\phi \Gamma_{gt},\]

where \(K_{gt}\) and \(\Pi_{gt}\) are the Divisia quantity and price variances; and \(\Gamma_{gt}\)
is the Divisia price-quantity covariance. We use (4.2) and (4.3) with \( \phi = -0.7 \) (a value consistent with previous estimates; see Selvanathan, 1987a) to evaluate (3.5).

Column 11 of Table 1 gives the corrected quantity-advertising correlation. As before, in 13 out of 20 cases the correlation is positive and its average value is 0.21, which is close to the average of \( \bar{V}_g \) given in column 9 of the same table. The reason why the means of \( \bar{V}_g \) and \( \bar{V}^* \) are approximately equal is that the mean of the price-advertising covariance \( C_{\bar{g}} \) is quite small and the mean of quantity variance \( K_g \) is reasonably large; see equations (4.2) and (4.3). However, a comparison of the individual entries shows that in some years the difference between \( \bar{V}_g \) and \( \bar{V}^* \) is very large; and in two cases they even have the opposite sign.

**The Second Approach**

We now abandon assumption (4.1) and evaluate \( \bar{V}_{gt} \) by using econometric estimates of \( \beta_i^* \) and \( \pi_{ij}^* \) from Selvanathan (1987a, Table 5.2). The results are presented in column 12 of Table 1. Now in 16 cases the correlation is positive with an average of 0.34, which is about 60 percent larger than before.
REFERENCES


